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Business Mathematics & Statistics

MTH302

Course Handouts

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Business Mathematics & Statistics

MTH 302



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MTH 302
LECTURE 1
COURSE OVERVIEW

COURSE TITLE
The title of this course is “BUSINESS MATHEMATICS AND STATISTICS”.

Instructor’s Resume
The instructor of the course is Dr. Zahir Fikri who holds a Ph.D. in Electric Power Systems Engineering from the Royal Institute of Technology, Stockholm, Sweden. The title of Dr. Fikri’s thesis was “Statistical Load Forecasting for Distribution Network Planning”.

Objective
The purpose of the course is to provide the student with a mathematical basis for personal and business financial decisions through eight instructional modules.
The course stresses business applications using **arithmetic**, **algebra**, **and ratio-proportion** and **graphing**.
Applications include **payroll**, **cost-volume-profit analysis** and **merchandising mathematics**.
The course also includes **Statistical Representation of Data, Correlation, Time Series and Exponential Smoothing, Elementary Probability and Probability Distributions.**
This course stresses **logical reasoning** and **problem solving skills**.

Access to **Microsoft Excel** software is required for the course.

Course Outcomes
Successful completion of this course will enable the student to:
1. Apply arithmetic and algebraic skills to everyday business problems.
2. Use ratio, proportion and percent in the solution of business problems.
3. Solve business problems involving commercial discount, markup and markdown.
4. Solve systems of linear equations graphically and algebraically and apply to cost volume-profit analysis.
5. Apply Statistical Representation of Data, Correlation, Time Series and Exponential Smoothing methods in business decision making
6. Use elementary probability theory and knowledge about probability distributions in developing profitable business strategies.

Unit Outcomes Resources/Tests/Assignments
Successful completion of the following units will enable the student to apply mathematical methods to business problems solving.
Required Student Resources (Including textbooks and workbooks)
Text: Selected books on Business Mathematics and Statistics.
Optional Resources
Handouts supplied by the professor.
Instructor’s Slides Online or CD based learning materials.
Prerequisites
The students are not required to have any mathematical skills. Basic knowledge of Microsoft Excel will be an advantage but not a requirement.
Evaluation
In order to successfully complete this course, the student is required to meet the following evaluation criteria:
Full participation is expected for this course
All assignments must be completed by the closing date.
Overall grade will be based on VU existing Grading Rules.
All requirements must be met in order to pass the course.
COURSE MODULES
The following are the main modules of this course:
Module 1
- Overview (Lecture 1)
- Perform arithmetic operations in their proper order (Lecture 2)

- Convert fractions their percent and decimal equivalents. (Lecture 2)
- Solve for any one of percent, portion or base, given the other two quantities. (Lecture 2)
- Using Microsoft Excel (Lecture 2)
  Calculate the gross earnings of employees paid a salary, an hourly wage or commissions. (Lecture 3)
- Calculate the simple average or weighted average given a set of values. (Lecture 4)
  Perform basic calculations of the percentages, averages, commission, brokerage and discount (Lecture 5)
- Simple and compound interest (Lecture 6)
- Average due date, interest on drawings and calendar (Lecture 6)

Module 2

- Exponents and radicals (Lecture 7)
- Solve linear equations in one variable (Lecture 7)
- Rearrange formulas to solve for any of its contained variables (Lecture 7)
- Solve problems involving a series of compounding percent changes (Lecture 8)
- Calculate returns from investments (Lecture 8)
- Calculate a single percent change equivalent to a series of percent changes (Lecture 8)
- Matrices (Lecture 9)
- Ratios and Proportions (Lecture10)
- Set up and manipulate ratios (Lecture11)
- Allocate an amount on a prorata basis using proportions (Lecture11)
- Assignment Module 1-2

Module 3

- Discounts (Lectures 12)
- Mathematics of Merchandising (Lectures 13-16)

Module 4

- Applications of Linear Equations (Lecture 17-18)
- Break-even Analysis (Lecture 19-22)
- Assignment Module 3-4
- Mid-Term Examination

Module 5

- Statistical data (Lectures 23)
- Measures of central tendency (Lectures 24-25)
- Measures of dispersion and skewness (Lectures 26-27)

Module 6

- Correlation (Lectures 28-29)
- Line Fitting (Lectures 30-31)
- Time Series and Exponential Smoothing (Lectures 31-33)
- Assignment Module 5-6

Module 7

- Factorials (Lecture 34)
- Permutations and Combinations (Lecture 34)
- Elementary Probability (Lectures 35-36)
- Patterns of probability: Binomial, Poisson and Normal Distributions (Lecture 37-40)

Module 8

- Estimating from Samples: Inference (Lectures 41-42)
- Hypothesis testing : Chi-Square Distribution (Lectures 43-44)
- Planning Production Levels: Linear Programming (Lecture 45)
- Assignment Module 7-8
- End-Term Examination

Note: The course modules are subject to change.

MARKING SCHEME

As per VU Rules

DESCRIPTION OF TOPICS

NO.	MAIN TOPIC	LECTURE	TOPICS	RECOMMENDED READING
Module 1	1.0 Applications of Basic Mathematics (Lectures 1-6)	1	• Overviewew (Lecture 1)	Reference 1
Module 1		2	• Course Overview • Arithmetic Operations & • Using Microsoft Excel	Reference 2, Lecture 2 Tool: Microsoft Excel
Module 1		3	• Calculate Gross Earnings • Using Microsoft Excel	Reference 2, Lecture 3 Tool: Microsoft Excel
Module 1		4	• Calculating simple or weighted averages • Using Microsoft Excel	Reference 2, Lecture 4 Tool: Microsoft Excel Reference 6
Module 1		5	• Basic calculations of percentages, averages, commission, brokerage and discount using • Microsoft Excel	Reference 2, Lecture 5 Reference 3, Ch 3 Tool: Microsoft Excel
Module 1		6	• Simple and compound interest • Average due date, interest on drawings and calendar	Reference 2, Lecture 6 Reference 3, Ch 3 Tool: Microsoft Excel
Module 2	2.0 Applications of Basic Algebra (Lectures 7-9)	7	• Exponents and radicals • Simplify algebraic expressions • Solve linear equations in one variable • Rearrange formulas to solve for any of its contained variables	Reference 2, Lecture 7 Reference 3, Ch 2 Tool: Microsoft Excel
		8	• Calculate returns from investments • Problems involving a series of compounding percent changes • Single percent change equivalent to a series of percent changes	Reference 2, Lecture 8 Reference 3, Ch 3 Tool: Microsoft Excel

		9	• Matrices	**Reference 2, Lecture 9 Reference 3, Ch 4 Tool: Microsoft Excel**
Module 2	**3.0 Applications of Ratio and Proportion (Lectures 10-11)**	10	• **Set up and manipulate ratios.** • **Set up and solve proportions.** • **Express percent differences using proportions.** • **Allocate an amount on a prorata basis using proportions.**	**Reference 2, Lecture 10 Reference 3, Ch 3 Tool: Microsoft Excel**
Module 2		11	• **Set up and manipulate ratios.** • **Allocate an amount on a prorata basis using proportions**	**Reference 2, Lecture 11 Reference 3, Ch 3 Tool: Microsoft Excel**
Module 3	**4.0 Merchandising and Financial Mathematics (Lectures 12-16)**	12	• **Calculate the net price of an item after single or multiple trade discounts.** • **Calculate an equivalent single discount rate given a series of discounts.**	**Reference 2, Lecture 12 Reference 3, Ch 3 Tool: Microsoft Excel**
Module 3		13	**Solve merchandising pricing problems involving markup and markdown.**	**Reference 2, Lecture 13 Reference 3, Ch 3 Tool: Microsoft Excel**
Module 3		14	**Financial Mathematics Part 1**	**Reference 2, Lecture 14 Reference 3, Ch 3 Reference 5, Ch 16 Tool: Microsoft Excel**
Module 3		15	**Financial Mathematics Part 2**	**Reference 2, Lecture 15 Reference 3, Ch 3 Reference 5, Ch 16 Tool: Microsoft Excel**
Module 3		16	**Financial Mathematics Part 3**	**Reference 2, Lecture 16 Reference 3, Ch 3 Reference 5, Ch**

				16 **Tool: Microsoft Excel**
Module 4	**5.0 Break-Even Analysis (Lectures 17-22)**	17	**Graph a linear equation in two variables.**	**Reference 2, Lecture 17** **Reference 3, Ch 3** **Reference 5, Ch 16 & 18** **Tool: Microsoft Excel**
Module 4		18	**Solve two linear equations with two unknowns**	**Reference 2, Lecture 18** **Reference 3, Ch 2** **Reference 5, Ch 1** **Tool: Microsoft Excel**
Module 4		19	• **Perform linear cost-volume profit and break-even analysis.** • **Using a break-even chart**	**Reference 2, Lecture 19** **Tool: Microsoft Excel**
Module 4		20	• **Perform linear cost-volume profit and break-even analysis.** • **Using the algebraic approach of solving the cost and revenue functions**	**Reference 2, Lecture 20** **Tool: Microsoft Excel**
Module 4		21	• **Perform linear cost-volume profit and break-even analysis.** • **Using the contribution margin approach**	**Reference 2, Lecture 21** **Tool: Microsoft Excel**
Module 4		22	• **Perform linear cost-volume profit and break-even analysis.** • **Using Microsoft Excel** • **Assignment Module 3-4** • **Mid-Term Examination**	**Reference 2, Lecture 22** **Tool: Microsoft Excel**
Module 5	**6. Statistical Representation of Data (Lectures 23-27)**	23	**Statistical Data**	**Reference 2, Lecture 23** **Reference 5, Ch 5** **Tool: Microsoft Excel**
Module 5		24	**Statistical Representation Measures of Central Tendency Part 1**	**Reference 2, Lecture 24** **Reference 4, Ch 3** **Reference 5, Ch 6** **Tool: Microsoft Excel**
Module 5		25	• **Statistical Representation** • **Measures of Central**	**Reference 2, Lecture 25** **Reference 4, Ch**

			Tendency Part 2	**3 Reference 5, Ch 6 Tool: Microsoft Excel**
Module 5		26	• **Measures of Dispersion and Skewness Part 1**	**Reference 2, Lecture 26 Reference 4, Ch 4 Reference 5, Ch 6 Tool: Microsoft Excel**
Module 5		27	• **Measures of Dispersion and Skewness Part 2**	**Reference 2, Lecture 27 Reference 4, Ch 4 Reference 5, Ch 6 Tool: Microsoft Excel**
Module 6	**7. Correlation, Time Series and Exponential Smoothing (Lectures 28-33)**	28	• **Correlation Part 1**	**Reference 2, Lecture 28 Reference 5, Ch 13 Tool: Microsoft Excel**
		29	• **Correlation Part 2**	**Reference 2, Lecture 29 Reference 5, Ch 13 Tool: Microsoft Excel**
		30	• **Line Fitting Part 1**	**Reference 2, Lecture 30 Reference 5, Ch 14 Tool: Microsoft Excel**
		31	• **Line Fitting Part 2**	**Reference 2, Lecture 31 Tool: Microsoft Excel**
		32	• **Time Series and** • **Exponential Smoothing Part 1**	**Reference 2, Lecture 32 Reference 5, Ch 15 Tool: Microsoft**

				Excel
		33	• **Time Series and** • **Exponential Smoothing Part 2** • **Assignment Module 5-6**	**Reference 2, Lecture 33** **Reference 5, Ch 15** **Tool: Microsoft Excel**
Module 7	**7. Elementary Probability (Lectures 34-38)**	34	• **Factorials** • **Permutations and Combinations**	**Reference 2, Lecture 34** **Reference 3, Ch 2** **Tool: Microsoft Excel**
Module 7		35	• **Elementary Probability Part 1**	**Reference 2, Lecture 35** **Reference 5, Ch 8** **Tool: Microsoft Excel**
Module 7		36	• **Elementary Probability Part 2**	**Reference 2, Lecture 36** **Reference 5, Ch 8** **Tool: Microsoft Excel**
Module 7		37	• **Patterns of probability: Binomial, Poisson and Normal Distributions Part 1**	**Reference 2, Lecture 39** **Reference 5, Ch 9** **Tool: Microsoft Excel**
Module 7		38	• **Patterns of probability: Binomial, Poisson and Normal Distributions Part 2**	**Reference 2, Lecture 40** **Reference 5, Ch 9** **Tool: Microsoft Excel**
Module 7		39	• **Patterns of probability: Binomial, Poisson and Normal Distributions Part 3**	**Reference 2, Lecture 41** **Reference 5, Ch 9** **Tool: Microsoft Excel**

Module 7		40	• **Patterns of probability: Binomial, Poisson and Normal Distributions Part 4**	**Reference 2, Lecture 41 Reference 5, Ch 9 Tool: Microsoft Excel**
Module 8	**8. Probability Distributions (Lectures 39-44) 9. Linear Programming (Lecture 45)**	41	• **Estimating from Samples: Inference Part 1**	**Reference 2, Lecture 42 Reference 5, Ch 10 Tool: Microsoft Excel**
Module 8		42	• **Estimating from Samples: Inference Part 2**	**Reference 2, Lecture 43 Reference 5, Ch 10 Tool: Microsoft Excel**
Module 8		43	• **Hypothesis testing : Chi-Square Distribution Part 1**	**Reference 2, Lecture 44 Reference 5, Ch 11 Tool: Microsoft Excel**
Module 8		44	• **Hypothesis testing : Chi-Square Distribution Part 2**	**Reference 2, Lecture 45 Reference 5, Ch 11 Tool: Microsoft Excel**
Module 8		45	• **Production Planning: Linear Programming** • **Assignment Module 7-8** • **End Term Examination**	**Reference 2, Lecture 45 Reference 5, Ch 18 Tool: Microsoft Excel**

Methodology

There will be 45 lectures each of 50 minutes duration as indicated above. The lectures will be delivered in a mixture of Urdu and English. The lectures will be heavily supported by slide presentations. The slides for a lecture will be made available on the VU website for the course a few days before the actual lecture is televised. This will allow students to carry out preparatory reading before the lecture. The course will be provided its own page on the VU's web site. This will be used to provide lecture and other supporting material from the course to the students. The page will have a link to a web-based discussion and bulletin board for the students. Teaching assistants will be assigned by VU to provide various forms of assistance such as grading, answering questions posted by students and preparation of slides.

Text and Reference Material

The course is based on material from different sources. Topics for reading will be indicated on course web site and in professor's handouts, also to be posted on the course web site. A list of reference books will also be posted and updated on the course web site.

The following material will be used by the students as reference:

Reference 1: Course Outline
2: Instructor's Power Point Slides
3: Business Mathematics & Statistics by Prof. Miraj Din Mirza
4: Elements of statistics & Probability by Shahid Jamal
5: Quantitative Approaches in Business studies by Clare Morris
6: Microsoft Excel Help File

Schedule of Lectures

Given above is the tentative schedule of topics to be covered. Minor changes may occur but these will be announced well in advance.

LECTURE 2
Applications of Basic Mathematics
Part 1

OBJECTIVES

The objectives of the lecture are to learn about:

- Different course modules
- Basic Arithmetic Operations
- Starting Microsoft (MS) Excel
- Using MS Excel to carry out arithmetic operations

COURSE MODULES

This course comprises 8 modules as under:

- Modules 1-4: Mathematics
- Modules 5-8: Statistics

Details of modules are given in handout for lecture 01.

BASIC ARITHMETIC OPERATIONS

Five arithmetic operations provide the foundation for all mathematical operations. These are:

- Addition
- Subtraction
- Multiplication
- Division
- Exponents

Example- Addition

12 + 5 = 17

Example- Subtraction

12 - 5 = 7

Example- Multiplication

12 x 5 = 60

Example- Exponent

(4)^2 = 16
(4)^1/2 = 2
(4)^-1/2 = 1/(4)^1/2 = ½ = 0.5

MICROSOFT EXCEL IN BUSINESS MATHEMATICS & STATISTICS

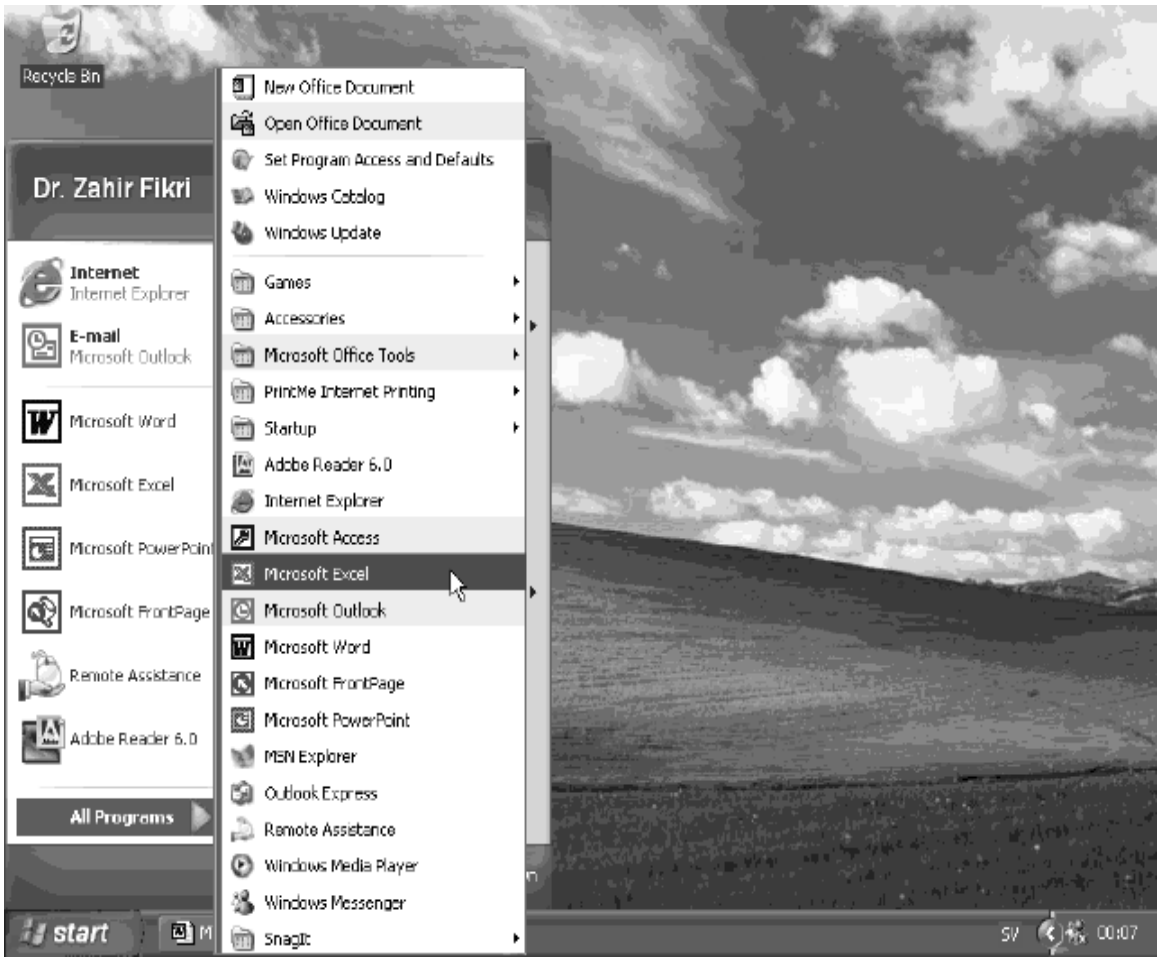
Microsoft Corporation's Spreadsheet software **Excel** is widely used in business mathematics and statistical applications.

This course is based on wide applications of **EXCEL 2002**. It is recommended that you install **EXCEL 2002 XP** software on your computer. If your computer has **Windows 2000** and **EXCEL 2000** even that version of EXCEL can be used as the applications we intend to learn can be done using the earlier version of **EXCEL**. Those of you who are still working with Windows 98 and have **EXCEL 97** installed are encouraged to migrate to newer version of **EXCEL** software.

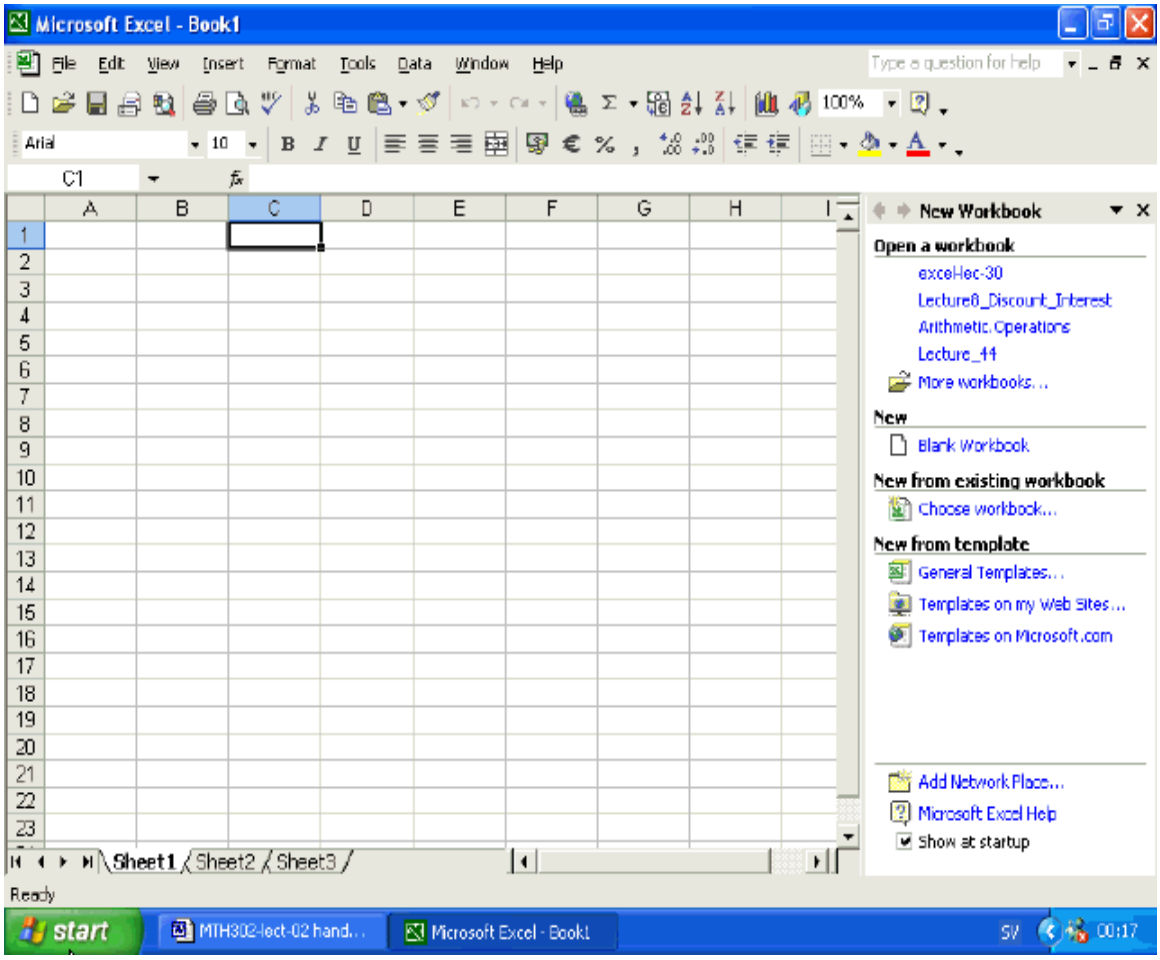
Starting EXCEL 2000 XP

EXCEL 2000 XP can be started by going through the following steps:
Click **Start** on your computer
Click **All Programs**
Click **Microsoft Excel**

The following slides show the operations:



The EXCEL window opens and a blank worksheet becomes available as shown below:



The slide shows a Workbook by the name book1 with three sheets: Sheet1, Sheet2 and Sheet3. The Excel Window has Column numbers starting from A and row numbers starting from 1. the intersection of a row and column is called a **Cell**. The first cell is A1 which is the intersection of column A and row 1. All cells in a Sheet are referenced by a combination of Column name and row number.

Example 1: B15 means cell in column B and row 15.

Example 2: A cell in row 12 and column C has reference C12.

A **Range** defines all cells starting from the leftmost corner where the range starts to the rightmost corner in the last row. The Range is specified by the starting cell, a colon and the ending cell.

Example 3: A **Range** which starts from A1 and ends at D15 is referenced by A1:D15 and has all the cells in columns A to D up to and including row 15.

A value can be entered into a cell by clicking that cell. The mouse pointer moves to the selected cell. Simply enter the value followed by the **Enter** key. The mouse pointer moves to the cell below.

If you make a mistake while entering the value select the cell again (by clicking it). Enter the new value. The old value is replaced by the new value.

If only one or more digits are to be changed then select the cell. Then double click the mouse. The blinking cursor appears. Either move the arrow key to move to the

digit to be changed or move the cursor to the desired position. Enter the new value and delete the undesired value by using the **Del** key.

I suggest that you learn the basic operations of entering, deleting and changing data in a worksheet.

About calculation operators in Excel

In Excel there are four different types of operators:
Arithmetic operators
Comparison operators
Text concatenation operator
Reference operators

The following descriptions are reproduced from Excel's Help file for your ready reference. In the present lecture you are directly concerned with arithmetic operators. However, it is important to learn that the comparison operators are used where calculations are made on the basis of comparisons. The text concatenation operator is used to combine two text strings. The reference operators include ":" and "," or ; as the case maybe. We shall learn the use of these operators in different worksheets. You should look through the Excel Help file to see examples of these functions. Selected material from Excel Help File relating to arithmetic operations is given in in a separate file.
The Excel arithmetic operators are as follows:
Addition. Symbol: + (Example: =5+4 Result: 9)
Subtraction. Symbol: - (Example: =5-4 Result: 1)
Multiplication. Symbol: * (Example: =5*4 Result: 20)
Division. Symbol: / (Example: =12/4 Result: 3)
Percent. Symbol: % (Example: =20% Result: 0.2)
Exponentiation: ^ (Example: =5^2 Result: 25)

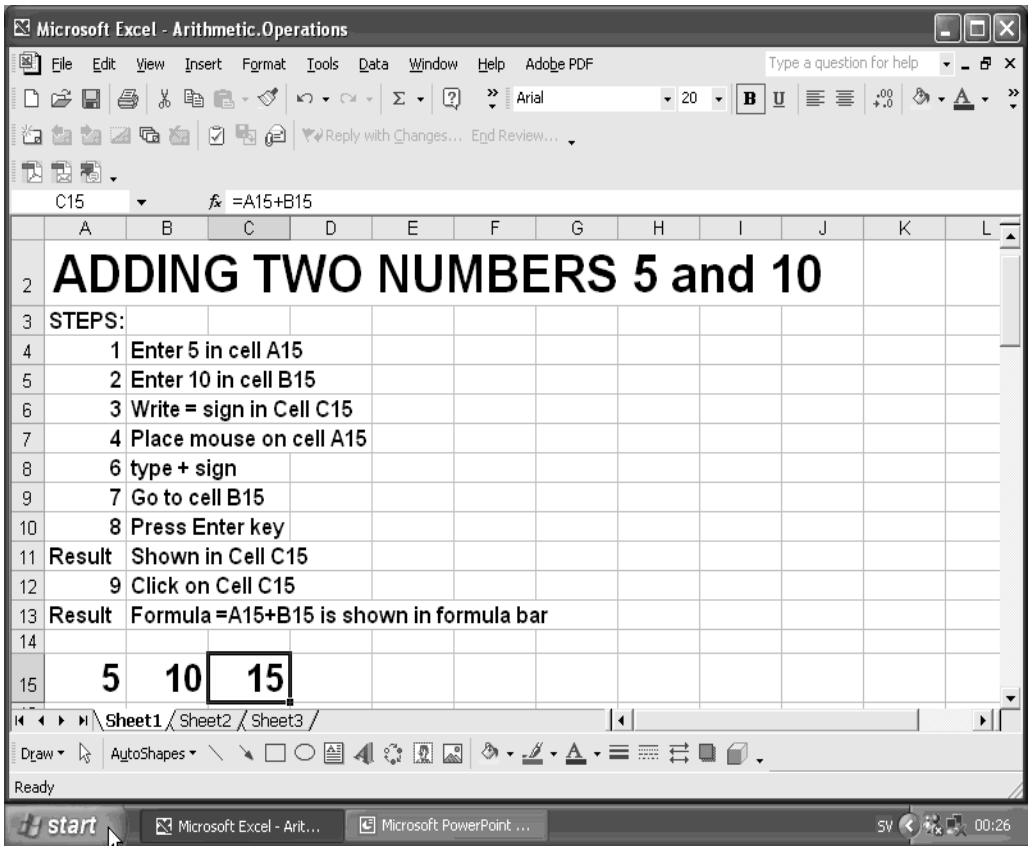
Excel Formulas for Addition

All calculations in Excel are made through formulas which are written in cells where result is required.
Let us do addition of two numbers 5 and 10.
We wish to calculate the addition of two numbers 10 and 5. Let us see how we can add these two numbers in Excel.

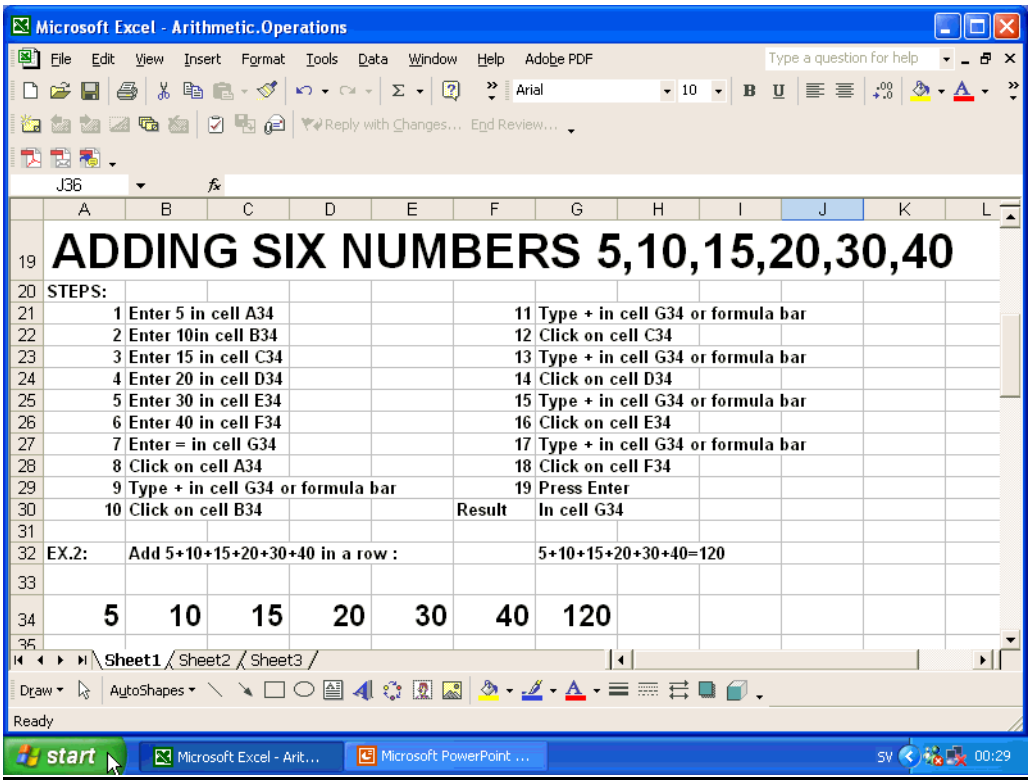
1. Open a blank worksheet.
2. Click on a cell where you would like to enter the number 10. Say cell A15.
3. Enter 10 in cell A15.
4. Click cell where you would like to enter the number 5. Say cell B15.
5. Click cell where you would like to get the sum of 10 and 5. Say cell C15.
6. Start the formula. Write equal sign = in cell C15.
7. After =, write "(" (left bracket) in cell C15.
8. Move mouse and left click on value 10 which is in cell A15. In cell C15, the cell reference A15 is written.
9. Write "+" after "A15" in cell C15.
10. Move mouse and left click on value 5 which is in cell B15. In cell C15, the cell reference B15 is written.
11. Write ") " (right bracket) in cell C15.
12. Press Enter key

The answer 15 is shown in cell C15.
If you click on cell C15, the formula "=A15+B15" is displayed the formula bar to the right of *fx* in the Toolbar.

The main steps along with the entries are shown in the slide below. The worksheet MTH302-lec-02 contains the actual entries.



The next slide shows addition of 6 numbers 5, 10, 15, 20, 30 and 40. The entries were made in row 34. The values were entered as follows:
Cell A34: 5
Cell B34: 10
Cell C34: 15
Cell D34: 20
Cell E34: 30
Cell F34: 40
The formula was written in cell G34. The formula was:
=5+10+15+20+30+40
The answer was 120.
You can use an Excel function SUM along with the cell range A34:F34 to calculate the sum of the above numbers. The formula in such a case will be:
=SUM(A34:F34)
You enter "=" followed by SUM, followed by "(". Click on the cell with value 5(reference: A34). Drag the mouse to cell with value 40(reference: F34) and drop the mouse. Enter ")" and then press the Enter key.



In the above two examples you learnt how formulas for addition are written in Excel.

Excel Formula for Subtraction

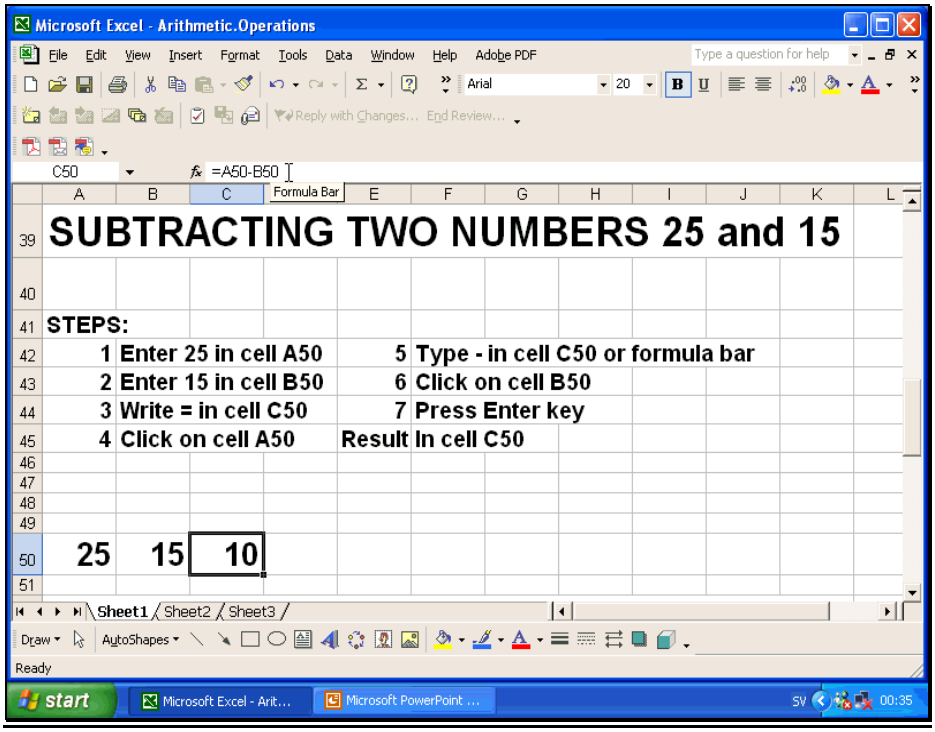
Excel formulas for subtraction are similar to those of addition but with the minus sign. Let us go through the steps for subtracting 15 from 25. Enter values in row 50 as follows:

Cell A50: 25
Cell B50: 15

Write the formula in cell C50 as follows:
=A50-B50

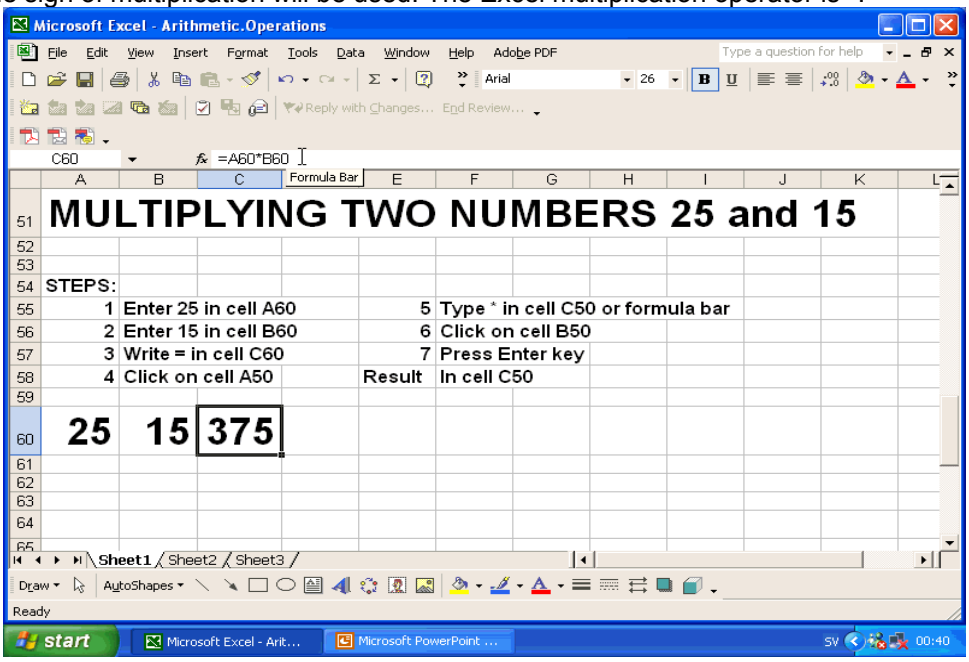
To write this formula, click cell C50, where you want the result. Enter "=". Click on cell with value 25 (reference:A50). Enter "-"(minus sign). Click on cell with value 15 (reference B50). Press enter key.

If you enter 15 first and 25 later, then the question will be to find result of subtraction 15-25.



Excel Formula for Multiplication

Excel formula for multiplication is also similar to the formula for addition. Only the sign of multiplication will be used. The Excel multiplication operator is *.



Let us look at the multiplication of two numbers 25 and 15. The entries will be made in row 60. Enter values as under:

Cell A50: 25

Cell B50: 15

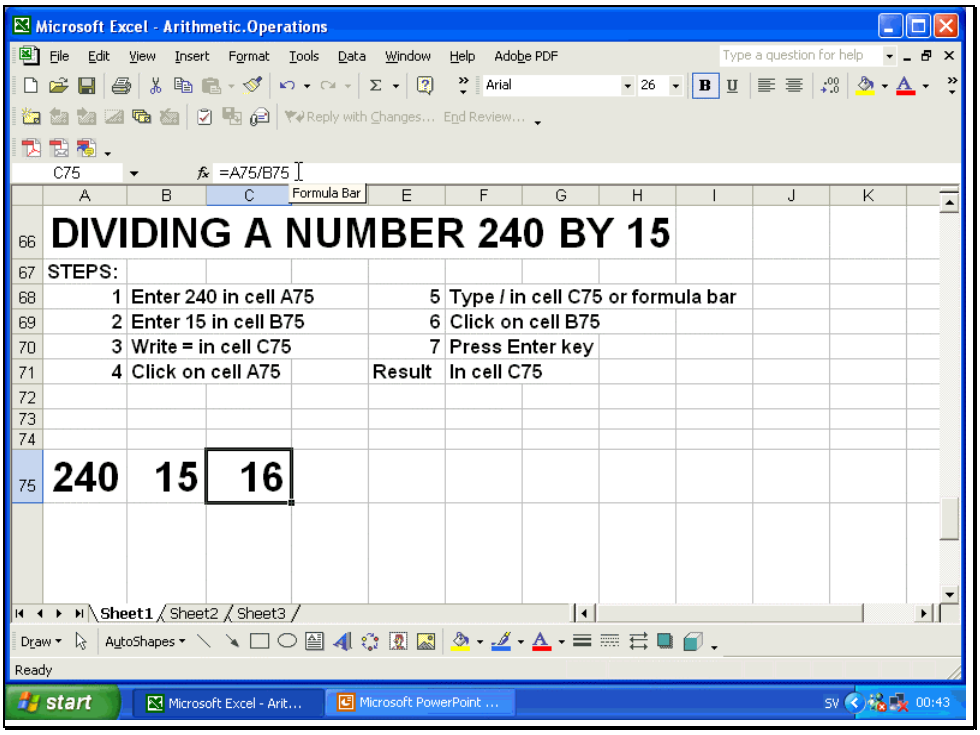
The formula for multiplication is:

=A50*B50

Click on cell C50 to write the formula in that cell. Enter "=". Click on cell with number 25 (reference: A50). Enter "*". Click on cell with number 15 (reference: B50). Press Enter key. The answer is 375 in cell C50.

Excel Formula for Division

The formula for division is similar to that of multiplication with the difference that the division sign "/" will be used.



Let us divide 240 by 15using Excel formula for division. Let us enter numbers in row 75 as follows:

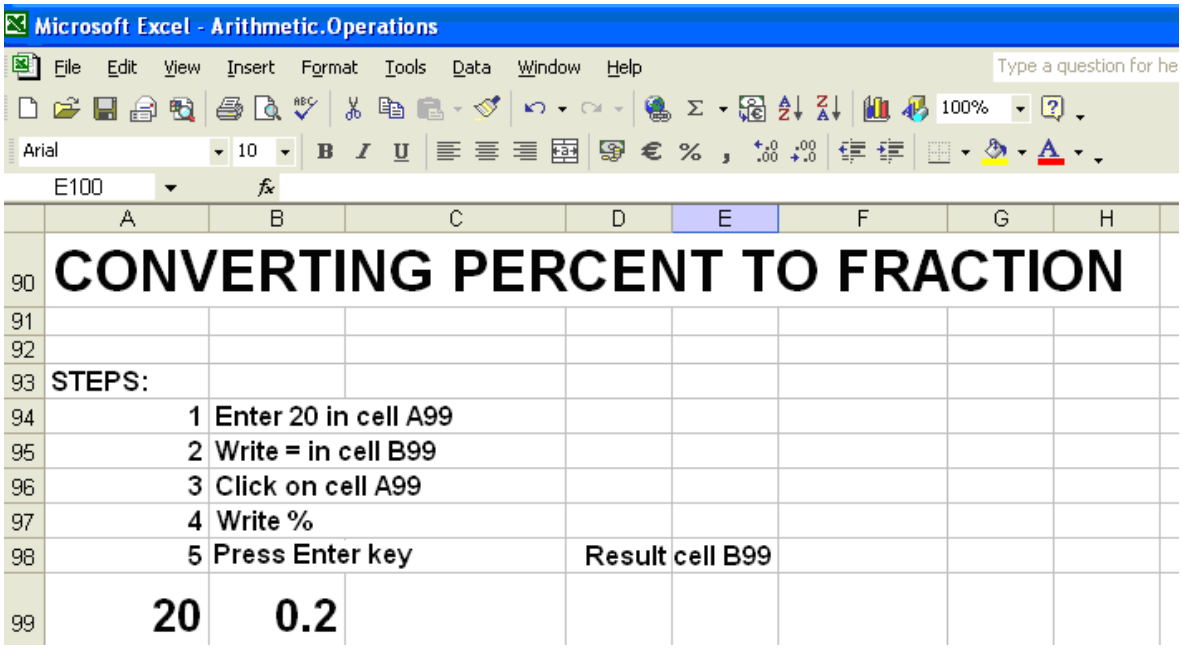
Cell A75: 240
Cell B75: 15

The formula for division will be written in cell C75 as under:
=A75/B75

The steps are as follows: Click the cell A75. Enter 240 in cell A75. Click cell B75. Enter 15. Click cell C75. Enter "=". Click on cell with value 240 (reference: A75). Enter "/". Click cell with number 15 (reference: B75). Press enter key. The answer 16 will be displayed in cell C75.

Excel Formula for Percent

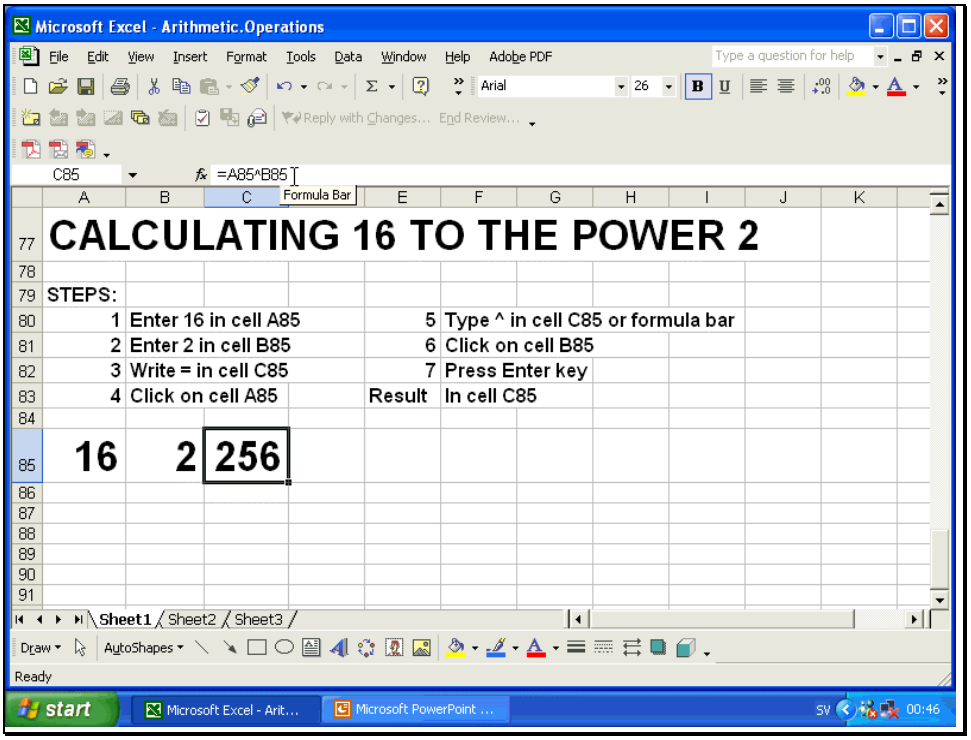
The formula for converting percent to fraction uses the symbol %. To convert 20% to fraction the formula is as under:
=20%
If you enter 20 in cell A99, you can write formula for conversion to fraction by doing the following:
Enter 2o in cell A99. In cell B99 enter "=". Click on cell A99. Enter"%". Press Enter key. The answer 0.2 is given in cell B99.



Excel Formula for Exponentiation

The symbol for exponentiation is ^. The formula for calculating exponents is similar to multiplication with the difference that the carat symbol ^ will be used.
Let us calculate 16 raised to the power 2 by Excel formula for exponentiation. The values will be entered in row 85.
The steps are:
Select Cell A85. Enter 16 in this cell.
Select cell B85 Enter 2 in this cell.
Select cell C85.
Enter"=".
Select cell with value 16 (reference:A85).
Enter "^".
Select number 2 (reference: B85)
Press Enter key.

The result 256 is displayed in cell C85.



Recommended Homework

Download worksheet MTH302-lec-02.xls from the course web site.
Change values to see change in results.
Set up new worksheets for each Excel operator with different values.
Set up worksheets with combinations of operations.

LECTURE 3
Applications of Basic Mathematics
Part 2

OBJECTIVES
The objectives of the lecture are to learn about:
- ***Evaluations***
- ***Calculate Gross Earnings***
- ***Using Microsoft Excel***

Evaluation
In order to successfully complete this course, the student is required to meet the evaluation criteria**:**

- ***Evaluation Criterion 1***
- ***Full participation is expected for this course***

- ***Evaluation Criterion 2***
- ***All assignments must be completed by the closing date***

- ***Evaluation Criterion 3***
- ***Overall grade will be based on VU existing Grading Rules***

- ***Evaluation Criterion 4***
- ***All requirements must be met in order to pass the course***

Grading
There will be a term exam and one final exam; there will also be 4 assignments. The final exam will be comprehensive.
These will contribute the following percentages to the final grade:
Mid Term Exam 35%
Final 50%
4 Assignments 15%

Collaboration
The students are encouraged to develop collaboration in studying this course. You are advised to carry out discussions with other students on different topics. It will be in your own interest to prepare your own solutions to Assignments. You are advised to make your original original submissions as copying other students' assignments will have negative impact on your studies.

ETHICS
Be advised that as good students your motto should be:
- ***No copying***
- ***No cheating***
- ***No short cuts***

Methodology

There will be 45 lectures each of 50 minutes duration.
The lectures will be delivered in a mixture of Urdu and Englis.

The lectures will be heavily supported by slide presentations.

The slides available on the VU website before the actual lecture is televised.
Students are encouraged to carry out preparatory reading before the lecture.

This course has its own page on the VU's web site.
There are lecture slides as well as other supporting material available on the web site.
Links to a web-based discussion and bulletin board will also been provided.

Teaching assistants will be assigned by VU to provide various forms of assistance such as grading, answering questions posted by students and preparation of slides

Text and Reference Material

This course is based on material from different sources.
Topics for reading will be indicated on course web site and in professor's handouts.
A list of reference books to be posted and updated on course web site. You are encouraged to regularly visit the course web site for latest guidelines for text and reference material.

Types of Employees

There may be three types of employees in a company:

- Regular employees drawing a monthly salary
- Part time employees paid on hourly basis
- Payments on per piece basis

To be able to understand how calculations of gross earnings are done, it is important to understand what gross earnings include.

GROSS EARNINGS/SALARY

Gross salary includes the following:

- ***Basic salary***
- ***Allowances***

Gross salary may include:

- **Basic salary**
- **House Rent**
- **Conveyance allowance**
- **Utilities allowance**

Accordance to the taxation rules if allowances are 50% of basic salary, the amount is treated as tax free. Any allowances that exceed this amount are considered taxable both for the employee as well as the company.

Example 1

The salary of an employee is as follows:
Basic salary = 10,000 Rs.
Allowances = 5,000 Rs.
What is the taxable income of employee?
Is any add back to the income of the company?
% Allowances = (5000/10000) x 100 =50%
Hence allowances are not taxable.

Total taxable income = 10,000 Rs.
Add back to the income of the company = 0

Example 2
The salary of an employee is as follows:
Basic salary = 10,000 Rs.
Allowances = 7,000 Rs.
What is the taxable income of employee?
Is any add back to the income of the company?
% Allowances = (7000/10000) x 100 =70%
Allowed non-taxable allowances = 50% = 0.5 x 10000 = 5,000 Rs.
Taxable allowances = 70% – 50% = 7000 - 5000 = 2,000 Rs.
Hence 2000 Rs. of allowances are taxable.
Total taxable income = 10,000 + 2000 = 12,000 Rs.
Add back to the income of the company = 20% allowances = 2,000 Rs.

Structure of Allowances
The common structure of allowances is as under:
- House Rent = 45 %
- Conveyance allowance = 2.5 %
- Utilities allowance = 2.5 %

Example 3
The salary of an employee is as follows:
Basic salary = 10,000 Rs.
What is the amount of allowances if House Rent = 45 %, Conveyance allowance = 2.5 % and Utilities allowance = 2.5 %?
House rent allowances = 0.45 x 10000 = 4,500 Rs.
Conveyance allowance = 0.025 x 10000 = 250 Rs.
Utilities allowance = 0.025 x 10000 = 250 Rs.
Thus total allowances are 4500+250+250 = 5000Rs

Provident Fund
According to local laws, a company can establish a Provident Trust Fund for the benefit of the employees. By law, $1/11^{th}$ of Basic Salary per month is deducted by the company from the gross earnings of the employee. An equal amount, i.e $1/11^{th}$ of basic salary per month, is contributed by the company to the Provident Fund to the account of the employee. Thus there is an investment of $2/11^{th}$ of **basic salary** on behalf of the employee in Provident Fund. The company can invest the savings in Provident Fund in Government Approved securities such as defense saving Certificates. Interest earned on investments in Provident Fund is credited to the account of the employees in proportion to their share in the Provident Fund.

Example 4
The salary of an employee is as follows:
Basic salary = 10,000 Rs.
Allowances = 5,000 Rs.
What is the amount of deduction on account of contribution to the Provident Trust Fund?
What is the contribution of the company?
What is the total saving of the employee per month on account of Provident Trust Fund?
Employee contribution to Provident Fund = 1/11 x 10000 = 909.1 Rs.
Company contribution to Provident Fund = 1/11 x 10000 = 909.1 Rs.
Total savings of employee in Provident Fund = 909.1 + 909.1 = 1,818.2 Rs.

Gratuity Fund
According to local laws, a company can establish a Gratuity Trust Fund for the benefit of the employees. By law, $1/11^{th}$ of Basic Salary per month is contributed by

the company to the Gratuity Fund to the account of the employee. Thus there is a saving of $1/11^{th}$ of basic salary on behalf of the employee in Gratuity Fund. The company can invest the savings in Gratuity Fund in Government Approved securities such as defence saving Certificates. Interest earned on investments in Gratuity Fund is credited to the account of the employees in proportion to their share in the Gratuity Fund.

Example 5

The salary of an employee is as follows:
Basic salary = 10,000 Rs.
Allowances = 5,000 Rs.
What is the contribution of the company on account of gratuity to the Gratuity Trust Fund?
Company contribution to Gratuity Fund
= Total savings of employee in Gratuity Fund = 1/11 x 10000 = 909.1 Rs.

Leaves

All companies have a clear leaves policy. The number of leaves allowed varies from company to company. Typical leaves allowed may be as under:

- ***Casual Leave = 18 Days per year***
- ***Earned Leave = 18 Days per year***
- ***Sick Leave = 12 Days per year***

Example 6

The salary of an employee is as follows:
Basic salary = 10,000 Rs.
Allowances = 5,000 Rs.
What is the cost on account of casual, earned and sick leaves per year if normal working days per month is 22? What is the total cost of leaves as percent of gross salary?
Gross salary = 10000 + 5000 = 15,000 Rs.
Cost of casual leaves per year = {18 / (22 x 12)} x 15000 x 12 = 12,272.7 Rs.
Cost of earned leaves per year = {18 / (22 x 12)} x 15000 x 12= 12,272.7 Rs
Cost of Sick leaves per year = {12 / (22 x 12) x 15000 x 12 = 8,181.8 Rs
Total cost of leaves per year = 12272.7 + 12272.7 + 8181.8 = 32,727.3 Rs.
Total cost of leaves as percent of gross salary = (32727.3/(12 x 15000))x 100 = 18.2%

Social Charges

Social charges comprise leaves, group insurance and medical. Typical medical/group insurance is about 5% of gross salary. Other social benefits may include contribution to employee's children's education, club membership, leave fare assistance etc. Such benefits may be about 5.8%. Leaves are 18.2% of gross salary (as calculated in above example)
Total social charges therefore may be = 18.2 + 5 + 5.8 = 29% of gross salary. Other companies may have more social benefits. The 29% social charges are quite common.

Example 7

The salary of an employee is as follows:
Basic salary = 10,000 Rs.
Allowances = 5,000 Rs.
What is the cost of the company on account of leaves (18.2%), group insurance/medical (5%) and other social benefits (5.8%)?

Leaves cost = 0.182 x 15000 = 2,730 Rs.
Group insurance/medical = 0.05 x 15000 = 750 Rs.
Other social benefits = 0.058 x 15000 = 870 Rs.
Total social charges = 2730 + 750 + 870 = 4,350 Rs.

SUMMARY

Summary of different components of salary is as follows:
Basic salary
Allowances 50 % of basic salary
Gratuity 9.09 % of basic salary
Provident Fund 9.09 % of basic salary
Social Charges 29 %

Gross remuneration is pay or salary, typically monetary payment for services rendered, as in an employment. It includes.

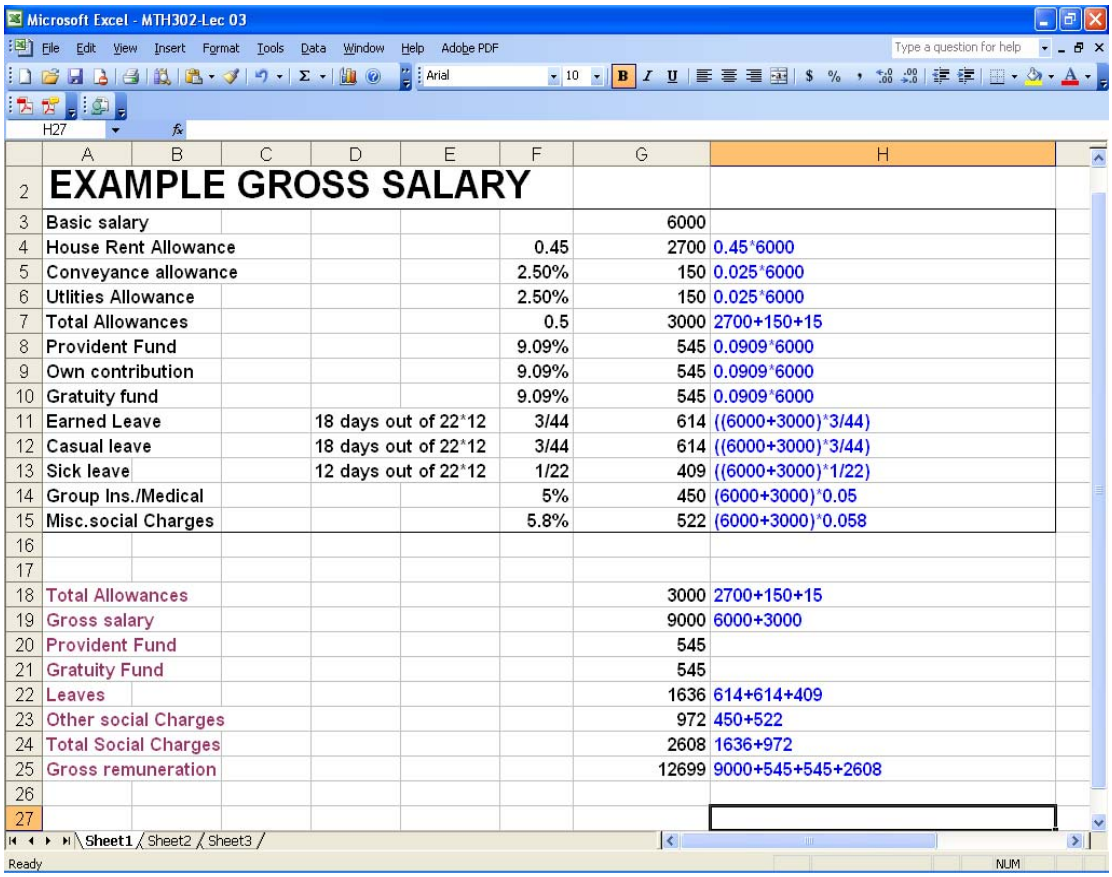
1. Basic Salary
2. House rent allowance
3. Conveyance allowance
4. Utilities
5. Provident fund
6. Gratuity fund
7. Leaves
8. Group insurance (medical etc)
9. Miscellaneous social charges.

Benefits can also include more factors and are not limited to the above list.

The purpose of the benefits is to increase the economic security of employees

Example 8

The salary of an employee is as follows:
Basic salary = 6,000 Rs.



Converting fraction to percent

Calculate percent by multiplying fraction by 100. and put the percent sign (%)

Percent = Fraction X 100

Example 9

Convert 0.1 to percent.

0.1 X 100 = 10%

Common Fraction

Common fraction is a fraction having an integer as a numerator and an integer as a denominator. For example ½, 10/100 are common fractions.

Converting percent into Common Fraction

Example 11

20%= 20/100= 0.2

Decimal fraction.

Any number written in the form: an integer followed by a decimal point followed by a (possibly infinite) string of digits. For example 2.5, 3.9 etc.

Converting percent into decimal fraction

Example

20% = 0.2

Percent

20% or 20/100=0.2

Percentage

Percentage is formed by multiplying a number called the base by a percent, called the rate. Thus

Percentage = Base x Rate

Example 13

What percentage is 20% ×̶ of 120?

Here, rate = 20% = 20/100 = 0.2

Base = 120

Percentage = 20/100 x 120

Or

0.2 X 120

= 24

Example 14

What Percentage is 6 % of 40?

Percentage

= Rate X Base

= 0.06 X 40

=̶2.4

Base

Base = Percentage/Rate

Example 15

Find base if

Rate = 24.0 % = 0.24

Percentage = 96

Base= 96/0.24=400

LECTURE 4
Applications of Basic Mathematics
Part 3

OBJECTIVES

The objectives of the lecture are to learn about:
Review Lecture 3
Calculating simple or weighted averages
Using Microsoft Excel

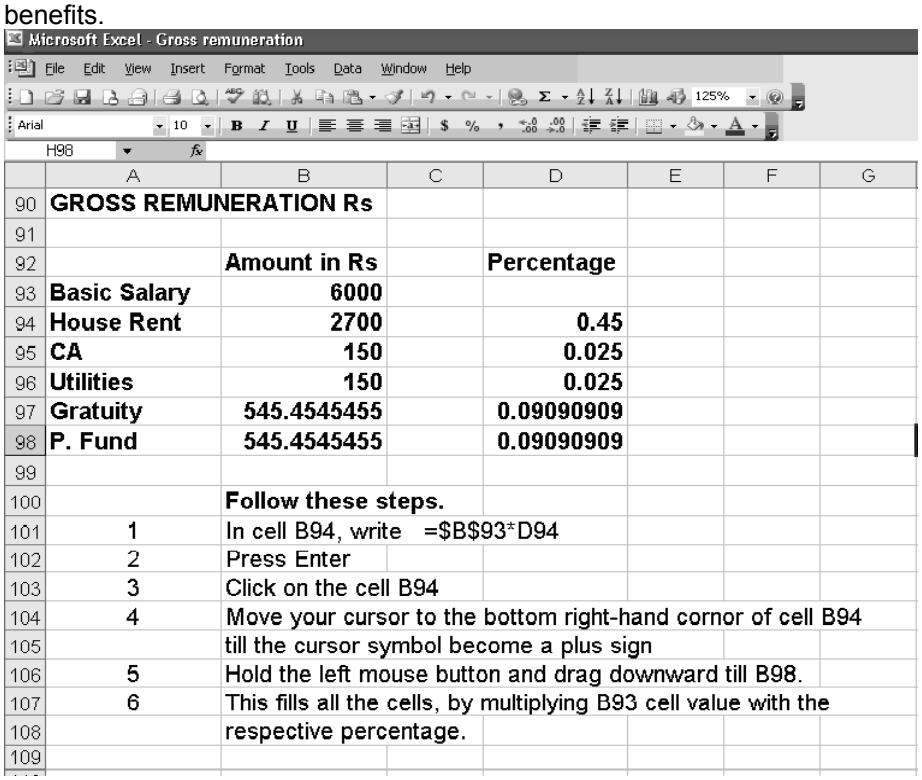
Gross Remuneration

The following slide shows worksheet calculation of Gross remuneration on the basis of 6000 Rs, basic salary.

As explained earlier, house rent is 45% of basic salary. Conveyance and Utilities Allowance are both 2.5% of basic salary. Both Gratuity and Provident fund are 1/11th of basic salary.

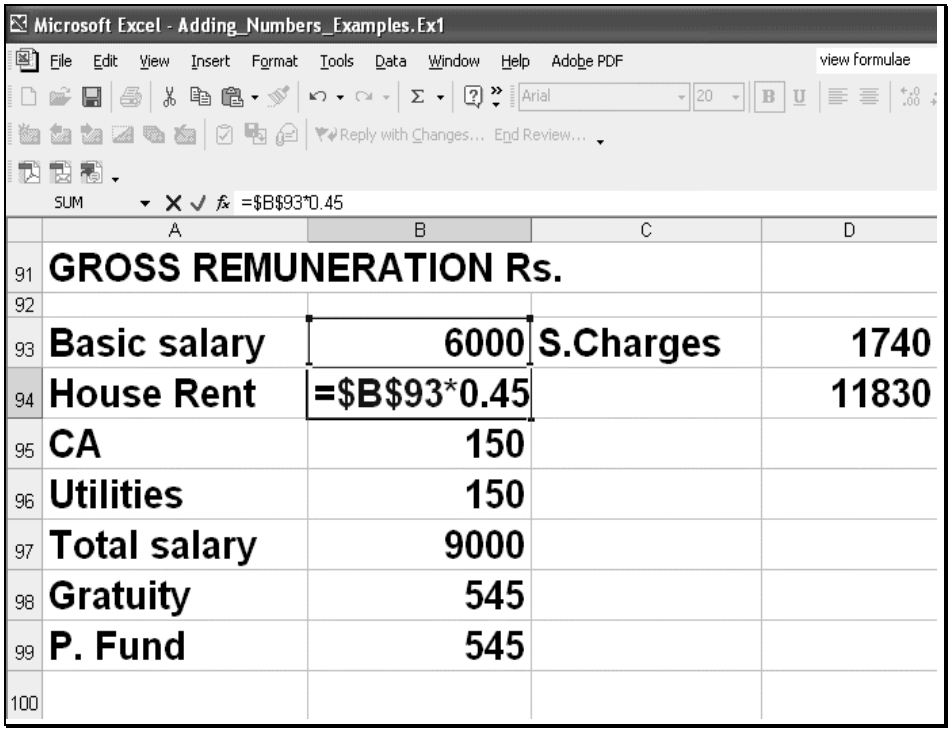
The arithmetic formulas are as follows: Excel formulas are within brackets.
Basic salary = 6000 Rs.
House rent = 0.45 x 6000 = 2700 Rs. (Excel formula: =$B$93*0.45)
Conveyance Allowance = 0.025 x 6000 = 150 Rs. (Excel formula: =$B$93*0.025)
Utilities allowance = 0.025 x 6000 = 150 Rs. (Excel formula: =$B$93*0.025)
Gross salary = 6000 + 2700 + 150 + 150 = 9000 Rs. (Excel formula: =SUM(B93:B96)
Gratuity = 1/11 x 6000 = 545 (Excel formula: =ROUND((1/11)*$B$93,0)

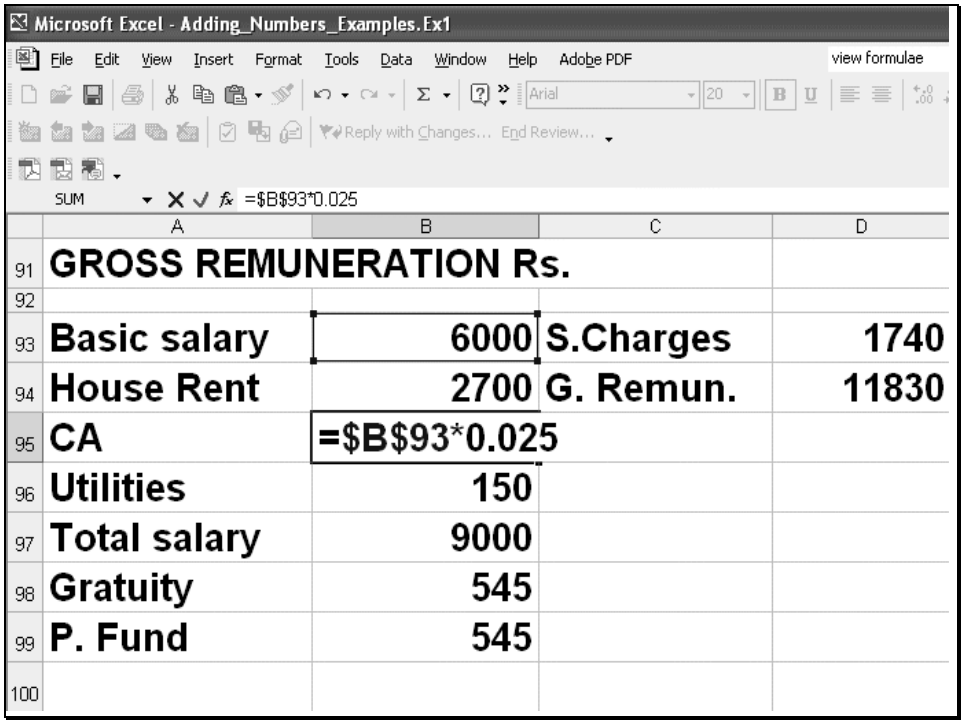
In the Excel formulas the $ sign is used before the row and column reference to fix the location of the cell. Since house rent, CA, utilities, gratuity and provident fund are calculated with respect to basic salary so by using $B$93, we fixes the location of cell B93. This feature can be used for quick and correct calculation of all allowances and benefits.

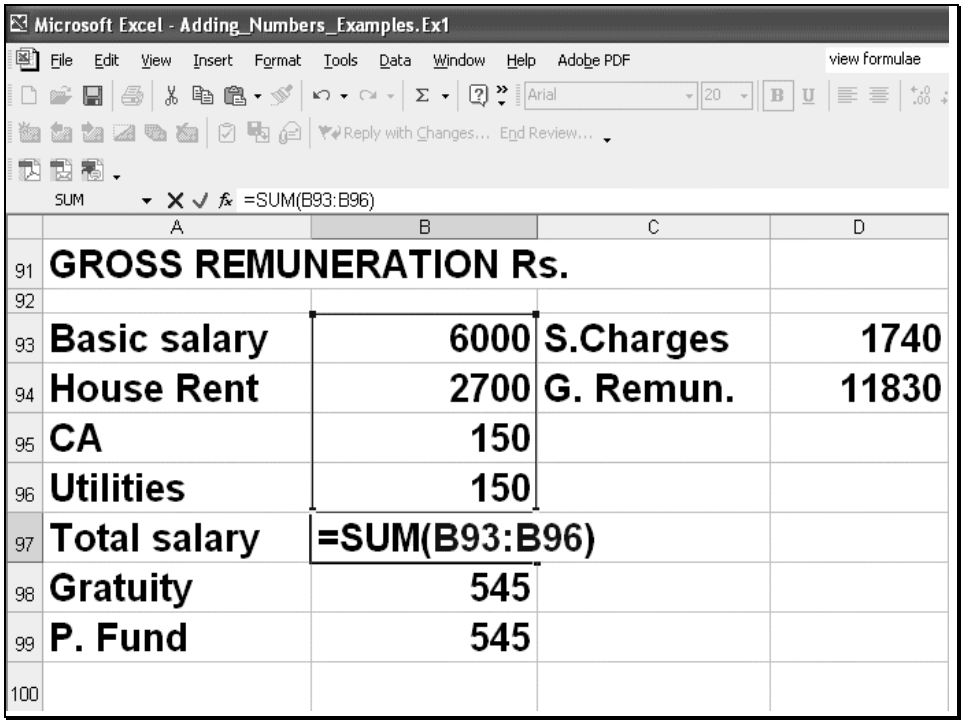


	A	B	C	D	E	F	G
90	**GROSS REMUNERATION Rs**						
91							
92		**Amount in Rs**		**Percentage**			
93	**Basic Salary**	**6000**					
94	**House Rent**	**2700**		**0.45**			
95	**CA**	**150**		**0.025**			
96	**Utilities**	**150**		**0.025**			
97	**Gratuity**	**545.4545455**		**0.09090909**			
98	**P. Fund**	**545.4545455**		**0.09090909**			
99							
100		**Follow these steps.**					
101	1	In cell B94, write =$B$93*D94					
102	2	Press Enter					
103	3	Click on the cell B94					
104	4	Move your cursor to the bottom right-hand cornor of cell B94					
105		till the cursor symbol become a plus sign					
106	5	Hold the left mouse button and drag downward till B98.					
107	6	This fills all the cells, by multiplying B93 cell value with the					
108		respective percentage.					
109							

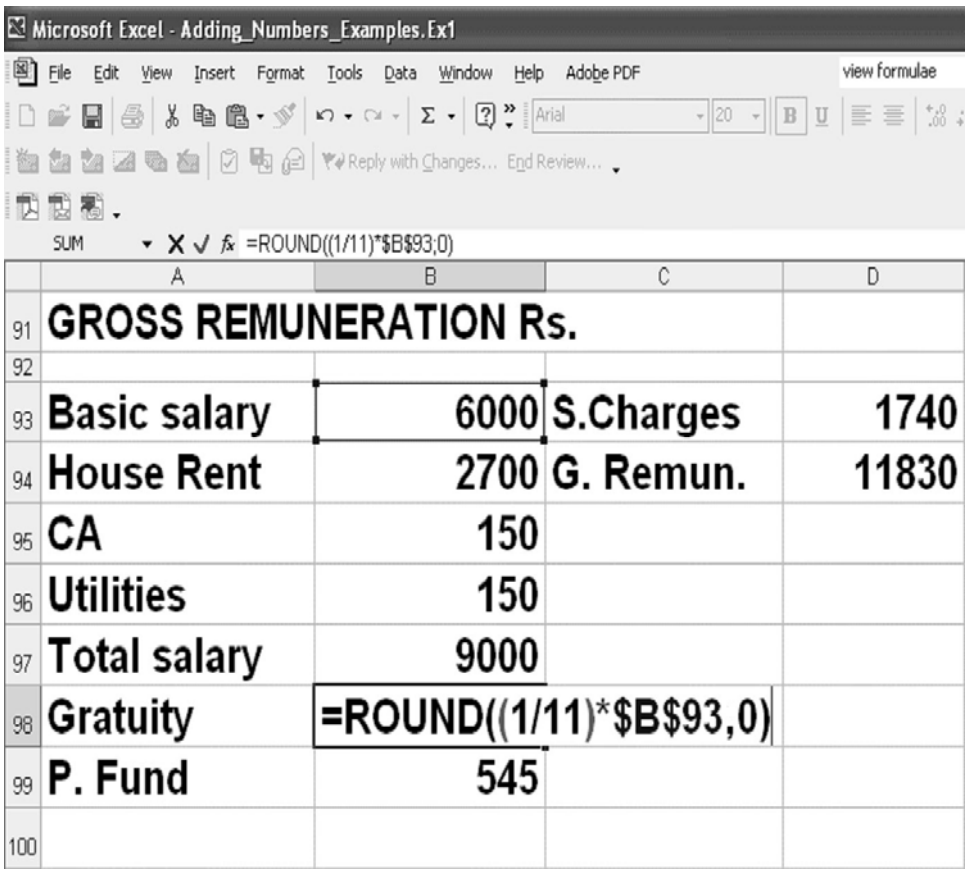
Now let us see cell by cell calculation.


Microsoft Excel - Adding_Numbers_Examples.Ex1
File Edit View Insert Format Tools Data Window Help Adobe PDF
view formulae
Reply with Changes... End Review...
SUM
=$B$93*0.45
GROSS REMUNERATION Rs.
Basic salary 6000 S.Charges 1740
House Rent =$B$93*0.45 11830
CA 150
Utilities 150
Total salary 9000
Gratuity 545
P. Fund 545

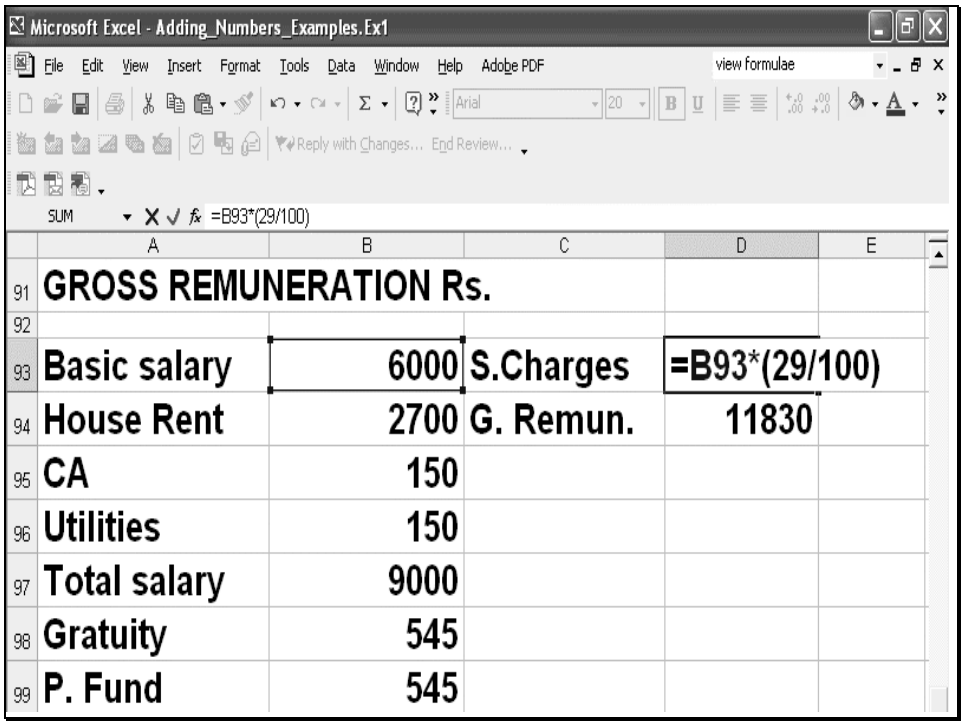

Microsoft Excel - Adding_Numbers_Examples.Ex1
File Edit View Insert Format Tools Data Window Help Adobe PDF
view formulae
Reply with Changes... End Review...
SUM
=$B$93*0.025
GROSS REMUNERATION Rs.
Basic salary 6000 S.Charges 1740
House Rent 2700 G. Remun. 11830
CA =$B$93*0.025
Utilities 150
Total salary 9000
Gratuity 545
P. Fund 545

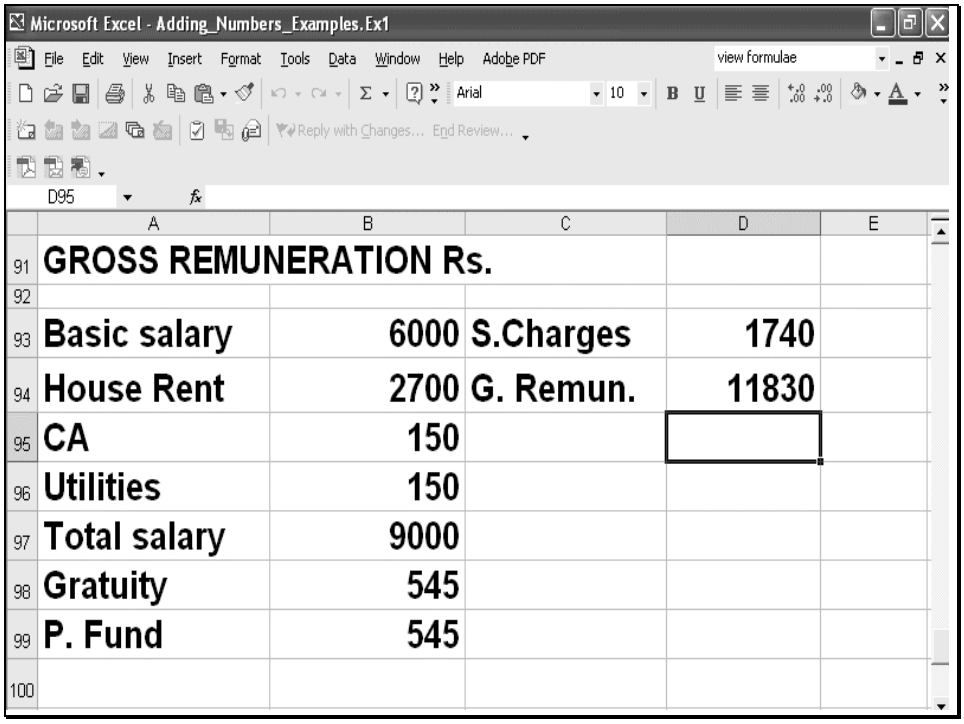


In Gratuity and provident calculations the function ROUND is used to round off values to desired number of decimals. In our case we used the value after the semicolon to indicate that no decimal is required. If you want 1 decimal use the value 1. for 2 decimals use 2 as the second parameter to the ROUND function. The first parameter is the expression for calculation 1/11*$B$93.



In the calculation for social charges the formula is B93*(29/100). Here 29/100 means 29% social charges. The $ sign was not used here to explain another feature of excel. If the formula in cell D93 is copied to cell E93 (say), the cell reference B93 in formula changes to C93. $B$93 would be needed to fix the value of basic salary in cell E93.





AVERAGE
Average (Arithmetic Mean) = Sum /N
Sum= Sum of all data values

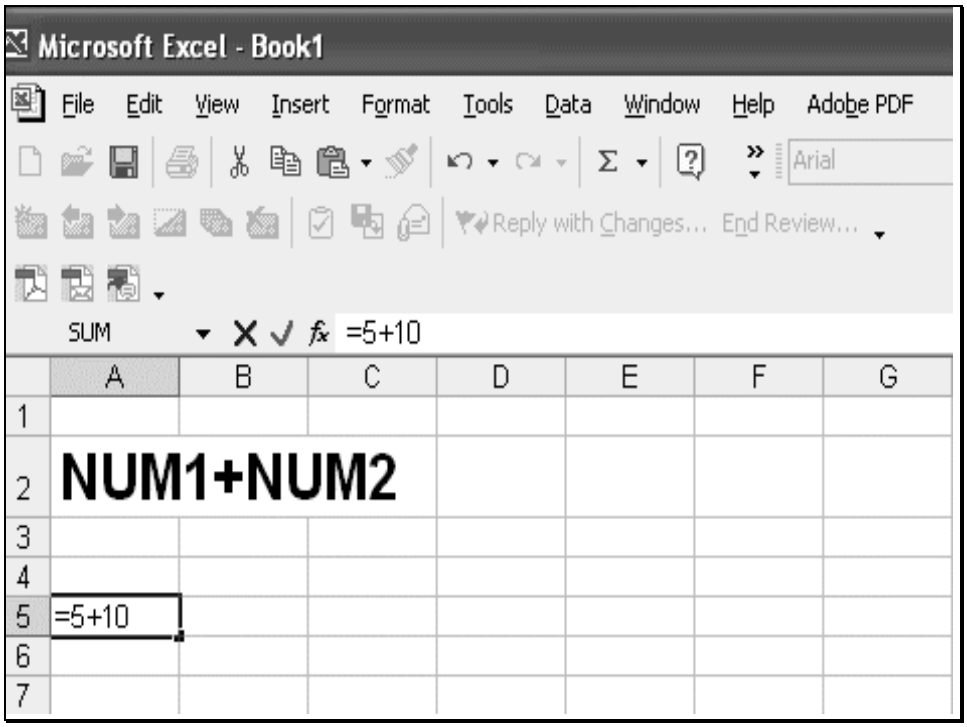
N= number of data values
EXAMPLE 1
Data: 10, 7, 9, 27, 2
Sum:
= 10+7+9+27+2 = 55
There are 5 data values
Average = 55/5 = 11

ADDING NUMBERS USING MICROSOFT EXCEL

1. Add numbers in a cell
2. Add all contiguous numbers in a row or column
3. Add noncontiguous numbers
4. Add numbers based on one condition
5. Add numbers based on multiple conditions
6. Add numbers based on criteria stored in a separate range
7. Add numbers based on multiple conditions with the Conditional Sum Wizard

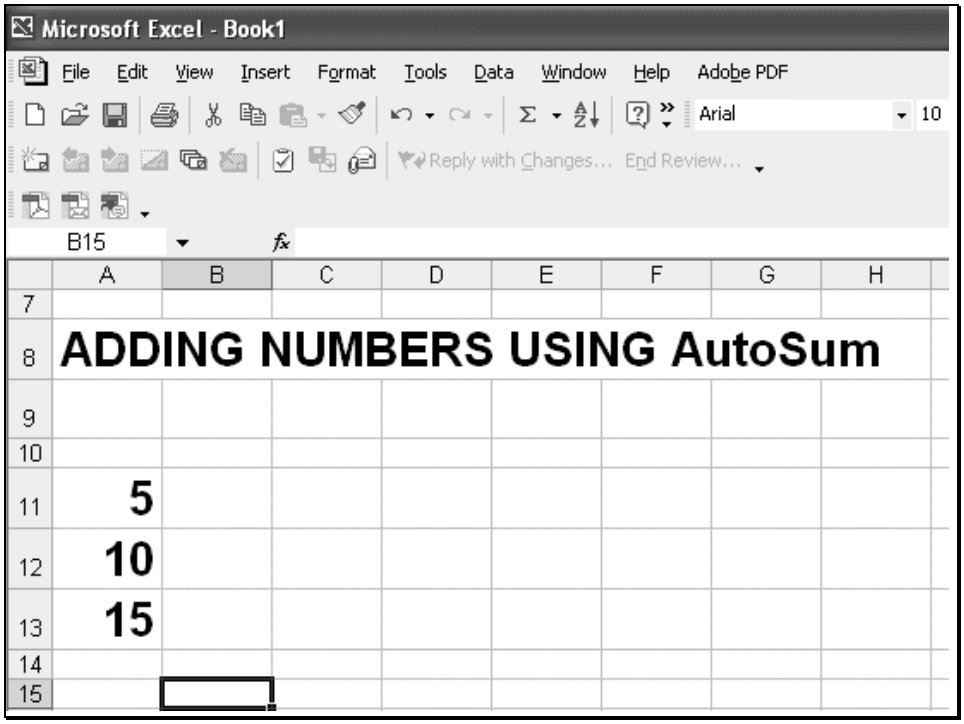
1. Add numbers in a cell

Type =5+10 in a cell
Result 15.
See Example ~~2~~



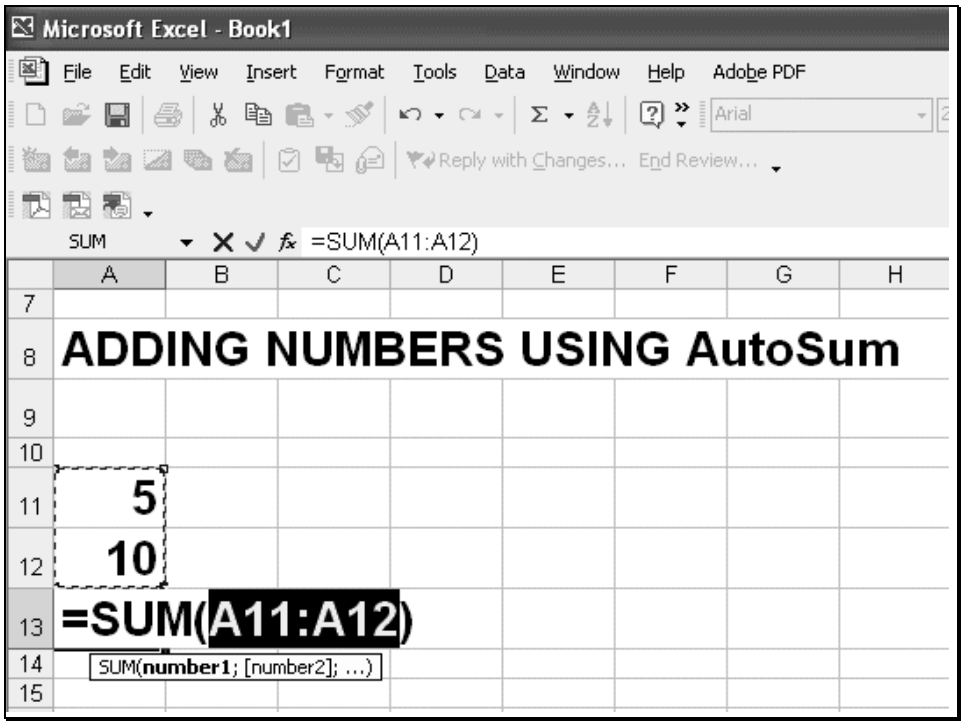
2. Add all contiguous numbers in a row or column using AutoSum

If data values are in contiguous cells of a column, click a cell next to last data value in the same column (If data values are in contiguous cells of a row then click a cell at right side of last data value)
Click AutoSum symbol, Σ , in tool bar
Press ENTER
This will add all the data values.
See Example



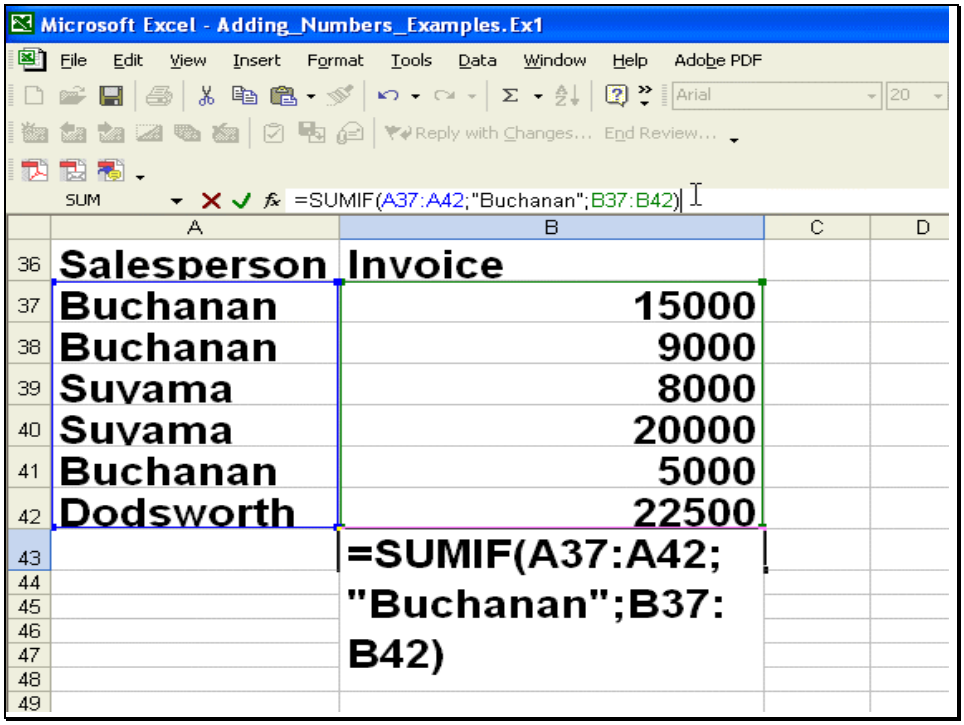
3. Add noncontiguous numbers

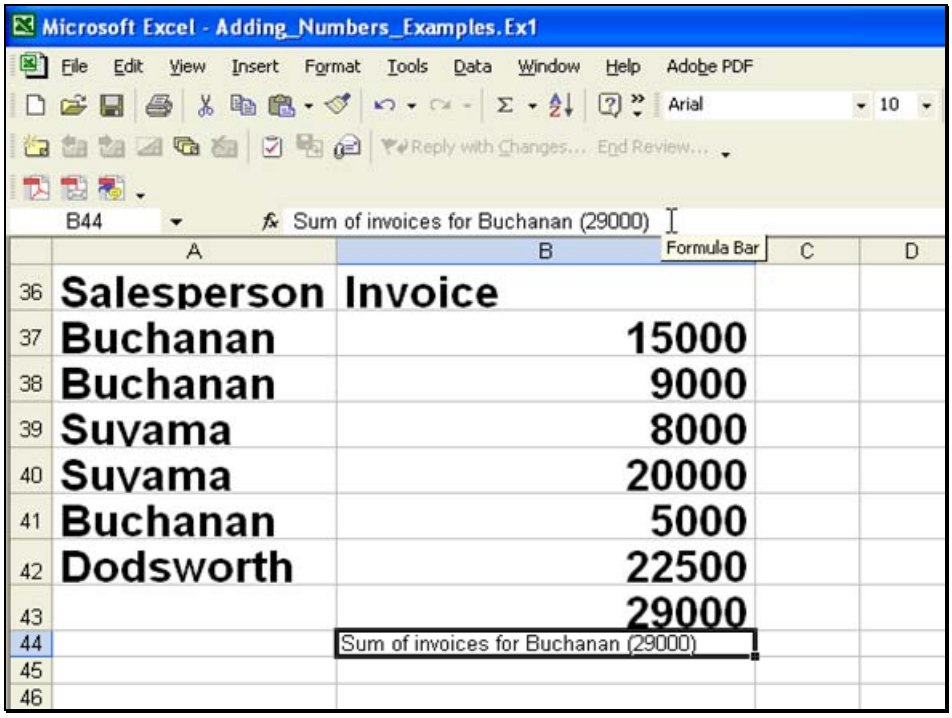
Use the SUM function See Example



4. Add numbers based on one condition

Use the SUMIF function to create a total value for one range, based on a value in another range.





5. Add numbers based on multiple conditions

Use the IF and SUM functions to do this task

6. Add numbers based on criteria stored in a separate range

Use the DSUM function to do this task

DSUM

Adds the numbers in a column of a list or database that match conditions you specify.

Syntax

DSUM(database,field,criteria)

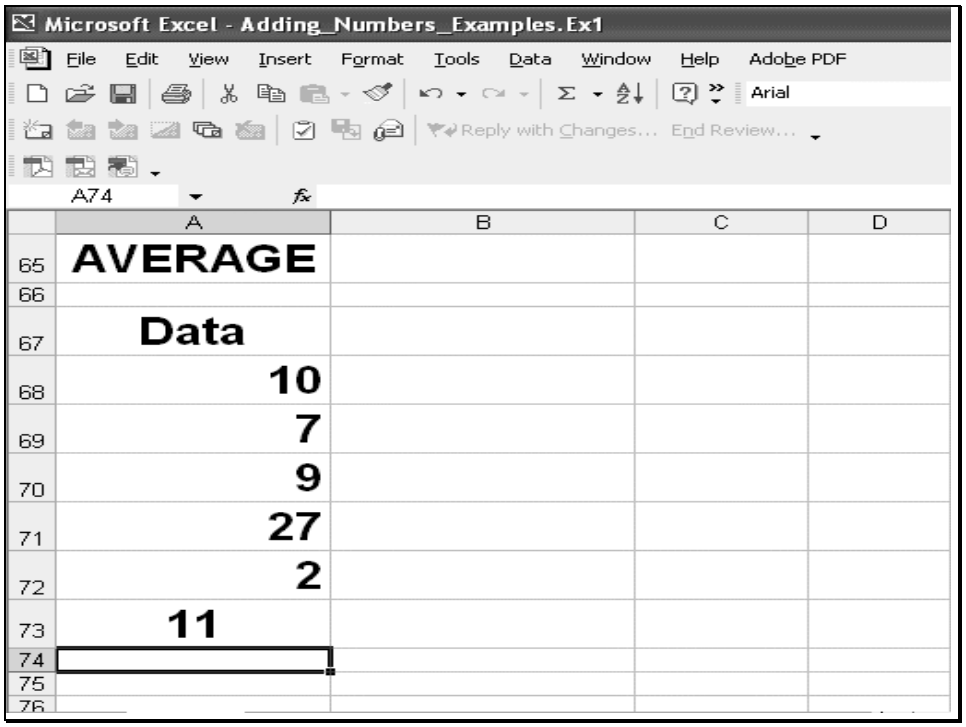
Database is the range of cells that makes up the list or database.

Field indicates which column is used in the function.

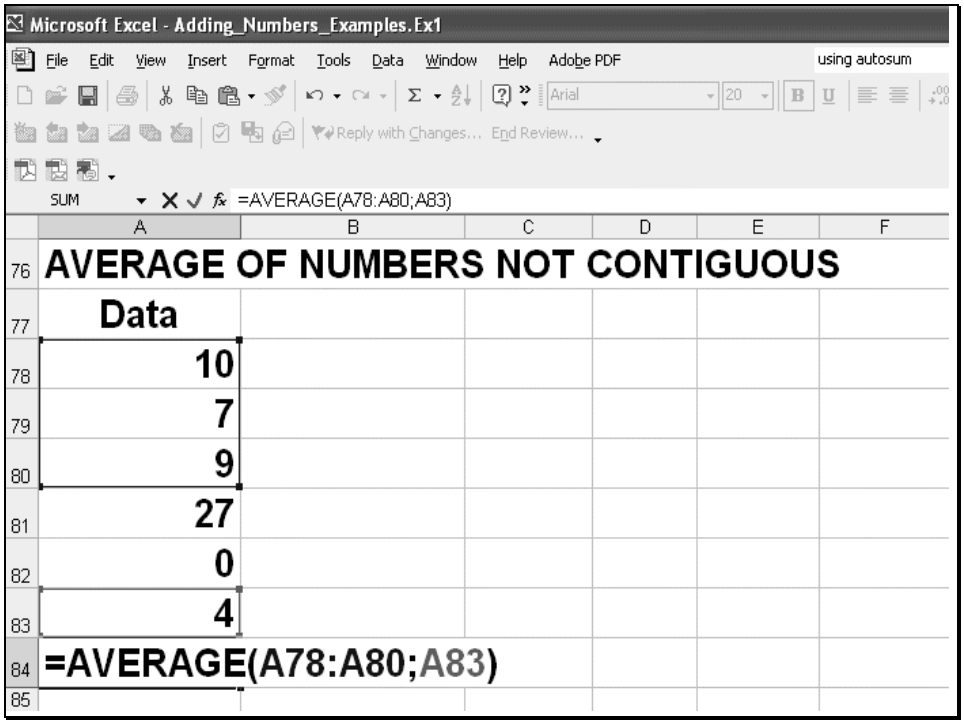
Criteria is the range of cells that contains the conditions you specify.
DSUM
EXAMPLE
=DSUM(A4:E10;"Profit";A1:F2)
The total profit from apple trees with a height between 10 and 16 (75)

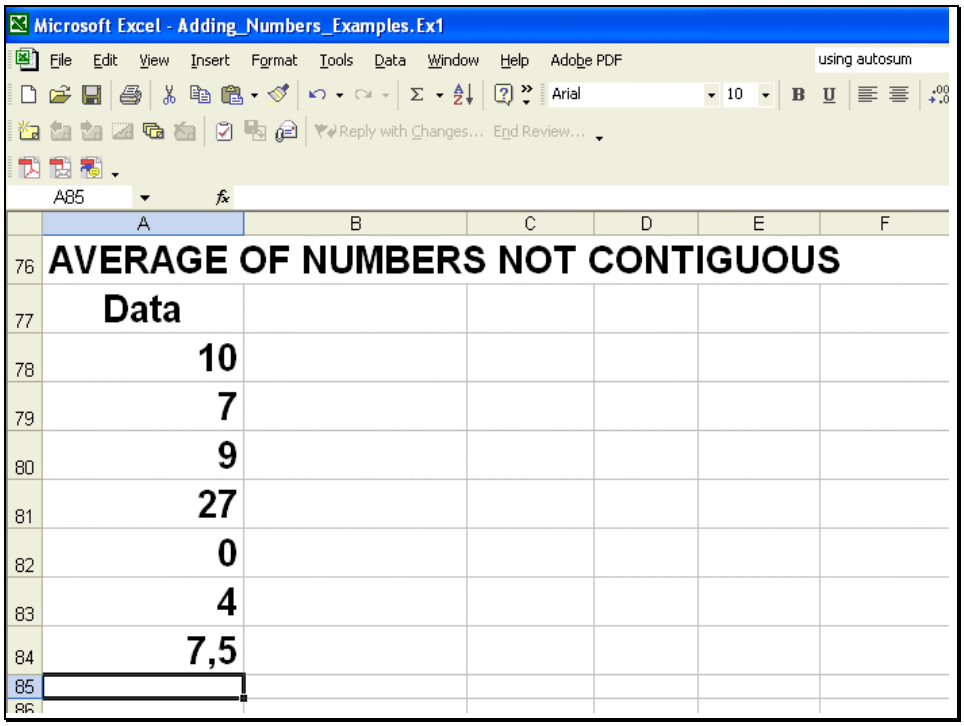
AVERAGE USING MICROSOFT EXCEL
AVERAGE
Returns the average (arithmetic mean) of the arguments.
Syntax
AVERAGE(number1,number2,...)
Number1, number2, ... are 1 to 30 numeric arguments for which you want the average.

Calculate the average of numbers in a contiguous row or column



Calculate the average of numbers not in contiguous row or column





WEIGHTED AVERAGE:

Weighted average is one type of arithmetic mean of a set of data, in which some elements of the set carry more importance (weight) than others.

If $\{x_1, x_2, x_3, ...x_n\}$ is a set of n number of data and $\{w_1, w_2, w_3, ...w_n\}$ are corresponding weights of the data then

Weighted average = $(x_1)(w_1) + (x_2)(w_2) + (x_3)(w_3) + ...+ (x_n)(w_n)$

Be careful about one thing that the weights should be in fraction.

Grades are often computed using a weighted average. Suppose the weightage of homework is 10%, quizzes 20%, and tests 70%.

Here weights of homework, quizzes, tests are already in fraction i-e10% = 0.1, 20% = 0.2, 70% = 0.7 respectively.

If Ahmad has a homework grade of 92, a quiz grade of 68, and a test grade of 81then

Ahmad's overall grade = (0.10)(92) + (0.20)(68) + (0.70)(81)

= 79.5

Let us see another example

Grade of Labor	Labor hours per unit of labor	Hourly wages (Rs)
Skilled	6	300
Semiskilled	3	200
Unskilled	1	100

Here weights (Labor hours per unit of labor) are not in fraction. So first we convert them to fraction. Total labor hours =6 + 3 + 1 =10

Grade of labor	Labor hours per unit of labor (in fraction)
Skilled	6/10 = 0.6
Semiskilled	3/10 = 0.3
Unskilled	1/10 = 0.1

Weighted average = (0.6)(300) + (0.3)(200) + (0.1)(100)

= 250 Rs per hour

LECTURE 5
Applications of Basic Mathematics
Part 4

OBJECTIVES

The objectives of the lecture are to learn about:

- ***Review of Lecture 4***
- ***Basic calculations of percentages, salaries and investments using Microsoft Excel***

PERCENTAGE CHANGE

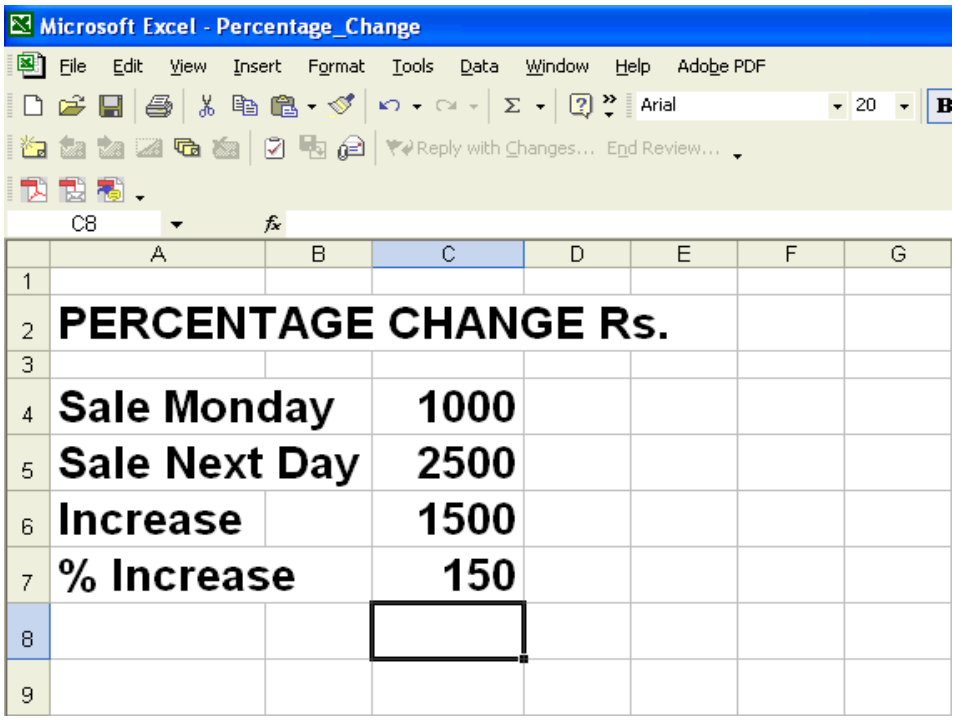
Monday's Sales were Rs.1000 and grew to Rs. 2500 the next day.
Find the percent change.

METHOD

Change = Final value – initial value
Percentage change = (Change / initial value) x 100%

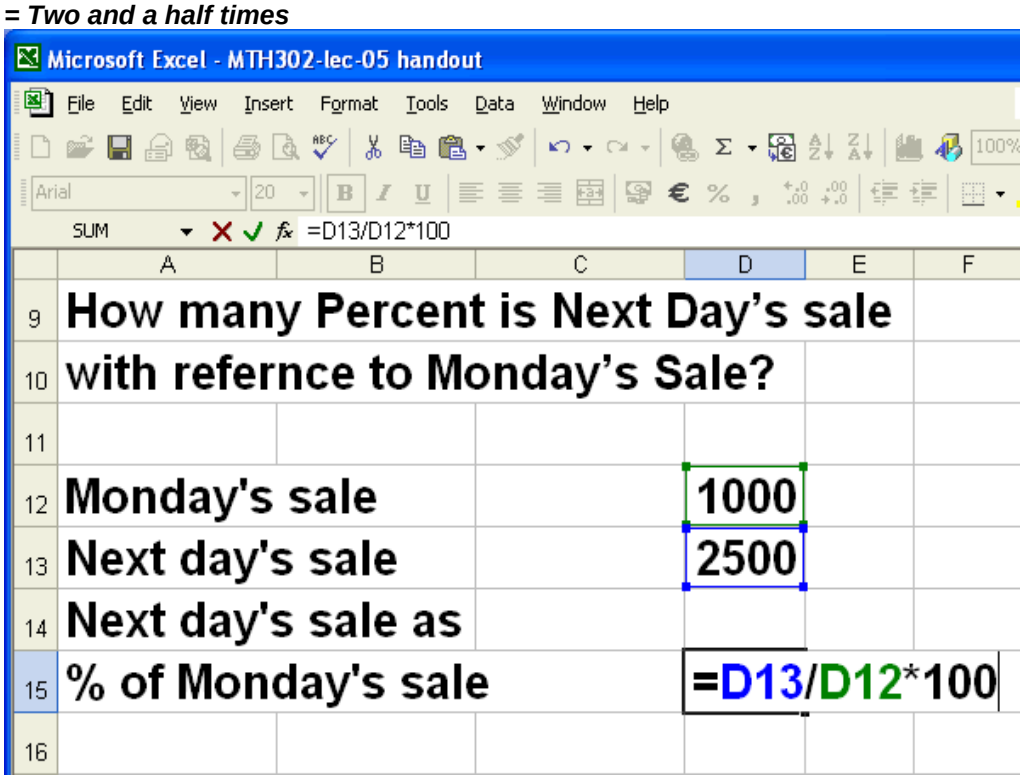
CALCULATION

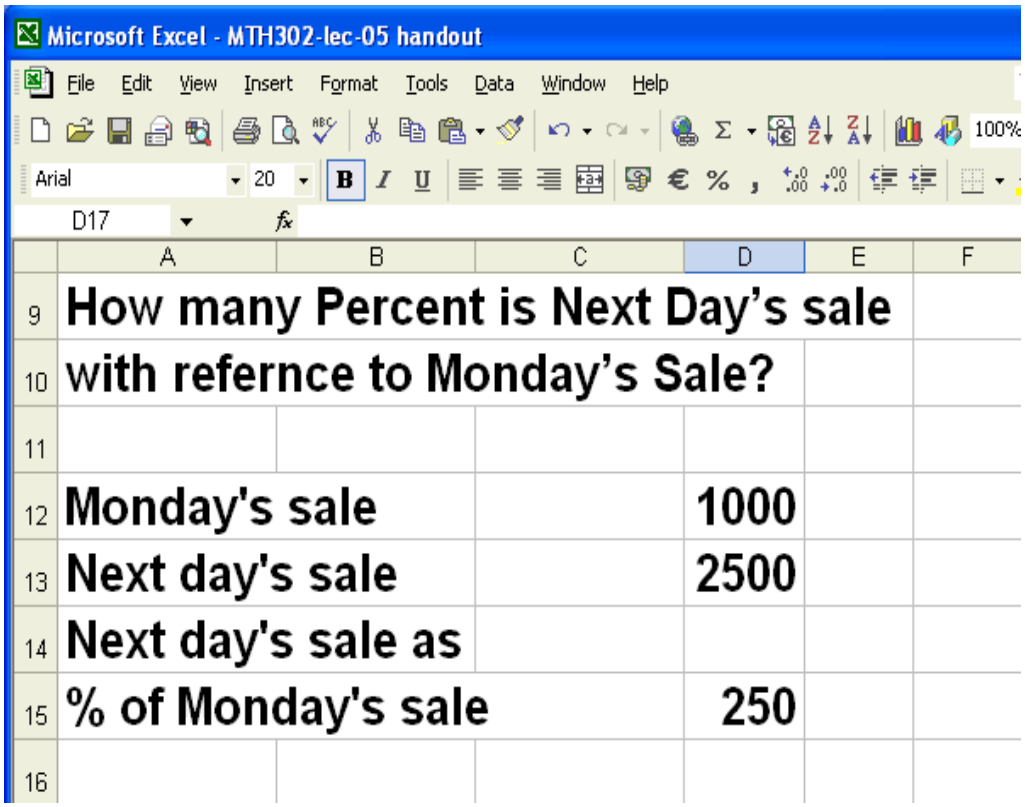
Initial value =1000
Final value = 2500
Change = 1500
% Change = (1500/1000) x 100 = 150%
The calculations using Excel are given below.
First the entries of data were made as follows:
Cell C4 = 1000
Cell C5 = 2500
In cell C6 the formula for increase was: =C5 – C4
The result was 1500.
In cell C7 the formula for percentage change was: = C6/C4*100
The result 150 is shown in the next slide.



EXAMPLE 1

How many Percent is Next Day's sale with reference to Monday's Sale?
Monday's sale= 1000
Next day's sale= 2500
Next day's sale as % = 2500/1000 x 100 = 250 %
= Two and a half times

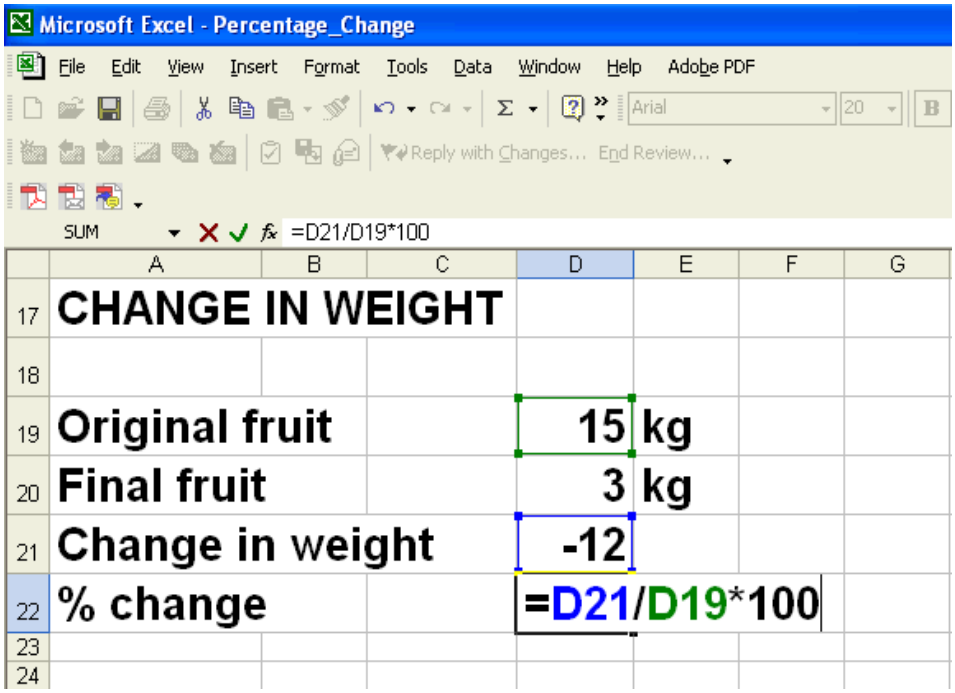


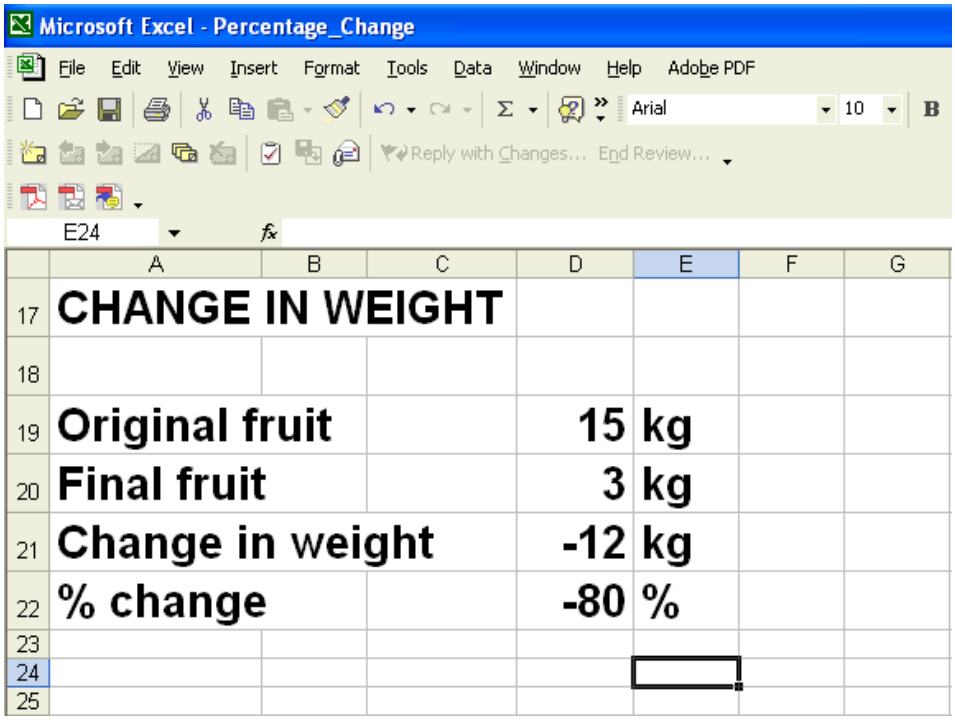


EXAMPLE 2

In the making of dried fruit, 15kg. of fresh fruit shrinks to 3 kg of dried fruit.
Find the percentage change.

Calculation
Original fruit = 15 kg
Final fruit = 3 kg
Change = 3-15 = -12
% change = - 12/15 x 100 = - 80 %
Size was reduced by 80%





Calculations in Excel were done as follows:

Data entry

Cell D19: 15
Cell D20: 3

Formulas

Formula for change in Cell D21: = D20 – D19
Formula for %change in Cell D22: = D21/D19*100

Results

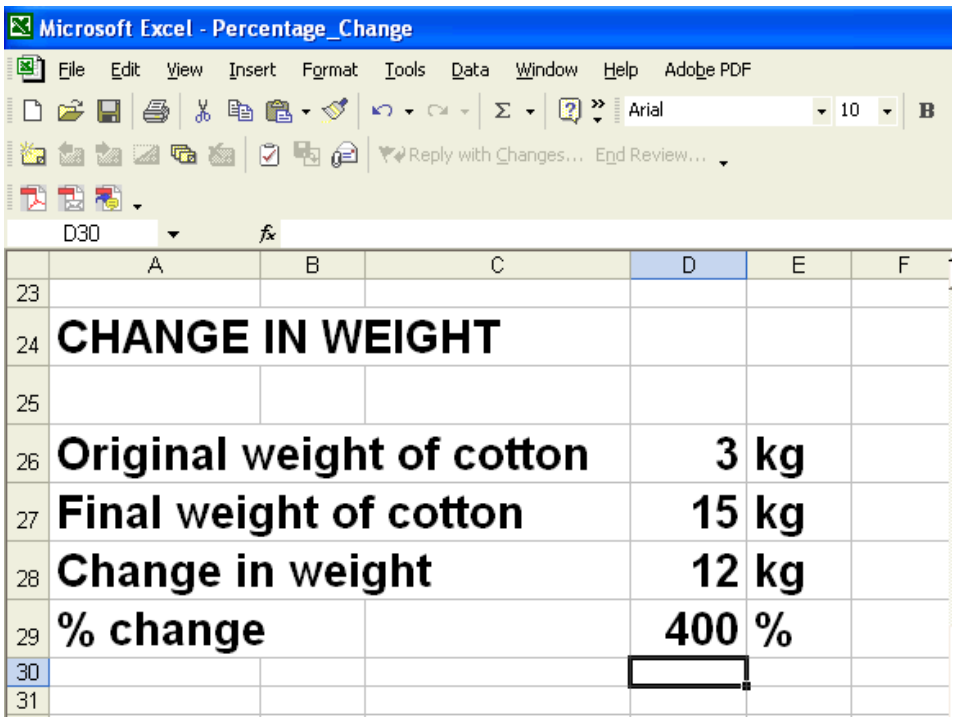
Cell D21 = -12 kg
Cell D22 = -80 %

EXAMPLE 3

After mixing with water the weight of cotton increased from 3 kg to 15 kg. Find the percentage change.

CALCULATION

Original weight = 3 kg
Final weight = 15 kg
Change = 15-3= 12
% change = 12/3 x 100 = 400 %
Weight increased by 400%



Calculations in Excel were done as follows:

Data entry

Cell D26: 3
Cell D27: 15

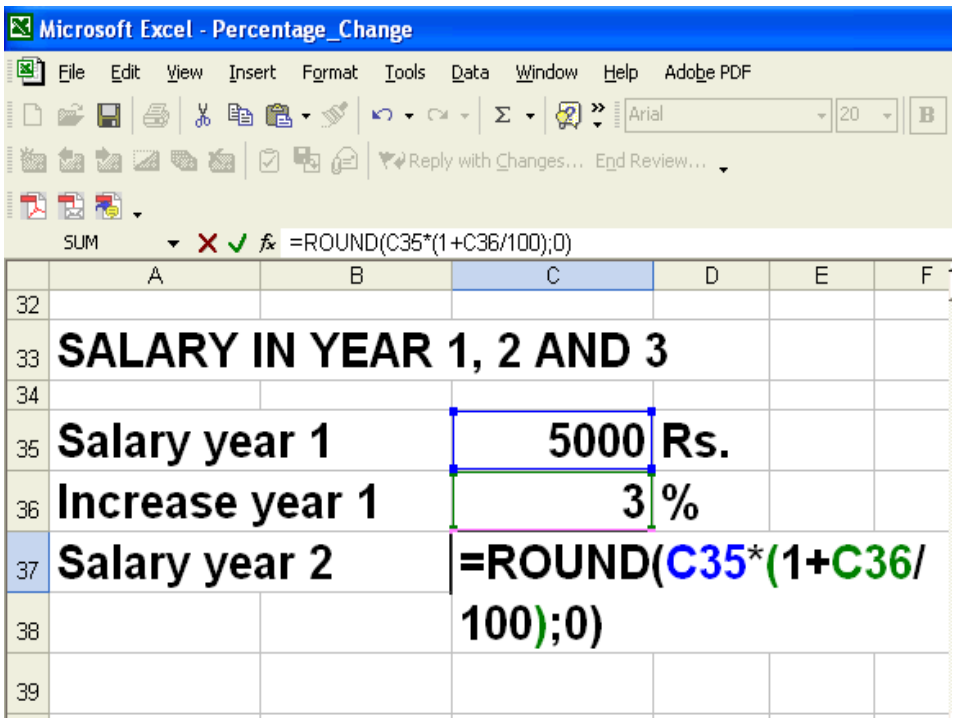
Formulas

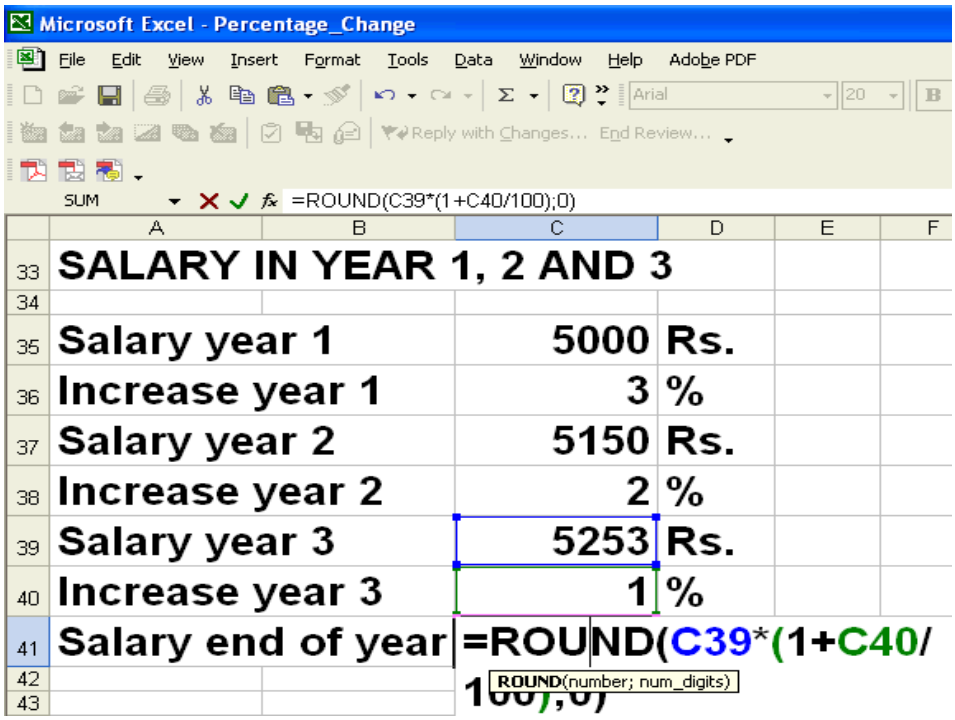
Formula for change in Cell D28: = D27 – D26
Formula for %change in Cell D29: = D28/D26*100

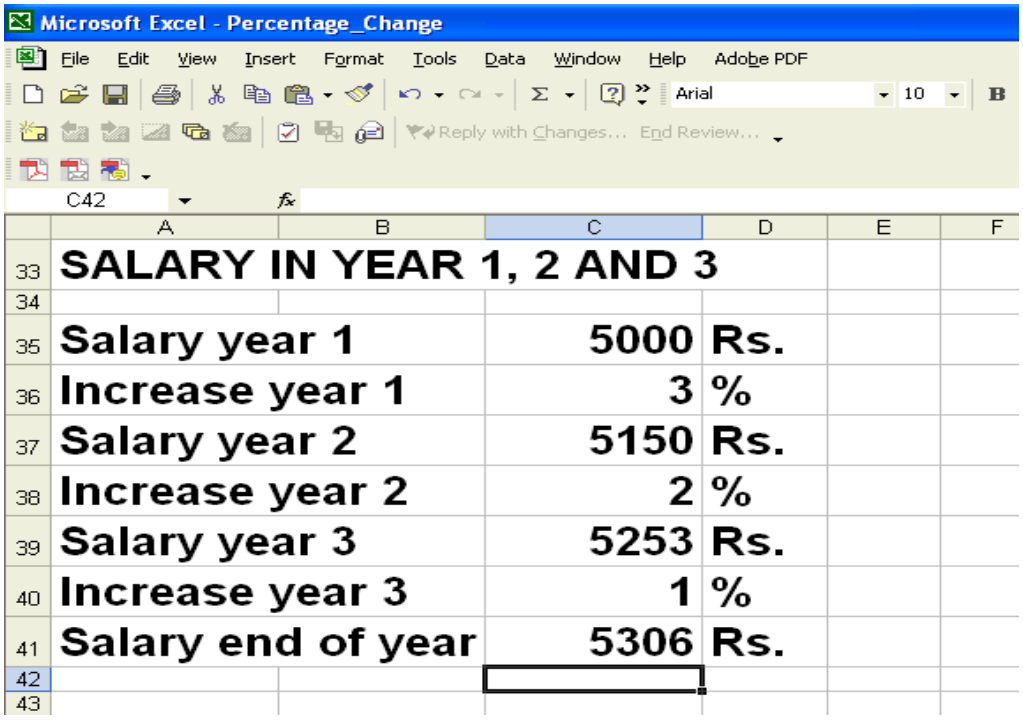
Results
Cell D28 = 12 kg
Cell D29 = 400 %

EXAMPLE 4
A union signed a three year collective agreement that provided for wage increases of 3%, 2%, and 1% in successive years
An employee is currently earning 5000 rupees per month
What will be the salary per month at the end of the term of the contract?

Calculation
= 5000(1 + 3%)(1 + 2%)(1 + 1%)
= 5000 x 1.03 x 1.02 x 1.01
= 5306 Rs.
Calculations using Excel are shown in the following slides.







Calculations in Excel were done as follows:

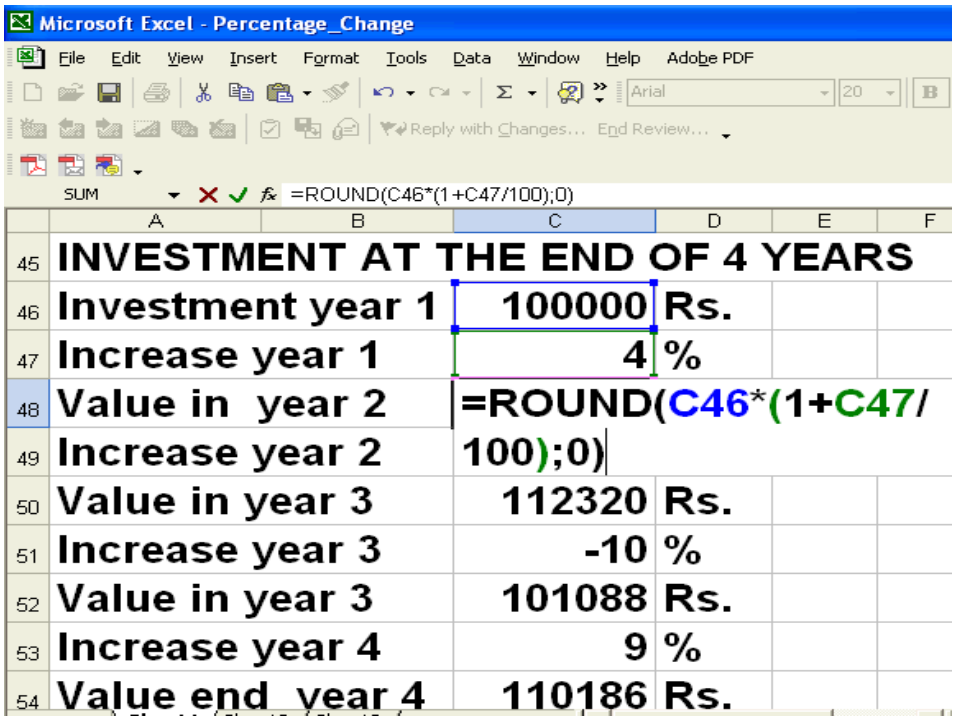
Data entry

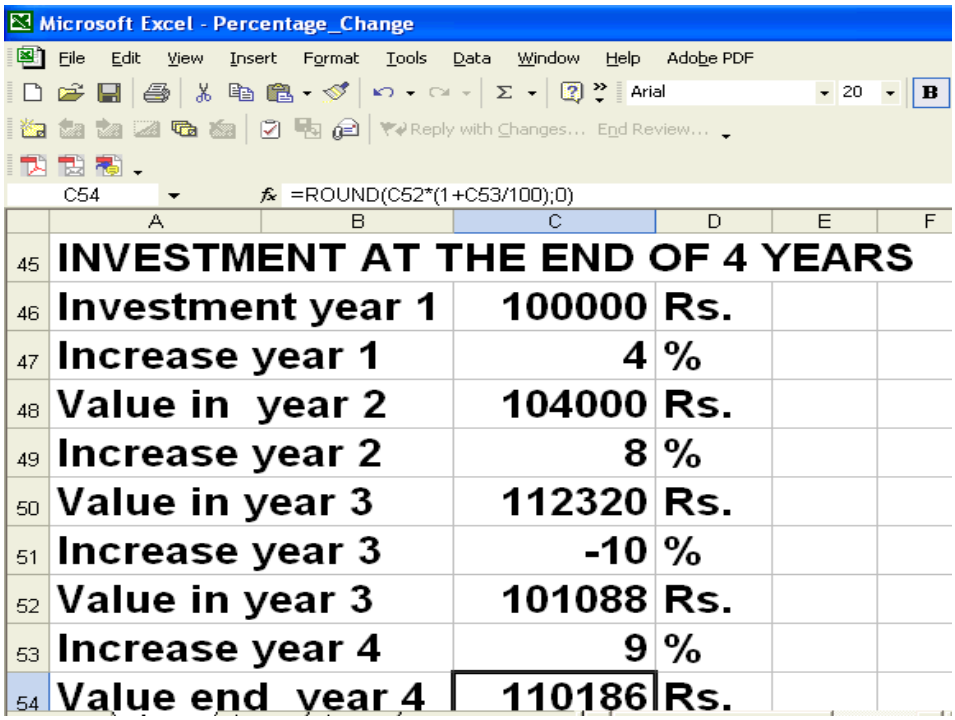
Cell C35: 5000
Cell C36: 3
Cell C38: 2
Cell C40: 1

Formulas
Formula for salary in year 2 in Cell C37: =ROUND(C35*(1+C36/100);0)
Formula for salary year 3 in Cell C39: =ROUND(C37*(1+C38/100);0)
Formula for salary at the end of year 3 in Cell C41: =ROUND(C39*(1+C39/100);0)

Results
Cell C37 = 5150 Rs.
Cell C39 = 5253 Rs.
Cell C41= 5306 Rs.

EXAMPLE 5
An investment has been made for a period of 4 years.
Rates of return for each year are 4%, 8%, -10% and 9% respectively.
If you invested Rs. 100,000 at the beginning of the term, how much will you have at the end of the last year?





	A	B	C	D	E	F
45	INVESTMENT AT THE END OF 4 YEARS					
46	Investment year 1		100000	Rs.		
47	Increase year 1		4	%		
48	Value in year 2		104000	Rs.		
49	Increase year 2		8	%		
50	Value in year 3		112320	Rs.		
51	Increase year 3		-10	%		
52	Value in year 3		101088	Rs.		
53	Increase year 4		9	%		
54	Value end year 4		110186	Rs.		

Calculations in Excel were done as follows:

Data entry
Cell C46: 100000
Cell C47: 4
Cell C49: 8
Cell C51: -10
Cell C53: 9

Formulas
Formula for value in year 2 in Cell C48: = ROUND(C46*(1+C47/100);0)
Formula for value in year 3 in Cell C50: = ROUND(C48*(1+C49/100);0)
Formula for value in year 4 in Cell C52: = ROUND(C50*(1+C51/100);0)
Formula for salary end of year 4 in Cell C54: = ROUND(C52*(1+C53/100);0)

Results
Cell C48 = 104000 Rs.
Cell C50 = 112320 Rs.
Cell C52 = 101088 Rs.
Cell C54 = 110186Rs.

LECTURE 6
Applications of Basic Mathematics
Part 5

OBJECTIVES

The objectives of the lecture are to learn about:

- ***Review Lecture 5***
- ***Discount***
- ***Simple and compound interest***
- ***Average due date, interest on drawings and calendar***

REVISION LECTURE 5

A chartered bank is **lowering** ***the*** **interest rate** ***on its loans*** **from** ***9%*** **to** ***7%.***

What will be the **percent decrease** ***in the interest rate on a given balance?***

A chartered bank is **increasing** ***the*** **interest rate** ***on its loans*** **from** ***7%*** **to** ***9%***

What will be the **percent increase** ***in the interest rate on a given balance?***

As we learnt in lecture 5, the calculation will be as follows:
Decrease in interest rate = 7-9 = -2 %
% decrease = -2/9 x 100 = -22.2 %
Increase in interest rate = 9-7 = 2 %
% increase = 2/7 x 100 = 28.6 %
The calculations in Excel are shown in the following slides:

DECREASE IN RATE
Data entry
Cell F4 = 9
Cell F5 = 7

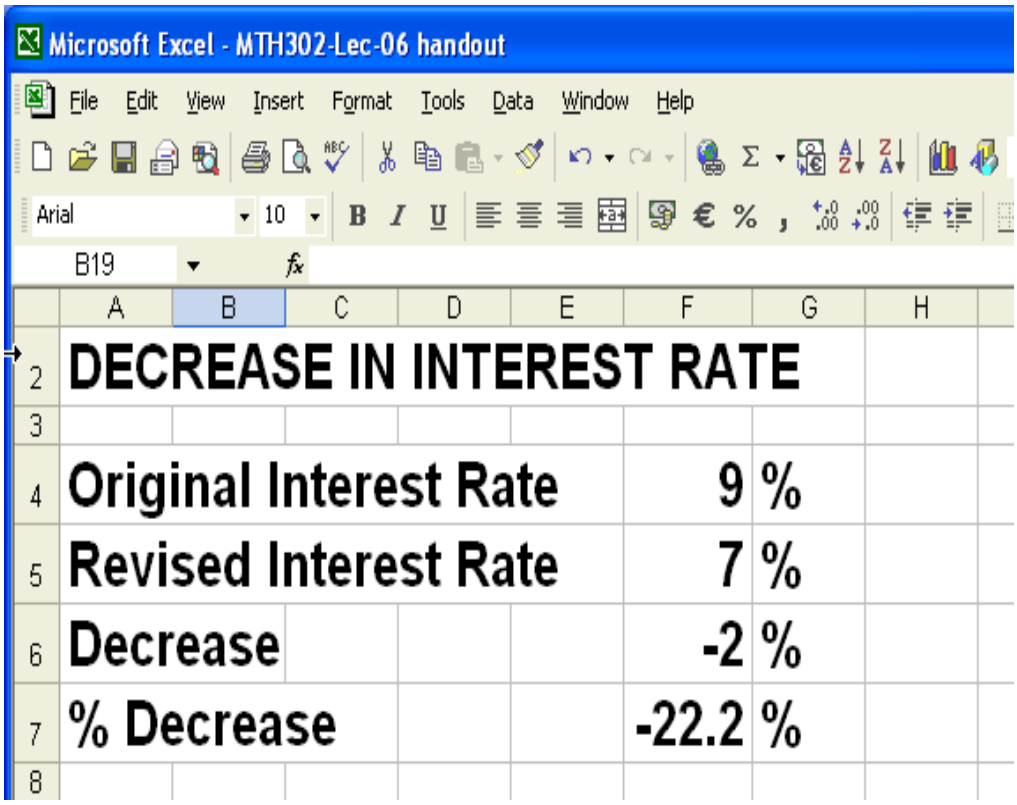
Formulas
Formula for decrease in Cell F6: = =F5-F4
Formula for % decrease in Cell F7: =F6/F4*100

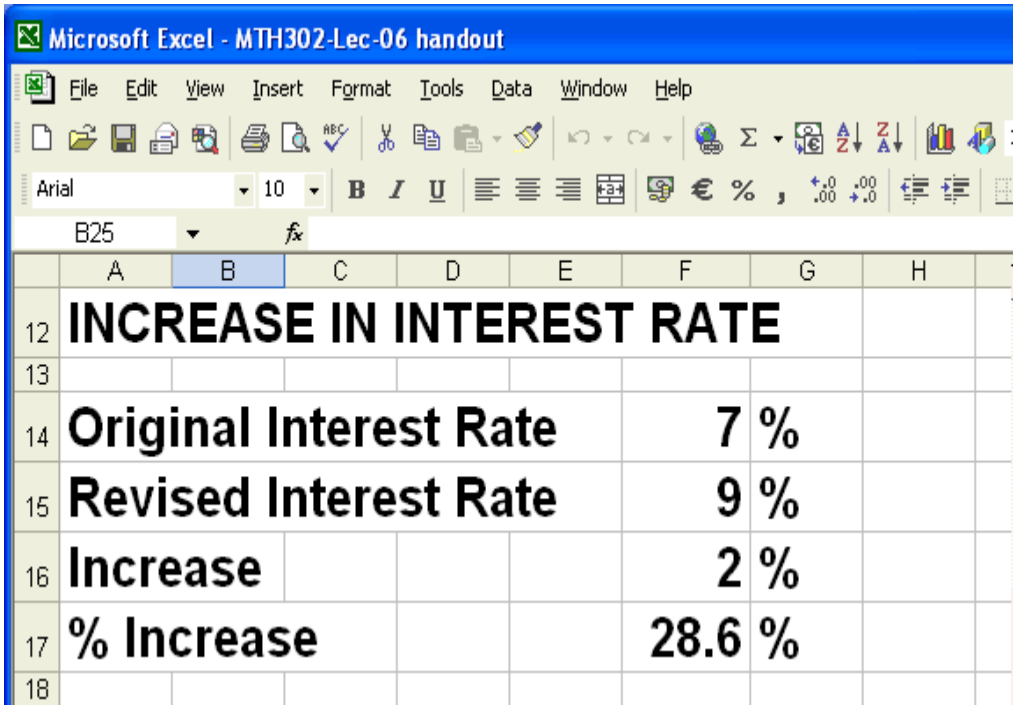
Results
Cell F6 = -2
Cell F7 = -22.2

INCREASE IN RATE
Data entry
Cell F14 = 7
Cell F15 = 9

Formulas
Formula for increase in Cell F16: =F15-F14
Formula for % increase in Cell F17: =F16/F14*100

Results
Cell F16 = 2
Cell F17= 28.6





The Definition of a Stock

Plain and simple, stock is a share in the ownership of a company. Stock represents a claim on the company's assets and earnings. As you acquire more stock, your ownership stake in the company becomes greater. Whether you say shares, equity, or stock, it all means the same thing.

Stock yield

With stocks, yield can refer to the rate of income generated from a stock in the form of regular dividends. This is often represented in percentage form, calculated as the annual dividend payments divided by the stock's current share price.

Earnings per share (EPS)

The EPS is the total profits of a company divided by the number of shares. A company with $1 billion in earnings and 200 million shares would have earnings of $5 per share.

Price-earnings ratio

A valuation ratio of a company's current share price compared to its per-share earnings.

Calculated as:

$$= \frac{\text{Market Value per Share}}{\text{Earnings per Share (EPS)}}$$

For example, if a company is currently trading at $43 a share and earnings over the last 12 months were $1.95 per share, the P/E ratio for the stock would be 22.05 ($43/$1.95).

Outstanding shares

Stock currently held by investors, including restricted shares owned by the company's officers and insiders, as well as those held by the public. Shares that have been repurchased by the company are not considered outstanding stock.

Net current asset value per share(NCAVPS)

NCAVPS is calculated by taking a company's current assets and subtracting the total liabilities, and then dividing the result by the total number of shares outstanding.

$$\text{NCAVPS} = \frac{(\text{Current Assets} - \text{Total Liabilities})}{\#\,\text{Of Shares Outstanding}}$$

Current Assets
The value of all assets that are reasonably expected to be converted into cash within one year in the normal course of business. Current assets include cash, accounts receivable, inventory, marketable securities, prepaid expenses and other liquid assets that can be readily converted to cash.

Liabilities
A company's legal debts or obligations that arise during the course of business operations.

Market value
The price at which investors buy or sell a share of stock at a given time

Face value
Original cost of a share of stock which is shown on the certificate. Also referred to as "par value." Face value is usually a very small amount that bears no relationship to its market price.

Dividend

Usually, a company distributes a part of the profit it earns as dividend.

For example: A company may have earned a profit of Rs 1 crore in 2003-04. It keeps half that amount within the company. This will be utilised on buying new machinery or more raw materials or even to reduce its borrowing from the bank. It distributes the other half as dividend.

Assume that the capital of this company is divided into 10,000 shares. That would mean half the profit -- ie Rs 50 lakh (Rs 5 million) -- would be divided by 10,000 shares; each share would earn Rs 500. The dividend would then be Rs 500 per share. If you own 100 shares of the company, you will get a cheque of Rs 50,000 (100 shares x Rs 500) from the company.

Sometimes, the dividend is given as a percentage -- i e the company says it has declared a dividend of 50 percent. It's important to remember that this dividend is a percentage of the share's face value. This means, if the face value of your share is Rs 10, a 50 percent dividend will mean a dividend of Rs 5 per share

BUYING SHARES
If you buy 100 shares at Rs. 62.50 per share with a 2% commission, calculate your total cost.

Calculation
100 * Rs. 62.50 = Rs. 6,250
0.02 * Rs. 6,250 = 125
Total = Rs. 6,375

RETURN ON INVESTMENT
Suppose you bought 100 shares at Rs. 52.25 and sold them after 1 year at Rs. 68. With a 1% commission rate of buying and selling the stock and 10 % dividend per share is due on these shares. Face value of each share is 10Rs. What is your return on investment?

Bought
100 shares at Rs. 52.25 = 5,225.00
Commission at 1% = 52.25
Total Costs =5,225 + 52.25 = 5,277.25

Sold
100 shares at Rs. 68 = 6,800.00
Commission at 1% = - 68.00
Total Sale = 6,800 – 68 = 6,732.00

Gain
Net receipts = 6,732.00
Total cost = - 5,277.25
Net Gain = 6,732 – 5,277.25 =1,454.75
Dividends (100*10/10) = 100.00
Total Gain = 1,454.75 + 100 = 1,554.75
Return on investment = 1,554.75/5,277.25*100
= 29.46 %

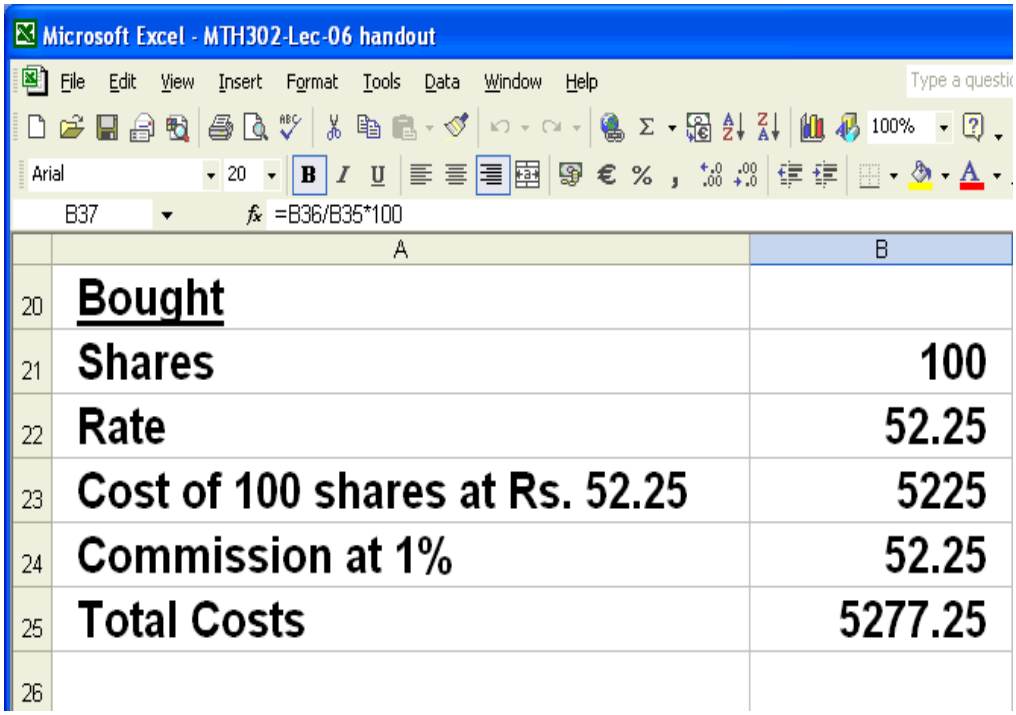
The calculations using Excel were made as follows:

BOUGHT
Data entry
Cell B21: 100
Cell B22: 52.25

Formulas

Formula for Cost of 100 shares at Rs. 52.25 in Cell B23: =B21*B22
Formula for Commission at 1% in Cell B24: =B23*0.01
Formula for Total Costs in Cell B25: =B23+B24

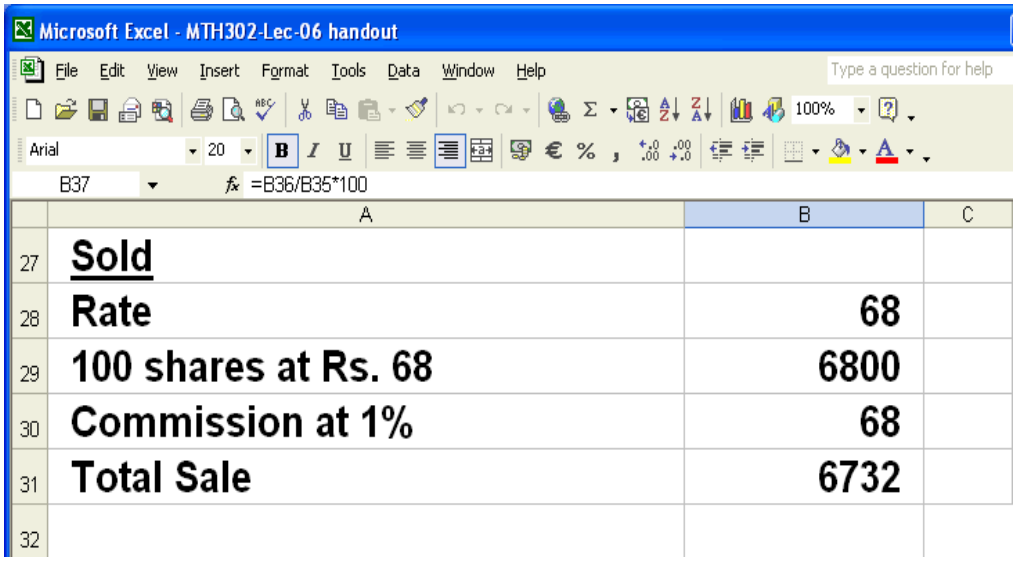
Results
Cell B23 = 5225
Cell B24 = 52.25
Cell B25 = 5277.25



SOLD
Data entry
Cell B28: 68

Formulas
Formula for sale of 100 shares at Rs. 68 in Cell B29: =B21*B28
Formula for Commission at 1% in Cell B30: =B29*0.01
Formula for Total Sale in Cell B31: =B29-B30

Results
Cell B29 = 6800
Cell B30 = 68
Cell B31 = 6732



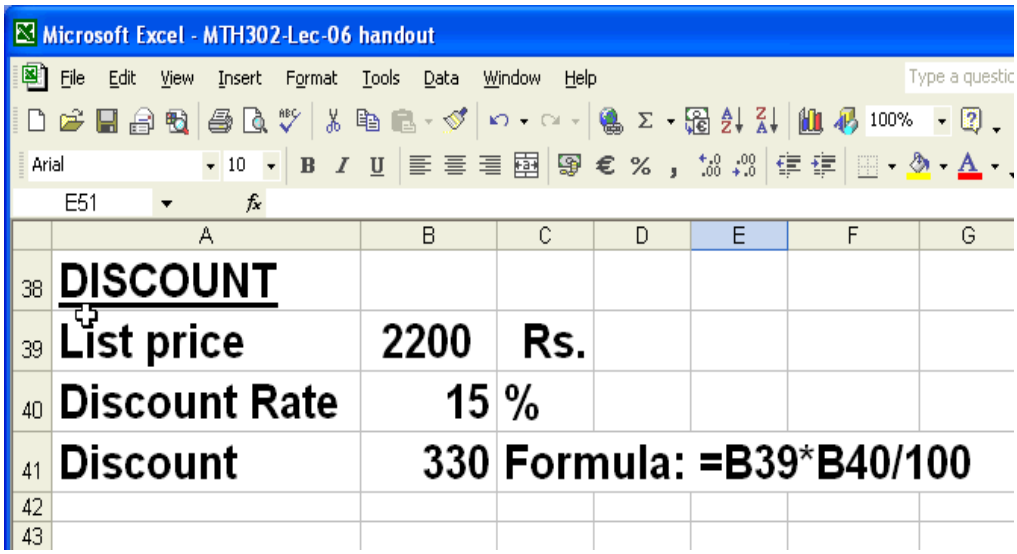
	A	B	C
27	**Sold**		
28	**Rate**	**68**	
29	**100 shares at Rs. 68**	**6800**	
30	**Commission at 1%**	**68**	
31	**Total Sale**	**6732**	
32			

GAIN
Formulas
Formula for Net receipts in Cell B34: =B31
Formula for Total cost in Cell B35: =B25
Formula for Net Gain in Cell B36: =B31-B25
Formula for % Gain in Cell B37: =B36/B35*100

Results
Cell B34 = 6732
Cell B35 = 5277.25
Cell B36 = 1454.75
Cell B37 = 27.57

DISCOUNT
Discount is Rebate or reduction in price.
Discount is expressed as % of list price.

Example
List price = 2200
Discount Rate = 15%
Discount?
= 2200 x 0.15= 330
Calculation using Excel along with formula is given in the following slide:



NET COST PRICE

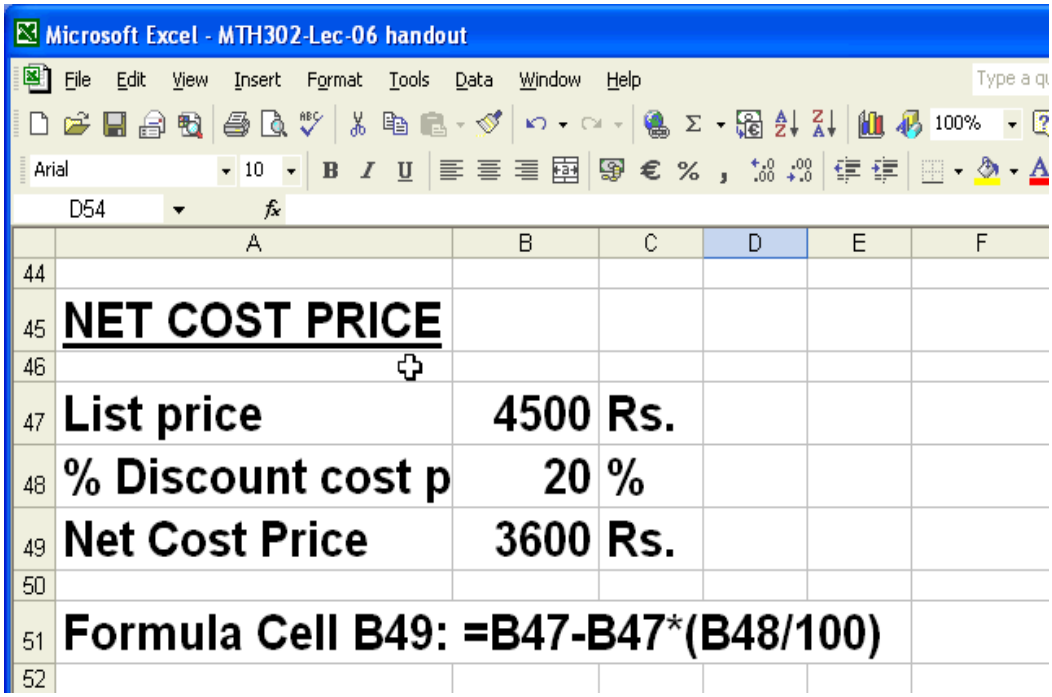
Net Cost Price = List price - Discount

Example

List price = 4,500 Rs.
Discount = 20 %
Net cost price?

Net cost price = 4,500 – 20 % of 4,500
= 4,500 – 0.2 x4,500
=4,500 – 900
= 3,600 Rs.

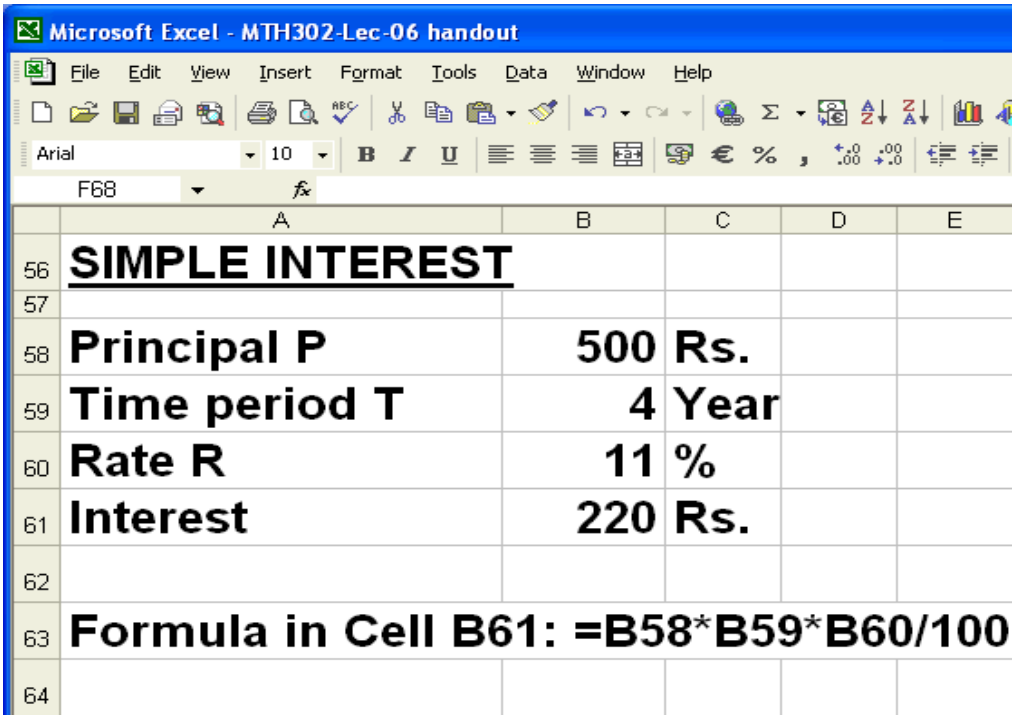
Calculation using Excel along with formula is given in the following slide:



SIMPLE INTEREST

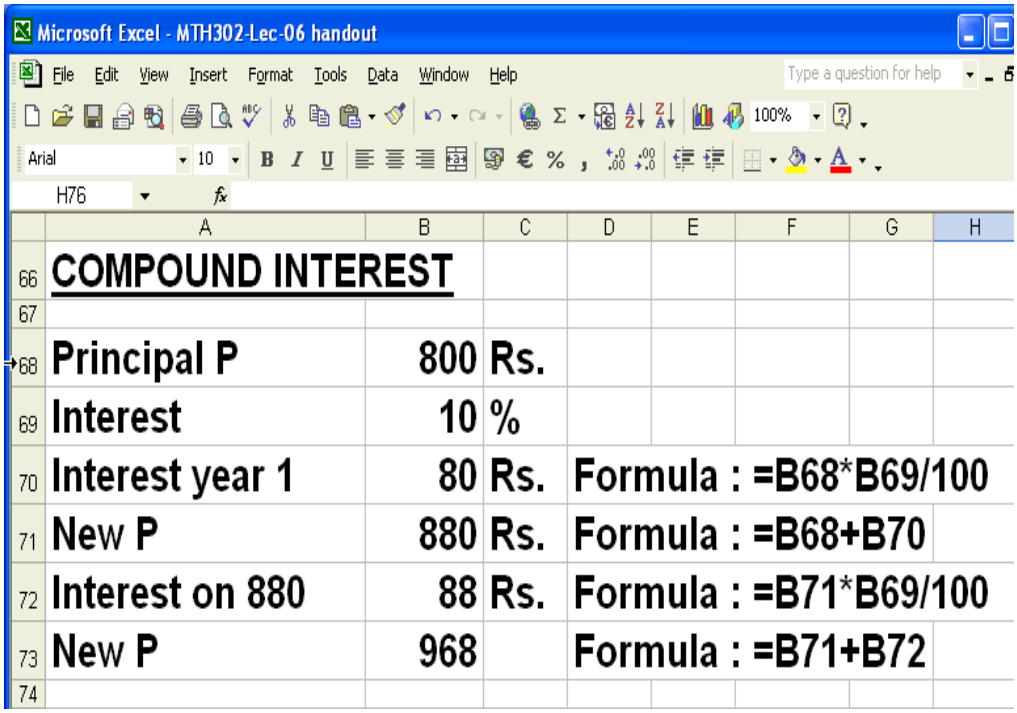
P = Principal
R = Rate of interest ~~percent~~ per annum
T = Time in years
I = Simple interest
then
I = P. R. T / 100
Thus total amount A to be paid at the end of T years = P + I

Example
P = Rs. 500
T = 4 years
R =11%
Find simple interest
I = P x T x R /100
= 500 x 4 x 11/100
= Rs. 220
Calculation using Excel along with formula is given in the following slide:



COMPOUND INTEREST
Compound Interest also attracts interest.

Example
P = 800
Interest year 1= 0.1 x 800= 80
New P = 800 + 80 = 880
Interest on 880 = 0.1 X 880 = 88
New P = 880 + 88 = 968
Calculation using Excel along with formula is given in the following slide:



CoCompound Interest Formula

S = Money accrued after n years also called compound amount

P = Principal

r = Rate of interest

n = Number of periods

S = P(1 + r/100)^ n

Compound interest = S - P

Example

Calculate compound interest earned on Rs. 750 invested at 12% per annum for 8 years.

S= P(1+r/100)^8

= 750(1+12/100)^8

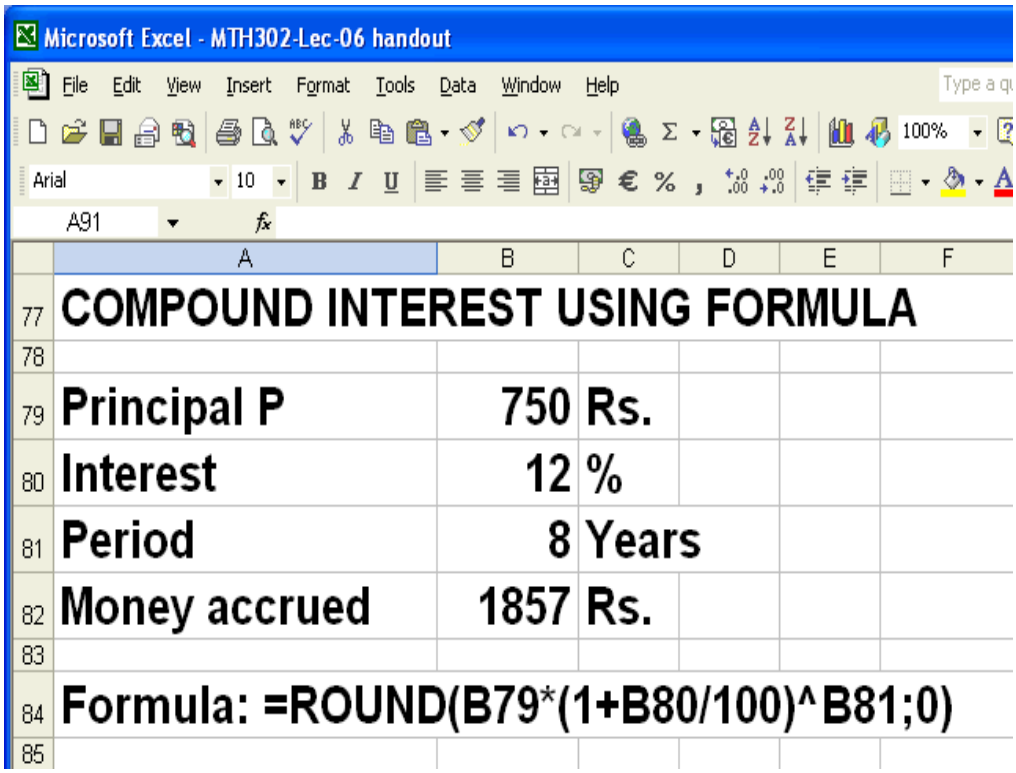
=1857 Rs

Compound interest = 1857 – 750 = 1107 Rs

3646.5= 3000(1+0.05)^n

3646.5/3000 = (1+0.05)^n

Calculation using Excel along with formula is given in the following slide



LECTURE 7
Applications of Basic Mathematics

OBJECTIVES

The objectives of the lecture are to learn about:

- Scope of Module 2
- Review of lecture 6
- Annuity
- Accumulated value
- Accumulation Factor
- Discount Factor
- Discounted value
- Algebraic operations
- Exponents
- Solving Linear equations

Module 2

Module 2 covers the following lectures:

- Linear Equations (Lectures 7)
- Investments (Lectures 8)
- Matrices (Lecture 9)
- Ratios & Proportions and Index Numbers (Lecture 10)

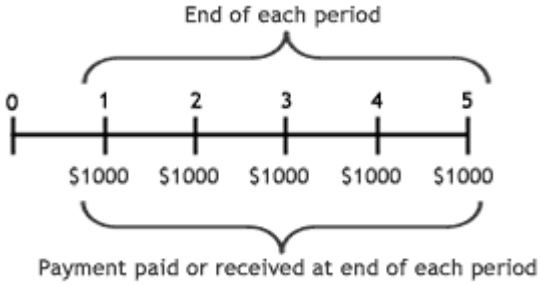
Annuity

It some point in your life you may have had to make a series of fixed payments over a period of time - such as rent or car payments - or have received a series of payments over a period of time, such as bond coupons. These are called annuities.
Annuities are essentially series of fixed payments required from you or paid to you at a specified frequency over the course of a fixed period of time.
An annuity is a type of investment that can provide a steady stream of income over a long period of time. For this reason, annuities are typically used to build retirement income, although they can also be a tool to save for a child's education, create a trust fund, or provide for a surviving spouse or heirs.
The most common payment frequencies are yearly (once a year), semi-annually (twice a year), quarterly (four times a year) and monthly (once a month).

Calculating the Future Value or accumulated value of an Annuity

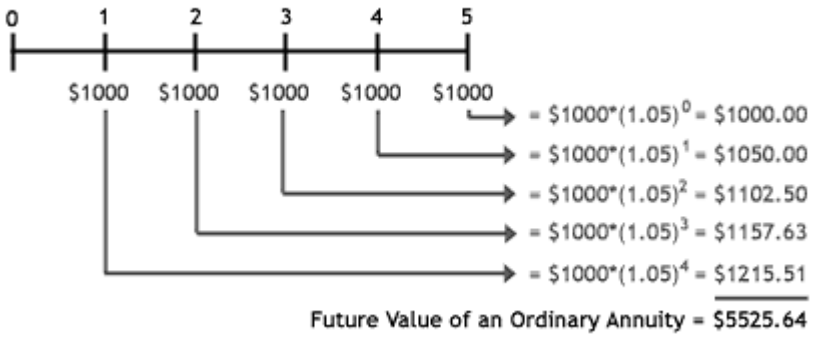
If you know how much you can invest per period for a certain time period, the future value of an ordinary annuity formula is useful for finding out how much you would have in the future by investing at your given interest rate. If you are making payments on a loan, the future value is useful for determining the total cost of the loan.

Let's now run through Example 1. Consider the following annuity cash flow schedule:



In order to calculate the future value of the annuity, we have to calculate the future value of each

cash flow. Let's assume that you are receiving $1,000 every year for the next five years, and you invested each payment at 5%. The following diagram shows how much you would have at the end of the five-year period:



Since we have to add the future value of each payment, you may have noticed that, if you have an annuity with many cash flows, it would take a long time to calculate all the future values and then add them together. Fortunately, mathematics provides a formula that serves as a short cut for finding the accumulated value of all cash flows received from an annuity:

$$FV_{\text{Ordinary Annuity}} = C * \left[\frac{(1+i)^n - 1}{i}\right]$$

C=Payment per period or amount of annuity
i = interest rate
n = number of payments

$((1 + i)^n - 1) / i)$ is called accumulation factor for n periods.

Accumulated value of n period = payment per period × accumulation factor for n periods

If we were to use the above formula for Example 1 above, this is the result:

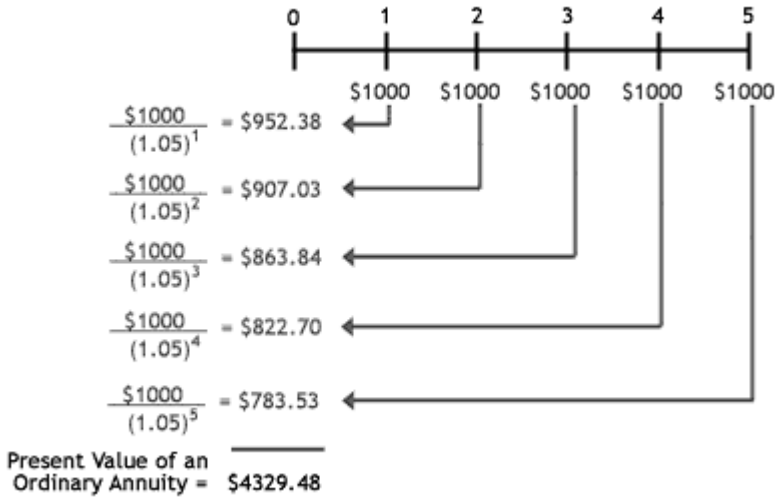
$$FV_{\text{Ordinary Annuity}} = \$1000 * \left[\frac{(1+0.05)^5 - 1}{0.05}\right]$$

=$1000*[5.53]
=$5525.63

Note that the $0.01 difference between $5,525.64 and $5,525.63 is due to a rounding error in the first calculation. Each of the values of the first calculation must be rounded to the nearest penny - the more you have to round numbers in a calculation the more likely rounding errors will occur. So, the above formula not only provides a short-cut to finding FV of an ordinary annuity but also gives a more accurate result.

Calculating the Present Value or discounted value of an Annuity
If you would like to determine today's value of a series of future payments, you need to use the formula that calculates the present value of an ordinary annuity.

For Example 2, we'll use the same annuity cash flow schedule as we did in Example 1. To obtain the total discounted value, we need to take the present value of each future payment and, as we did in Example 1, add the cash flows together.



Again, calculating and adding all these values will take a considerable amount of time, especially if we expect many future payments. As such, there is a mathematical shortcut we can use for PV of ordinary annuity.

$$PV_{\text{Ordinary Annuity}} = C * \left[\frac{1-(1+i)^{-n}}{i}\right]$$

C = Cash flow per period
i = interest rate
n = number of payments.
(1 – (1 + i)-n) / i is called discount factor for n periods.
Thus Discounted value of n period = payment per period × discount factor for n period
The formula provides us with the PV in a few easy steps. Here is the calculation of the annuity represented in the diagram for Example 2:

$$PV_{\text{Ordinary Annuity}} = \$1000 * \left[\frac{1-(1+0.05)^{-5}}{0.05}\right]$$

= $1000*[4.33]
= $4329.48

NOTATIONS

The following notations are used in calculations of Annuity:
R = Amount of annuity
N = Number of payments
I = Interest rater per conversion period
S = Accumulated value
A = Discounted or present worth of an annuity

ACCUMULATED VALUE

The accumulated value S of an annuity is the total payments made including the interest. The formula for Accumulated Value S is as follows:

S = r ((1+i)^n – 1)/i

Accumulation factor for n payments = ((1 + i)^n – 1) / i
It may be seen that:
Accumulated value = Payment per period x Accumulation factor for n payments

The discounted or present worth of an annuity is the value in today's rupee value. As an example if we deposit 100 rupees and get 110 rupees

(i.e. 10 % interest on 100 Rs. which is 100*10/100 = 10 Rs. so total amount is 100+10 = 110 or
simply 100(1+10/100) = 100(1+ 0.1) = 100(1.1) = 110)

after one year, the Present Worth or of 110 rupees will be 100. Here 110 will be future value of 100 at the end of year 1.

The amount 110, if invested again, can be Rs. 121 after year 2.

(i.e. 10 % of 110 is 110*10/100 = 11, so total amount is 110+11 = 121)

The present value of Rs. 121, at the end of year 2, will also be 100.

DISCOUNT FACTOR AND DISCOUNTED VALUE

When future value is converted into present worth, the rate at which the calculations are made is called Discount factor rate. In the previous example 10% was used to make the calculations. This rate is called Discount Rate. The present worth of future payments is called Discounted Value.

EXAMPLE 1. ACCUMULATION FACTOR (AF) FOR n PAYMENTS

Calculate Accumulation Factor and Accumulated value when:
rate of interest i = 4.25 %
Number of periods n = 18
Amount of Annuity R = 10,000 Rs.
Accumulation Factor AF = ((1 + 0.0425)^18-1)/0.0425 = 26.24
Accumulated Value S = 10,000x 26.24 = 260,240 Rs

EXAMPLE 2. DISCOUNTED VALUE (DV)

In the above example calculate the value of all payments at the beginning of term of annuity i-e present value or discounted value.
Discount rate = 4.25%
Number of periods = 18
Amount of annuity= 10000 Rs
Value of all payments at the beginning of term of Annuity or discounted value
= Payment per period x Discount Factor (DF)
Formula for Discount Factor = (1-1/(1+i)^n)/i
= (1-1/(1+0.0425)^18))/0.0425= 12.4059

Discounted value = 10000 × 12.4059 = 124059 Rs

EXAMPLE 3. DISCOUNTED VALUE (DV)

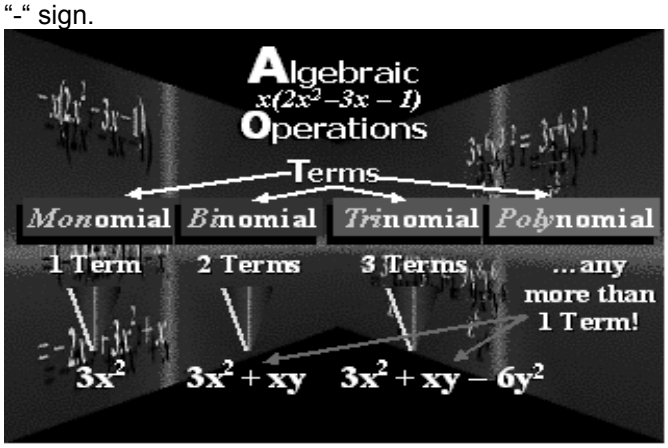
How much money deposited now will provide payments of Rs. 2000 at the end of each half-year for 10 years if interest is 11% compounded six-monthly.
Amount of annuity = 2000Rs
Rate of interest = i = 11% / 2 = 0.055
Number of periods = n = 10 × 2 = 20
DISCOUNTED VALUE = 2,000 x ((1-1 / (1+0.055)^20) / 0.055)
= 2,000 x11.95
=23,900.77

ALGEBRAIC OPERATIONS

Algebraic Expression indicates the mathematical operations to be carried out on a combination of NUMBERS and VARIABLES.

The components of an algebraic expression are separated by Addition and Subtraction.

In the expression $2x^2 – 3x -1$ the components $2x^2$, 3x and 1 are separated by minus "-" sign.



In algebraic expressions there are four types of terms:

- Monomial, i.e. 1 term (Example: $3x^2$)
- Binomial, i.e. 2 terms (Example: $3x^2+xy$)
- Trinomial, i.e. 3 terms (Example: $3x^2+xy-6y^2$)
- Polynomial, i.e. more than 1 term (Binomial and trinomial examples are also polynomial)

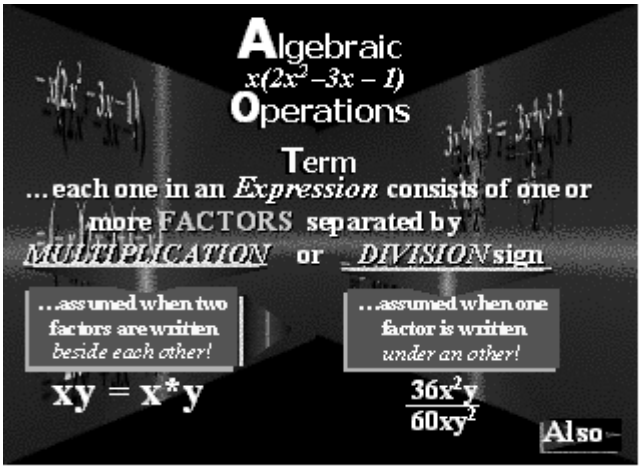
Algebraic operations in an expression consist of one or more FACTORs separated by MULTIPLICATION or DIVISION sign.

Multiplication is assumed when two factors are written beside each other.

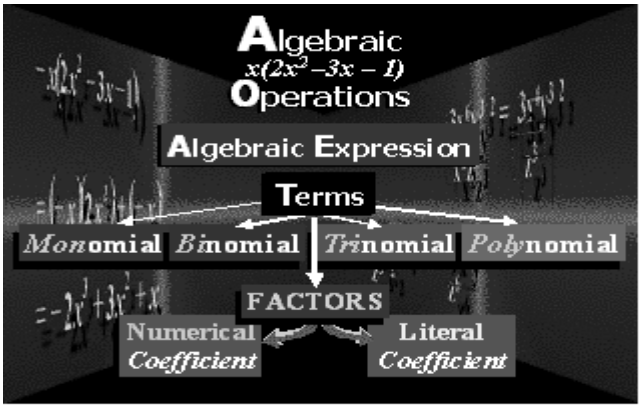
Example: xy = x*y

Division is assumed when one factor is written under an other.

Example: $36x^2y / 60xy^2$



Factors can be further subdivided into NUMERICAL and LITERAL coefficients.



There are two steps for Division by a monomial.

1. Identify factors in the numerator and denominator
2. Cancel factors in the numerator and denominator

Example:

$36x^2y / 60xy^2$

36 can be factored as 3 x 12.

60 can be factored as 5 x 12

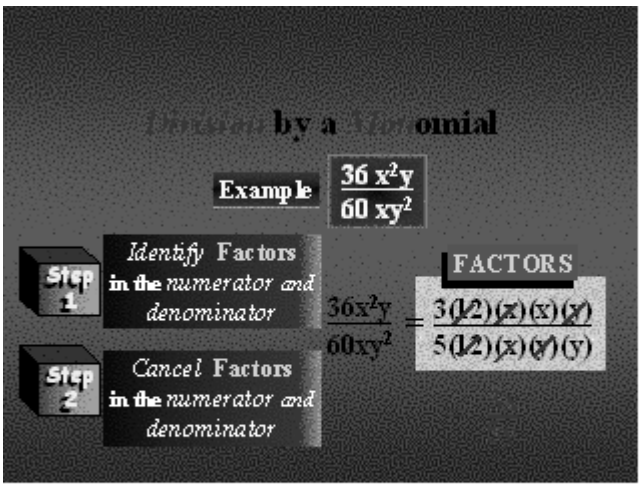
x^2y can be factored as (x)(x)(y)

xy^2 can be factored as (x)(y)(y)

Thus the expression is converted to: 3 x 12(x)(x)(y)/ 5 x 12(x)(y)(y)

12x(x)(y) in both numerator and denominator cancel each other. The result is:

3(x)/5(y)



Another example of division by a monomial is $(48a^2 – 32ab)/8a$.

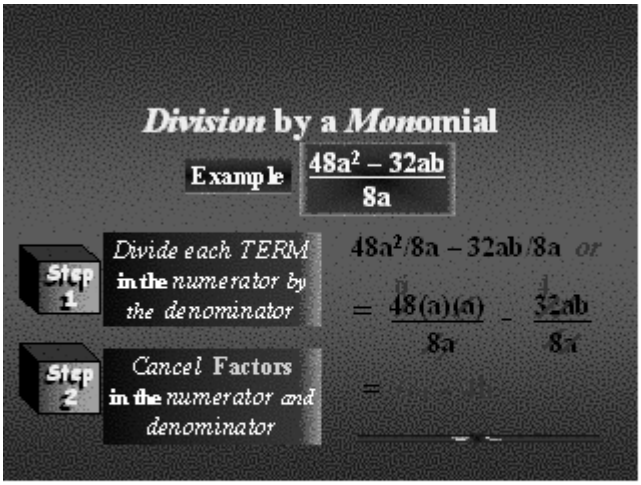
Here the steps are:

1. Divide each term in the numerator by the denominator
2. Cancel factors in the numerator and denominator

$48a^2 / 8a = 8x6(a)(a) / 8a = 6(a)$

$32(a)(b) / 8(a) = 4x8(a)(b) / 8(a) = 4(b)$

The answer is $6(a) – 4(b)$.

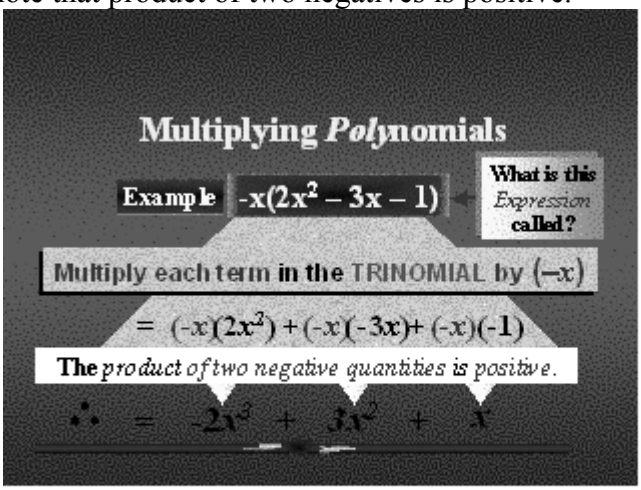


How to multiply polynomials? Look at the example $-x(2x^2 - 3x - 1)$. Here each term in the trinomial $2x^2 - 3x - 1$ is multiplied by $-x$.

$= (-x)(2x^2) + (-x)(-3x) + (-x)(-1)$

$= -2x^3 + 3x^2 + x$

Please note that product of two negatives is positive.



$(3x^6y^3 / x^2z^3)^2$

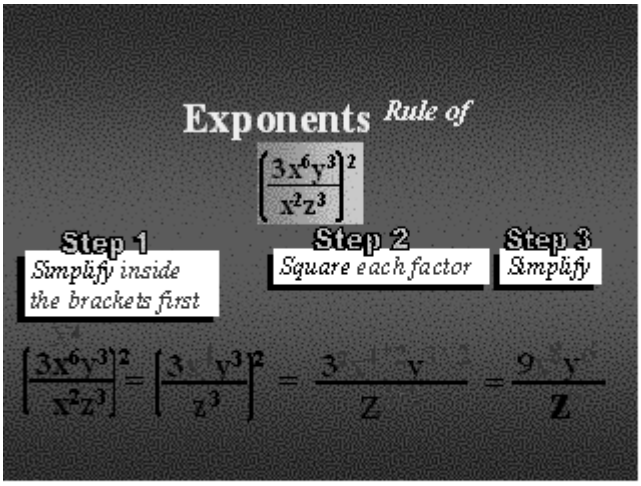
Exponent of a term means calculating some power of that term. In the example we are required to work out exponent of $3x^6y^3 / x^2z^3$ to the power of 2. The steps in this calculation are:

1. Simplify inside the brackets first.
2. Square each factor
3. Simplify

In the first step, the expression $3x^6y^3 / x^2z^3$ is first simplified to $(3x^4)(y^3)/z^3$.

In the next step we take squares. The resulting expression is:

$(3^2)(x^{4*2})(y^{3*2})/z^{3*2} = 9x^8 y^6 /z^6$

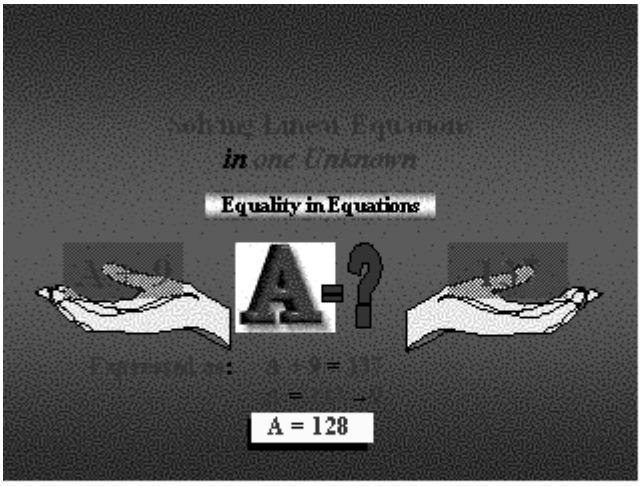


LINEAR EQUATION

If there is an expression A + 9 = 137, how do we calculate the value of A?

A = 137 – 9 = 128

As you see the term 9 was shifted to the right of the equality.



To solve linear equations:

1. Collect like terms
2. Divide both sides by numerical coefficient.

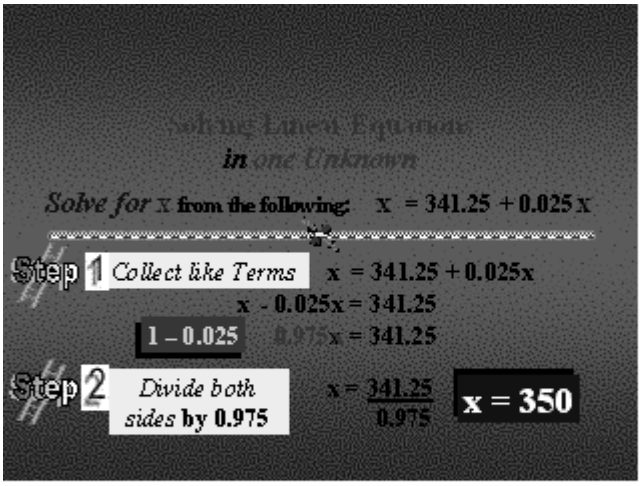
Step 1: x = 341.25 + 0.025x

x – 0.025x = 341.25

x(1-0.025) = 341.25

0.975x = 341.25

Step 2. x = 341.25/0.975 = 350


Solving Linear Equations
in one Unknown
Solve for x from the following: x = 341.25 + 0.025 x
Step 1
Collect like Terms
x = 341.25 + 0.025x
x - 0.025x = 341.25
1 – 0.025
0.975x = 341.25
Step 2
Divide both sides by 0.975
x = 341.25 / 0.975
x = 350

LECTURE 8

Compound Interest
Calculate returns from investments
Annuities
Excel Functions

OBJECTIVES

The objectives of the lecture are to learn about:

- Review of lecture 7
- Compound Interest
- Calculate returns from investments
- Annuities
- Excel Functions

CUMIPMT

Returns the cumulative interest paid on a loan between start_period and end_period. If this function is not available, and returns the #NAME? error, install and load the Analysis ToolPak add-in.
The syntax is as follows:
CUMIPMT(rate,nper,pv,start_period,end_period,type)

Rate: interest rate.
Nper: total number of payment periods
Pv: present value.
Start_period: first period in the calculation
End_period: last period in the calculation
Type: timing of the payment

Type	Timing
0 (zero)	Payment at the end of the period
1	Payment at the beginning of the period

CUMIPMT-EXAMPLE

Following is an example of CUMIPMT function. In this example, in the first case the objective is to find total interest paid in the second year of payments for periods 13 to 24. Please note there are 12 periods per year. The second case is for the first payment period.

In the first formula, the Annual interest rate 9% is cell A2 (not shown here). The Years of the loan are given in cell A3. The Present value is in cell A4. For the Start period the value 13 was entered. For the End period, the value 24 has been specified. The value of Type is 0, which means that the payment will be at the end of the period. Please note that the annual interest is first divided by 12 to arrive at monthly interest. Then the Years of the loan are multiplied by 12 to get total number of months in the Term of the loan. The answer is (-11135.23).

In the second formula, which gives Interest paid in a single payment in the first month 1 was specified as the Start period. For the End period also the value 1 was entered.This is because only 1 period is under study. All other inputs were the same. The answer is (-937.50).

Data **Description**
9% Annual interest rate
30 Years of the loan
125,000 Present value
=CUMIPMT(A2/12,A3*12,A4,13,24,0)Total interest paid in the second year of payments, periods 13 through 24 (-11135.23)
=CUMIPMT (A2/12,A3*12,A4,1,1,0)Interest paid in a single payment in the first month (-937.50)

CUMPRINC
The CUMPRINC function returns the cumulative principal paid on a loan between two periods.
The syntax is as under:

CUMPRINC(rate,nper,pv,start_period,end_period,type)

Rate: interest rate.
Nper: total number of payment periods.
Pv: present value
Start_period: period in the calculation. Payment
End_period: last period in the calculation
Type: timing of the payment (0 or 1 as above)

CUMPRINC EXAMPLE
Following is an example of CUMPRINC function. In this example, in the first case the objective is to find the total principal paid in the second year of payments, periods 13 through 24. Please note there are 12 periods per year. The second case is for the principal paid in a single payment in the first month.
In the first formula, the Interest rate per annum 9% is in cell A2 (not shown here). The Term in years (30) is given in cell A3. The Present value is in cell A4. For the Start period the value 13 was entered. For the End period, the value 24 has been specified. The value of Type is 0, which means that the payment will be at the end of the period. Please note that the interest is first divided by 12 to arrive at monthly interest. Then the years of loan are multiplied by 12 to get total number of months in the term of the loan. The answer is (-934.1071).

In the second formula, which gives the principal paid in a single payment in the first month 1 was specified as the start period. For the end period also the value 1 was entered.This is because only 1 period is under study. All other inputs were the same. The answer is (-68.27827).

EXAMPLE
Data Description
9.00% Interest rate per annum
30 Term in years
125,000 Present value
=CUMPRINC(A2/12,A3*12,A4,13,24,0)The total principal paid in the second year of payments, periods 13 through 24 (-934.1071)
=CUMPRINC(A2/12,A3*12,A4,1,1,0)The principal paid in a single payment in the first month (-68.27827)

EFFECT
Returns the effective annual interest rate. As you see there are only two inputs, namely, the nominal interest Nominal_rate and the number of compounding periods per year Npery.
EFFECT(nominal_rate,npery)
Nominal_rate: nominal interest rate
Npery: number of compounding periods per year

EFFECT-EXAMPLE

Here Nominal_rate = 5.25% in cell A2. Npery =4 in cell A3. The answer is 0.053543 or 5.3543%. You should round off the value to 2 decimals. 5.35%.
5.25% Nominal interest rate
4 Number of compounding periods per year
=EFFECT(A2,A3)
Effective interest rate with the terms above (0.053543 or 5.3543 percent)

FV
Returns the future value of an investment. There are 5 inputs, namely, Rate the interest rate, Nper number of periods, Pmt payment per period, Pv present value and Type.
FV(rate,nper,pmt,pv,type)
Rate: interest rate per period
Nper: total number of payment periods
Pmt: payment made each period.
Pv: present value, or the lump-sum amount
Type: number 0 or 1 due

FV-EXAMPLE 1
In the formula, there are 5 inputs, namely, Rate 6% in cell A2 as the interest rate, 10 as Nper number of periods in cell A3, -200 (notice the minus sign) as Pmt payment per period in cell A4, -500 (notice the minus sign) as Pv present value in cell A4 and 1 as Type in cell A6. The answer is (2581.40).

Data	Description	EXAMPLE 1
6%	Annual interest rate	
10	Number of payments	
-200	Amount of the payment	
-500	Present value	
1	Payment is due at the beginning of the period	

=FV(A2/12, A3, A4, A5, A6)
Future value of an investment with the above terms (2581.40)
=FV(rate,nper,pmt,pv,type)

FV-EXAMPLE 2

In the formula, there are 3 inputs, namely, Rate 12% in cell A2 as the interest rate, 12 as Nper number of periods in cell A3, -1000 (notice the minus sign) as Pmt payment per period in cell A4. Pv present value and Type are not specified. Both are not required as we are calculating the Future value of the investment. The answer is (12682.50).

EXAMPLE 2

12% Annual interest rate
12 Number of payments
-1000 Amount of the payment
=FV(A2/12, A3, A4)
Future value of an investment with the above terms (12,682.50)
FV(rate,nper,pmt,pv,type)

FV-EXAMPLE 3

In the formula, there are 4 inputs, namely, Rate 11% in cell A2 as the interest rate, 35 as Nper number of periods in cell A3, -2000 (notice the minus sign) as Pmt payment per period in cell A4, 1as Type in cell A5. The value of Pv was omitted by entering a blank for the value (note the double commas",,". The answer is (82846.25).

EXAMPLE 3

11% Annual interest rate
35 Number of payments
-2000 Amount of the payment
1 Payment is due at the beginning of the period
=FV(A2/12, A3, A4,, A5)
Future value of an investment with the above terms (82,846.25)

FV(rate,nper,pmt,pv,type)

FV SCHEDULE

Returns the future value of an initial principal after applying a series of compound interest rates.
FVSCHEDULE(principal, schedule)
Principal: present value
Schedule: an array of interest rates to apply

FV SCHEDULE-EXAMPLE
In this example, the Principal is 1. The compound rates {0.09, 0.11,0.1} are given within curly brackets. The answer is (1.33089).
FVSCHEDULE(principal,schedule)
=FVSCHEDULE(1,{0.09,0.11,0.1})
Future value of 1 with compound interest rates of 0.09,0.11,0.1 (1.33089)

IPMT
Returns the interest payment for an investment for a given period.
IPMT(rate,per,nper,pv,fv,type)
Rate: interest rate per period
Per: period to find the interest
Nper: total number of payment periods
Pv: present value, or the lump-sum amount
Fv: future value, or a cash balance
Type: number 0 or 1

ISPMT
Calculates the interest paid during a specific period of an investment
ISPMT(rate,per,nper,pv)
Rate: interest rate
Per: period
Nper: total number of payment periods
Pv: present value. For a loan, pv is the loan amount

NOMINAL
Returns the annual nominal interest rate.
NOMINAL(effect_rate,npery)
Effect_rate: effective interest rate
Npery: number of compounding periods per year

NPER
Returns the number of periods for an investment.
NPER(rate, pmt, pv, fv, type)
Rate: the interest rate per period.
Pmt: payment made each period
Pv: present value, or the lump-sum amount
Fv: future value, or a cash balance
Type: number 0 or 1 (due)

NPV
Returns the net present value of an investment based on a series of periodic cash flows and a discount rate. Its syntax is
NPV(rate,value1,value2,…)
Rate: is the rate of discount over the length of one period.
Valu1,value2,… are 1 to 29 arguments representing the payments and income.

PMT
Returns the periodic payment for an annuity.
PMT(rate,nper,pv,fv,type)
Rate: interest rate
Nper: total number of payments
Pv: present value
Fv: future value

Type: number 0 (zero) or 1

PPMT

Returns the payment on the principal for an investment for a given period.
PPMT(rate,per,nper,pv,fv,type)
Rate: interest rate per period.
Per: period and must be in the range 1 to nper
Nper: total number of payment periods
Pv: the present value
Fv: future value (0)
Type: the number 0 or 1 (due)

PV

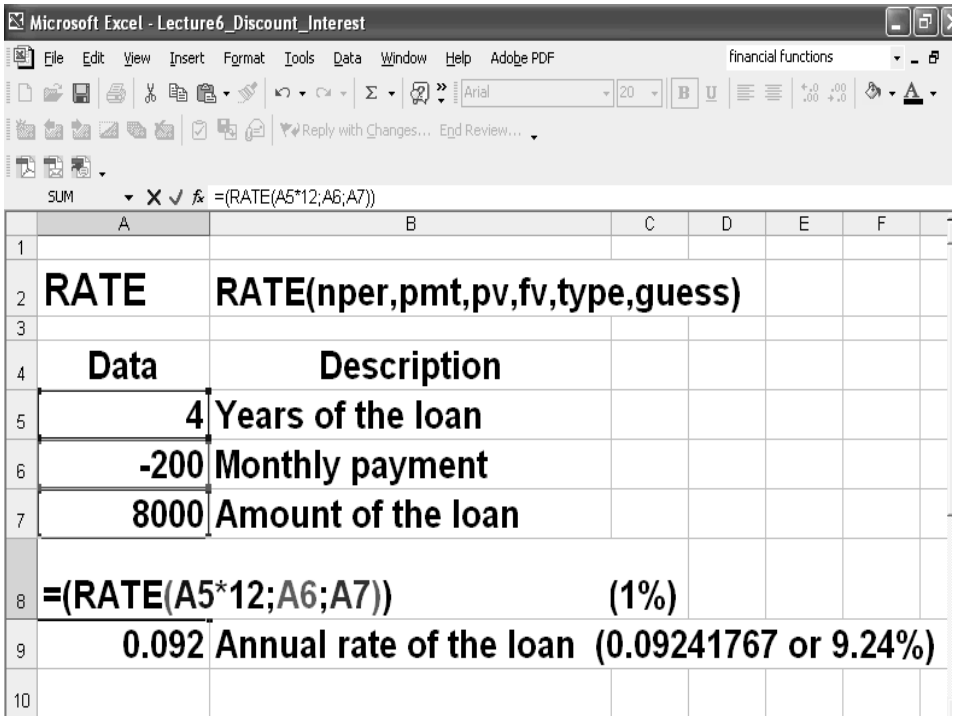
Returns the present value of an investment.
PV(rate,nper,pmt,fv,type)
Rate: interest rate per period
Nper: total number of payment periods in an annuity
Pmt: payment made each period and cannot change over the life of the annuity
Fv: future value, or a cash balance
Type: number 0 or 1 and indicates when payments are due

RATE

Returns the interest rate per period of an annuity.
RATE(nper,pmt,pv,fv,type,guess)
Nper: total number of payment periods
Pmt: payment made each period
Pv: present value
Fv: future value, or a cash balance (0)
Type: number 0 or 1 (due)
Guess: (10%)

RATE-EXAMPLE

Three inputs are specified. 4 as years of loan in cell A5, -200 as monthly payment in cell A6 and 8000 as amount of loan in cell A7. The answer is 0.09241767 or 9.24%.



LECTURE 9
Matrix and its dimension
Types of matrix

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 8
- Matrices

QUESTIONS

Every student wonders why he or she should study matrices. There are many important questions:
Where can we use Matrices?
Typical applications?
What is a Matrix?
What are Matrix operations?
Excel Matrix Functions?
There are many applications of matrices in business and industry especially where large amounts of data are processed daily.

TYPICAL APPLICATIONS

Practical questions in modern business and economic management can be answered with the help of matrix representation in:

- Econometrics
- Network Analysis
- Decision Networks
- Optimization
- Linear Programming
- Analysis of data
- Computer graphics

WHAT IS A MATRIX?

A Matrix is a rectangular array of numbers. The plural of matrix is matrices.
Matrices are usually represented with capital letters such as Matrix A, B, C.
For example

$$A=\begin{bmatrix}-1 & 9\\ -3 & 4\end{bmatrix} \quad B=\begin{bmatrix}47 & 62 & 70 & 56\\ 52 & 33 & 28 & 45\end{bmatrix} \quad C=\begin{bmatrix}1 & -4\\ 0 & 3\\ 5 & -2\end{bmatrix}$$

The numbers in a matrix are often arranged in a meaningful way. For example, the order for school clothing in September is illustrated in the table, as well as in the corresponding matrix.

	Size				
	Youth	S	M	L	XL
Sweat Pants	0	10	34	40	12
Sweat Shirts	18	25	29	21	7
Shorts	19	13	48	36	9
T-shirts	27	7	10	24	14

The data in the above table can be entered in the shape of a matrix as follows:

$$\begin{bmatrix} 0 & 10 & 34 & 40 & 12 \\ 18 & 25 & 29 & 21 & 7 \\ 19 & 13 & 48 & 36 & 9 \\ 27 & 7 & 10 & 24 & 14 \end{bmatrix}$$

DIMENSION

Dimension or Order of a Matrix = Number of Rows x Number of Columns

Example

Matrix T has dimensions of 2x3 or the order of matrix T is 2x3. '×' is just the notation, it do not mean to multiply both of them.

$$T = \begin{bmatrix} 6 & 2 & -1 \\ -4 & 0 & 7 \end{bmatrix} \begin{matrix} \leftarrow \text{row1} \\ \leftarrow \text{row2} \end{matrix}$$

$$\nearrow \quad \uparrow \quad \nwarrow$$

col 1 col 2 col 3

ROW, COLUMN AND SQUARE MATRIX

Suppose n = 1,2,3,4,…….

A matrix with dimensions 1xn is referred to as a row matrix

For example, matrix A is a 1x4 **row matrix**.

A matrix with dimensions nx1 is referred to as a column matrix.

For example, matrix B in the middle is a 2x1 **column matrix**.

A matrix with dimensions nxn is referred to as a square matrix.

For example, matrix C is a 3x3 **square matrix**.

$$A = \begin{bmatrix} 12 & 17 & 10 & 9 \end{bmatrix} \quad B = \begin{bmatrix} -2 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 2 & -3 & 7 \\ 6 & 8 & -1 \\ -5 & 0 & 4 \end{bmatrix}$$

IDENTITY MATRIX

An identity matrix is a square matrix with 1's on the main diagonal from the upper left to the lower right and 0's off the main diagonal. An identity matrix is denoted as I. Some examples of identity matrices are shown below. The subscript indicates the size of the identity matrix. For example, I_n, represents an identity matrix with dimensions $n \times n$.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

MULTIPLICATIVE IDENTITY

With real numbers, the number 1 is referred to as a multiplicative identity because it has the unique property that the product a real number and 1 is that real number. In other words, 1 is called a multiplicative identity because for any real number n, $1 \cdot n = n$ and $n \cdot 1 = n$. With matrices, the identity matrix shares the same unique property as the number 1. In other words, a 2×2 identity matrix is a multiplicative inverse because for any 2×2 matrix A, $I_2 \cdot A = A$ and $A \cdot I_2 = A$

Example

Given the 2×2 matrix, A = $\begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$

$I_2 \cdot A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$

$A \cdot I_2 = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$

Work
r1c1 = 1(2) + 0(-3) = 2
r2c1 = 0(2) + 1(-3) = -3
r1c1 = 2(1) + -1(0) = 2
r2c1 = -3(1) + 4(0) = -3
r1c2 = 1(-1) + 0(4) = -1
r2c2 = 0(-1) + 1(4) = 4
r1c2 = 2(0) + -1(1) = -1
r2c2 = -3(0) + 4(1) = 4
where 'r' is for row and 'c' is for column.

LECTURE 10
MATRICES

OBJECTIVES
The objectives of the lecture are to learn about:
- **Review Lecture 9**
- **Matrices**

EXAMPLE 1
An athletic clothing company manufactures T-shirts and sweat shirts in four differents sizes, small, medium, large, and x-large. The company supplies two major universities, the U of R and the U of S. The tables below show September's clothing order for each university

University of S's September Clothing Order

	S	M	L	XL
T-shirts	100	300	500	300
sweat shirts	150	400	450	250

University of R's September Clothing Order.

	S	M	L	XL
T-shirts	60	250	400	250
sweat shirts	100	200	350	200

Matrix Representation
The above information can be given by two matrices S and R as shown below.

$$S = \begin{bmatrix} 100 & 300 & 500 & 300 \\ 150 & 400 & 450 & 250 \end{bmatrix}$$

$$R = \begin{bmatrix} 60 & 250 & 400 & 250 \\ 100 & 200 & 350 & 220 \end{bmatrix}$$

MATRIX OPERATIONS
The matrix operations can be summarized as under:

- **Organize and interpret data using matrices**
- **Use matrices in business applications**
- **Add and subtract two matrices**
- **Multiply a matrix by a scalar**
- **Multiply two matrices**
- **Interpret the meaning of the elements within a product matrix**

PRODUCTION
The clothing company production in preparation for the universities' September orders is shown by the table and corresponding matrix P below.

	S	M	L	XL
T-shirts	300	700	900	500
sweat shirts	300	700	900	500

$$P = \begin{bmatrix} 300 & 700 & 900 & 500 \\ 300 & 700 & 900 & 500 \end{bmatrix}$$

ADDITION AND SUBTRACTION OF MATRICES

The sum or difference of two matrices is calculated by adding or subtracting the corresponding elements of the matrices.
To add or subtract matrices, they must have the same dimensions.

PRODUCTION REQUIREMENT

Since the U of S ordered 100 small T-shirts and the U of R ordered 60, then altogether 160 small T-shirts are required to supply both universities. Thus, to calculate the total number of T-shirts and sweat shirts required to supply both universities, add the corresponding elements of the two order matrices as shown below.

$$\begin{bmatrix} 100 & 300 & 500 & 300 \\ 150 & 400 & 450 & 250 \end{bmatrix} + \begin{bmatrix} 60 & 250 & 400 & 250 \\ 100 & 200 & 350 & 220 \end{bmatrix}$$

$$= \begin{bmatrix} 160 & 550 & 900 & 550 \\ 250 & 600 & 800 & 470 \end{bmatrix}$$

OVERPRODUCTION

Since the company produced 300 small T-shirts and the received orders for only 160 small T-shirts, then the company produced 140 small T-shirts too many. Thus, to determine the company's over-production, subtract the corresponding elements of the total order matrix from the production matrix as shown below.

$$\begin{bmatrix} 300 & 700 & 900 & 500 \\ 300 & 700 & 900 & 500 \end{bmatrix} - \begin{bmatrix} 160 & 550 & 900 & 550 \\ 250 & 600 & 800 & 470 \end{bmatrix} = \begin{bmatrix} 140 & 150 & 0 & -50 \\ 50 & 100 & 100 & 30 \end{bmatrix}$$

MULTIPLY A MATRIX BY A SCALAR

Given a matrix A and a number c, the **scalar multiplication** cA is computed by multiplying the scalar c by every element of A .For example:

$$2\begin{bmatrix} 1 & 8 & -3 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 2 \times 1 & 2 \times 8 & 2 \times -3 \\ 2 \times 4 & 2 \times -2 & 2 \times 5 \end{bmatrix} = \begin{bmatrix} 2 & 16 & -6 \\ 8 & -4 & 10 \end{bmatrix}$$

MULTIPLICATION OF MATRICES

To understand the reasoning behind the definition of matrix multiplication, let us consider the following example.

Competing Companies, A and B, sell juice in 591 mL, 1 L and 2 L plastic bottles at prices of Rs.1.60, Rs.2.30 and Rs.3.10, respectively. The table below summarizes the sales for the two companies during the month of July.

	591mL	1L	2L
Company A	20,000	5,500	10,600
Company B	18,250	7,000	11,000

What is total revenue of Company A?
What is total revenue of Company B?
Matrices may be used to illustrate the above information.
As shown at the right, the sales can be written as
a 2X3 matrix, S, the selling prices can be written as a column matrix, P, and the total revenue for each company can be expressed as a column matrix, R.

$$S = \begin{bmatrix} 20,000 & 5,500 & 10,600 \\ 18,250 & 7,000 & 11,000 \end{bmatrix} \qquad P = \begin{bmatrix} 1.60 \\ 2.30 \\ 3.10 \end{bmatrix} \qquad R = \begin{bmatrix} 77,510 \\ 79,400 \end{bmatrix}$$

Since revenue is calculated by multiplying the number of sales by the selling price, the total revenue for each company is found by taking the product of the sales matrix and the price matrix.

$$\begin{bmatrix} 20,000 & 5,500 & 10,600 \\ 18,250 & 7,000 & 11,000 \end{bmatrix} \times \begin{bmatrix} 1.60 \\ 2.30 \\ 3.10 \end{bmatrix} = \begin{bmatrix} 77,510 \\ 79,400 \end{bmatrix}$$

Consider how the first row of matrix S and the single column P lead to the first entry of R.

$$\begin{bmatrix} 20,000 & 5,500 & 10,600 \\ 18,250 & 7,000 & 11,000 \end{bmatrix} \times \begin{bmatrix} 1.60 \\ 2.30 \\ 3.10 \end{bmatrix}$$

$$\Rightarrow 20,000(1.60) + 5,500(2.30) + 10,600(3.10) \Rightarrow \begin{bmatrix} 77,510 \\ 79,400 \end{bmatrix}$$

Product of First Entries; Product of Second Entries; Product of Third Entries

With the above in mind, we define the product of a row and a column to be the number obtained by multiplying corresponding entries (first by first, second by second, and so on) and adding the results.

MULTIPLICATION RULES

If matrix A is a $m \times n$ matrix and matrix B is a $n \times p$ matrix, then the product AB is the $m \times p$ matrix whose entry in the i-th row and the j-th column is the product of the i-th row of matrix A and the j-th row of matrix B.
The product of a row and a column is the number obtained by multiplying corresponding elements (first by first, second by second, and so on).

To multiply matrices, the number of columns of A must equal the number of rows of B.

For example
Given the matrices below, decide if the indicated product exists. And, if the product exists, determine the dimensions of the product matrix.

$$A=\begin{bmatrix}-5 & 0 & 4\\ 12 & -3 & 10\\ 7 & 15 & -1\end{bmatrix}$$

$$B=\begin{bmatrix}1 & 4\\ 9 & -1\\ -5 & 7\end{bmatrix}$$

MULTIPLICATION CHECKS
The table below gives a summary whether it is possible to multiply two matrices. It may be noticed that the product of matrix A and matrix B is possible as the number of columns of A are equal to the number of rows of B. The product BA is not possible as the number of columns of b are not equal to rows of A.

Product	**Dimensions of the Matrices**	**Does a product exist?** (Is it possible to multiply the given matrices in this order?)	**Dimensions of Product Matrix**
AB	$A = 3\times 3 \quad B = 3\times 2$ ↑ ↑ inner dimensions	Yes, the product exists since the inner dimensions match (# of columns of *A* = # of rows of *B*).	3×2
BA	$B = 3\times 2 \quad A = 3\times 3$ ↑ ↑ inner dimensions	No, the product does not exist since the inner dimensions do not match (# of columns of *B* ≠ # of rows of *A*).	n/a

MULTIPLICATIVE INVERSES
Real Numbers
Two non-zero real numbers are multiplicative inverses of each other if their products, in both orders, is 1. Thus,

the multiplicative inverse of a real number, x is $\frac{1}{x}$ or x^{-1} since $x \cdot \frac{1}{x} = 1$ and $\frac{1}{x} \cdot x = 1$.

Example:

The multiplicative inverse of 5 is $\frac{1}{5}$ since

$5 \cdot \frac{1}{5} = 1$ and $\frac{1}{5} \cdot 5 = 1$

Matrices

Two 2×2 matrices are inverses of each other if their products, in both orders, is a 2×2 identity matrix. Thus, the multiplicative inverse of a 2×2 matrix, A is A^{-1} since $A\cdot A^{-1}=I_2$ and $A^{-1}\cdot A=I_2$

Example:

The multiplicative inverse of a matrix, $\begin{bmatrix}2 & 1\\ 5 & 3\end{bmatrix}$ is $\begin{bmatrix}3 & -1\\ -5 & 2\end{bmatrix}$

$$\begin{bmatrix}2 & 1\\ 5 & 3\end{bmatrix}\begin{bmatrix}3 & -1\\ -5 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}3 & -1\\ -5 & 2\end{bmatrix}\begin{bmatrix}2 & 1\\ 5 & 3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

LECTURE 11
MATRICES

OBJECTIVES

The objectives of the lecture are to learn about:

- **Review Lecture 10**
- **Matrix functions in Excel**
- **Set up and manipulate ratios.**
- **Allocate an amount on a prorata basis using proportions.**

MATRIX FUNCTIONS IN MS EXCEL

The Matrix Functions in Microsoft Excel are as follows:

1. **MINVERSE**
2. **MDETERM**
3. **MMULT**

MINVERSE

Returns the inverse matrix for the matrix stored in an array.

Syntax

MINVERSE(**array**)

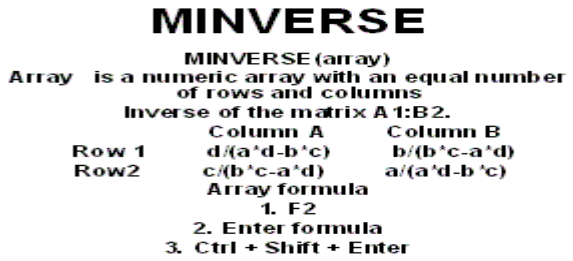
array is a numeric array with an equal number of rows and columns.

Remarks

- Array can be given as a cell range, such as A1:C3; as an array constant, such as {1,2,3;4,5,6;7,8,9}; or as a name for either of these.
- If any cells in array are empty or contain text, MINVERSE returns the #VALUE! error value.
- MINVERSE also returns the #VALUE! error value if array does not have an equal number of rows and columns.
- Formulas that return arrays must be entered as array formulas.
- Inverse matrices, like determinants, are generally used for solving systems of mathematical equations involving several variables. The product of a matrix and its inverse is the identity matrix — the square array in which the diagonal values equal 1, and all other values equal 0.
- As an example of how a two-row, two-column matrix is calculated, suppose that the range A1:B2 contains the letters a, b, c, and d that represent any four numbers. The following table shows the inverse of the matrix A1:B2.

	Column A	**Column B**
Row 1	d/(a*d-b*c)	b/(b*c-a*d)
Row 2	c/(b*c-a*d)	a/(a*d-b*c)

- MINVERSE is calculated with an accuracy of approximately 16 digits, which may lead to a small numeric error when the cancellation is not complete.
- Some square matrices cannot be inverted and will return the #NUM! error value with MINVERSE. The determinant for a noninvertable matrix is 0.



MINVERSE-EXAMPLE

Find the inversion or multiplicative inverse of following matrix

$$\begin{bmatrix} 4 & -1 \\ 2 & 0 \end{bmatrix}$$

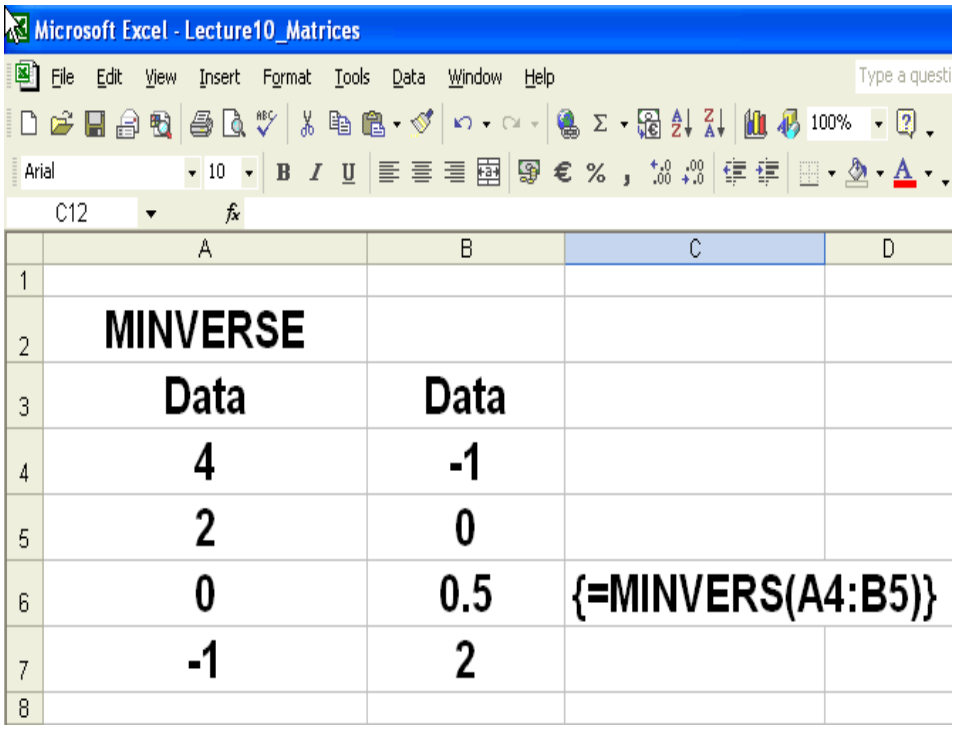
The excel formula in the example must be entered as an array formula. Otherwise a single result, 0, will appear.

Please note that formulas that return arrays must be entered as array formulas.

The steps of finding multiplicative inverse of above matrix is as follows:

1. **Enter data of array to be inverted, in Cells A4:B5. (i-e in cells A4, B4, A5, B5)**
2. **Click on cell A6.**
3. **Keeping left mouse button pressed, drag it to cell B7.Four cells A6, B6, A7, B7 will be selected.**
4. **Press F2 from your keyboard. (Key** F2 **is selected to enter Edit Mode in the active cell. It's a keyboard shortcut. Even if you don't press F2, you can write the formula).**
5. **Type the formula =MINVERSE(A4:B5). This will appear in cell A6.**
6. **Press Ctrl, Shift, Enter keys simultaneously from your keyboard.**
7. **Multiplicative inverse of the matrix will appear in cells A6, A7, B6, B7.**

Now click any cell among A6:B7, in the formula bar you can see curly brackets round the formula. That is, {=MINVERSE(A4:B5)}. This shows that you have gone through the right procedure of entering array formula.



MDETERM

Returns the matrix determinant of an array.

Syntax

MDETERM(**array**)

array is a numeric array with an equal number of rows and columns.

Remarks

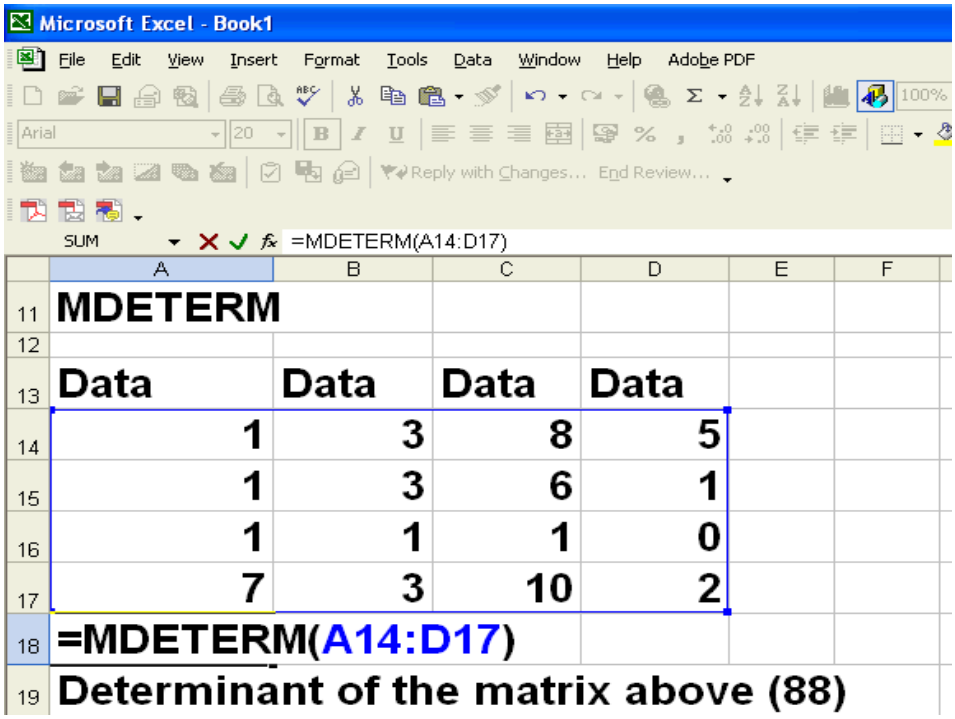
- Array can be given as a cell range, for example, A1:C3; as an array constant, such as {1,2,3;4,5,6;7,8,9}; or as a name to either of these.
- If any cells in array are empty or contain text, MDETERM returns the #VALUE! error value.
- MDETERM also returns #VALUE! if array does not have an equal number of rows and columns.
- The matrix determinant is a number derived from the values in array. For a three-row, three-column array, A1:C3, the determinant is defined as:

MDETERM(A1:C3) = A1*(B2*C3-B3*C2) + A2*(B3*C1-B1*C3) + A3*(B1*C2-B2*C1)

- Matrix determinants are generally used for solving systems of mathematical equations that involve several variables.
- MDETERM is calculated with an accuracy of approximately 16 digits, which may lead to a small numeric error when the calculation is not complete. For example, the determinant of a singular matrix may differ from zero by 1E-16.

MDETERM-EXAMPLE

The example shows an array of dimension 4 x 4 in cell range A14:D17. The formula was entered in cell A18. The result of this calculation is 88.



There are other ways also for using this function. For example you can enter the matrix as an array constant.

=MDETERM({3,6,1;1,1,0;3,10,2})	Determinant of the matrix as an array constant (1)

You can calculate the determinant of the matrix in the array constant.

=MDETERM({3,6;1,1})	Determinant of the matrix in the array constant (-3)

Unequal number of rows and columns results in an error.

=MDETERM({1,3,8,5;1,3,6,1})	Returns an error because the array does not have an equal number of rows and columns (#VALUE!)

MMULT

Returns the matrix product of two arrays. The result is an array with the same number of rows as array1 and the same number of columns as array2.

Syntax

MMULT(**array1**,**array2**)

Array1, array2 are the arrays you want to multiply.

Remarks

- The number of columns in array1 must be the same as the number of rows in array2, and both arrays must contain only numbers.
- Array1 and array2 can be given as cell ranges, array constants, or references.

- If any cells are empty or contain text, or if the number of columns in array1 is different from the number of rows in array2, MMULT returns the #VALUE! error value.
- The matrix product array a of two arrays b and c is:

$$a_{ij} = \sum_{k=1}^{n} b_{ik} c_{kj}$$

where i is the row number, and j is the column number.

- Formulas that return arrays must be entered as array formulas.

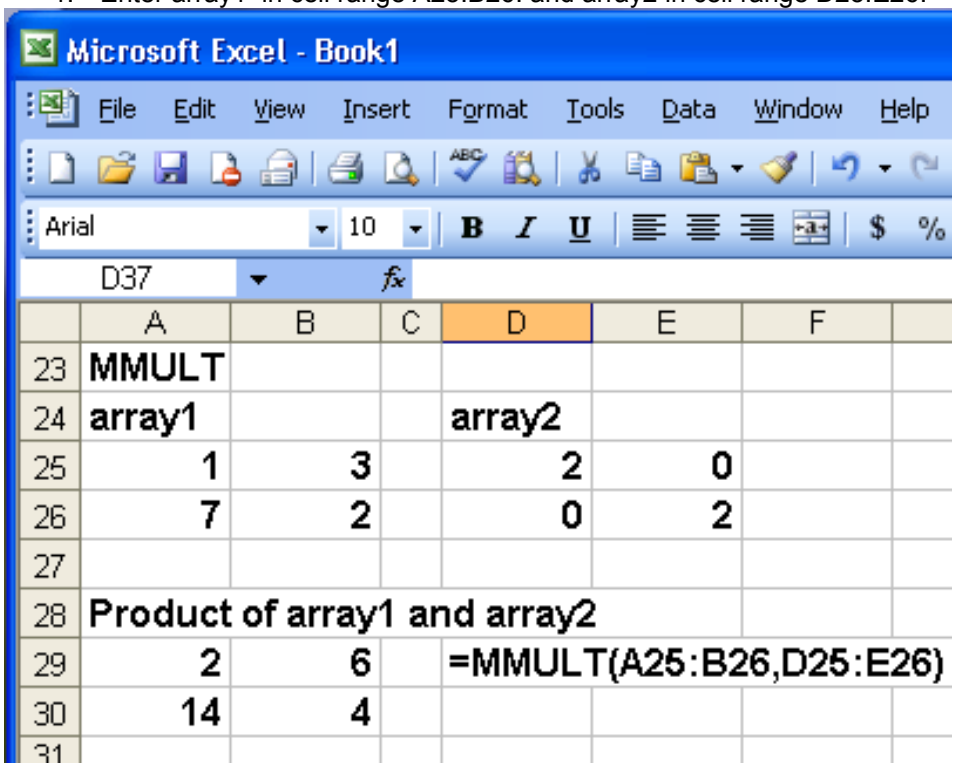
MMULT-EXAMPLE

Let

$$A = \begin{bmatrix} 1 & 3 \\ 7 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}. \; Find \;\; AB$$

To find the product AB, follow these steps.

1. Enter array1 in cell range A25:B26. and array2 in cell range D25:E26.



2. Find the dimension of AB matrix. Here as A and B are 2×2 matrices so AB is also a 2×2 matrix.
3. Click on cell A29.
4. Keeping left mouse button pressed, drag it to cell B30.Four cells A29, B29, A30, B30 will be selected.
5. Press F2 from your keyboard. (Key **F2** is selected to enter Edit Mode in the active cell. It's a keyboard shortcut. Even if you don't press F2, you can write the formula).
6. Type the formula =MMULT(
7. Select array1.
8. Put comma.
9. Select array2.
10. Close bracket
11. Press Ctrl, Shift, Enter keys simultaneously from your keyboard.
12. Product AB will appear in cells A29, A30, B29, B30.

RATIO

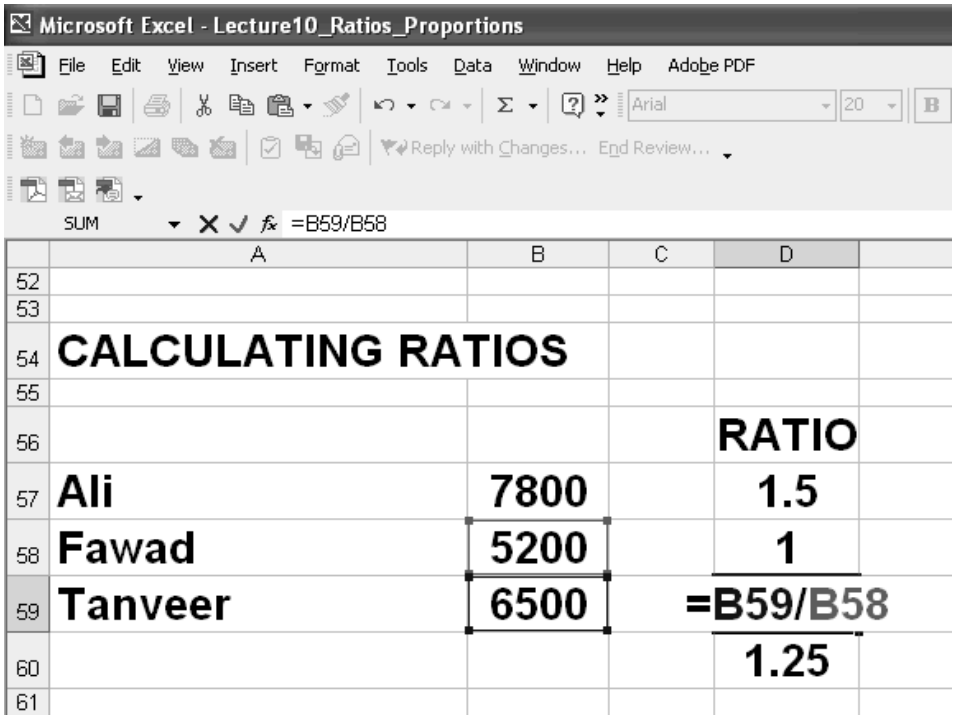
A Ratio is a comparison between things. If in a room there are 30 men and 15 women then the ratio of men to women is 2 to 1. This is written as 2:1 and read as "two is to one". ":" is the notation for a ratio.
Be careful, order matters. A ratio of 2:1 is not the same as 1:2.
In the form of fraction, we can write 2:1 as 2/1.

The method of calculating ratios is as under:
1. Find the minimum value
2. Divide all the values by the smallest value.

In the above example, the smallest value was 15. Division gives 30/15 = 2 for men and 15/15 = 1 for women. The ratio is therefore 2:1 for men and women.

RATIO-EXAMPLE

Three friends Ali, Fawad and Tanveer are doing business together. To set up the business Ali invested Rs. 7800, Fawad Rs. 5,200 and Tanveer Rs. 6,500 What is the ratio of their investments. As discussed above the smallest value is 5200Rs. All values are divided by 5200. The results are 1.5 for Ali, 1 for Fawad and 1.25 for Tanveer. The answer is:
1.5 : 1 : 1.25.



This example can be solved in Excel. The formula is as under:
Cell D57: =B57 / B58
Cell D58: =B58 / B58
Cell D59: =B59 / B58
The result for cell D59 is shown in cell D60, because the cell D59 is used to display the formula.

PROPORTION

A proportion is an equation with a ratio on each side. It is a statement that two ratios are equal.

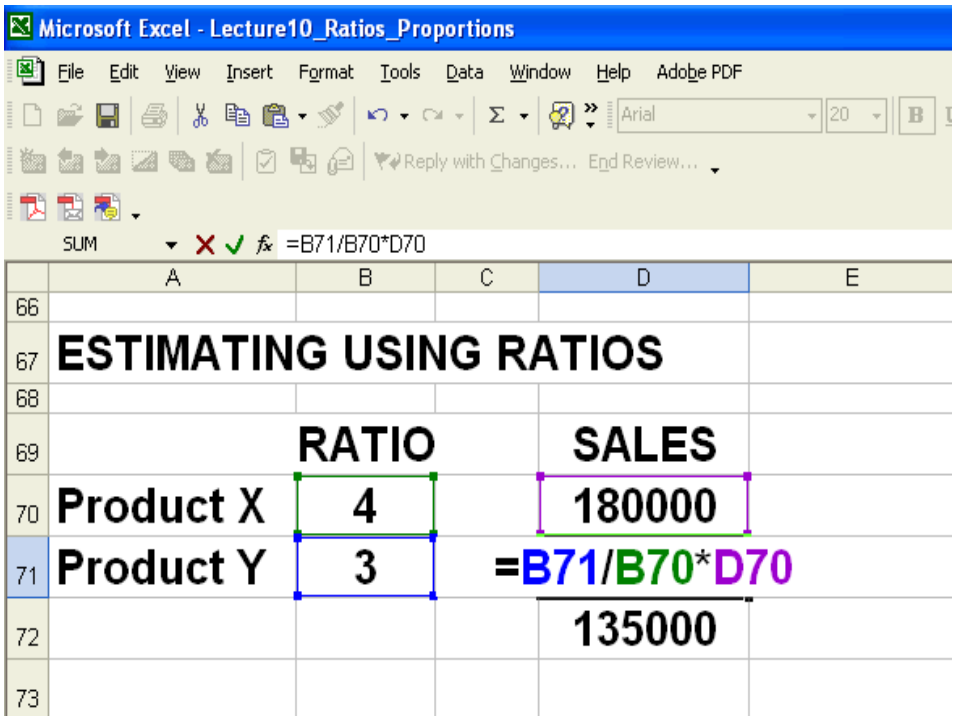
3:4 = 6:8 OR 3/4 = 6/8 is an example of a proportion.

When one of the four numbers in a proportion is unknown, cross products may be used to find the unknown number. This is called solving the proportion or **ESTIMATING USING RATIO**.

EXAMPLE
Ratio of sales of Product X to sales of Product Y is 4:3. The sales of product X is forecasted at Rs. 180,000. What should be the Sales of product Y to maintain the ratio of sales between the two products.

CALCULATION
Ratio sales X : Y = 4 : 3
Insert the value for forecasted sale for X.
180,000 : Y = 4 : 3
It can be rewritten as:
180,000/Y = 4/3
Cross – multiply
180,000 x 3 = 4 x Y
Rewrite to bring the unknown to the left of the equality
4 x Y = 180,000 x 3
Solve
Y = (180,000 x 3)/4
Y = 135,000 Rs

Calculations using EXCEL
In cells B70 and B71 the ratios of Product X and Y were entered.
The value of forecast of product X was entered in cell D70.
Before writing down the formula in excel, it was derived as follows:
1 Ratio of X = (cell B70)
2 Ratio of y = (cell B71)
3 Sale of X = (cell D70)
4 Sale of Y = (cell D71)
Now Ratio X: Y = (cell B70)/ (cell B71)
Ratio of sales = (cell D70)/ (cell D71)
Cross-multiply.
(cell B70) x (cell D71) = (cell B71) x (cell D70)
Cell D71 is unknown. Hence:
(cell D71) = (cell B71) x (cell D70)/ (cell B70)
Or
(cell D71) = (cell B71)/ (cell B70) * (cell D70)
Thus the formula was:
=B71/B70*D70
Please note that actually we are using the ratio Y to X as it is easier to think of ratio of unknown to the known.


Microsoft Excel - Lecture10_Ratios_Proportions
File Edit View Insert Format Tools Data Window Help Adobe PDF
Reply with Changes... End Review...
SUM
=B71/B70*D70
ESTIMATING USING RATIOS
RATIO
SALES
Product X
4
180000
Product Y
3
=B71/B70*D70
135000

ESTIMATING USING RATIO-EXAMPLE 2

In a 500 bed hospital there are 200 nurses and 150 other staff. If the hospital extends by a new wing for 100 beds, then what additional staff is needed? Let 500 beds B_1 and 100 beds B_2. Staff nurses N_1 is 200 and other staff O_1 is 150. What is the value of N_2 and O_2 for B_2. Obviously the ratio of beds will be used. As pointed out above, think of the ratio of unknown to known. In other words ratio $B_2:B_1$ or B_2/B_1. Ratio of nurses would be N_2/N_1. Ratio of other staff would be O_2/O_1.

Now :

$N_2/N_1 = B_2/B_1$ Or $N_2 = (B_2/B_1)*N_1$ or $N_2 = (100/500)*200 = 40$ Nurses

$O_2/O_1 = B_2/B_1$ Or $N_2 = (B_2/B_1)*O_1$ or $O_2 = (100/500)*150 = 30$ other staff.

Calculation

Beds : Nurses : Other staff
500 : 200 : 150
100 : X? : Y?

Nurses

500 : 200 = 100 : X

500 X = 200 x 100

X = (200 x 100)/500 = 40

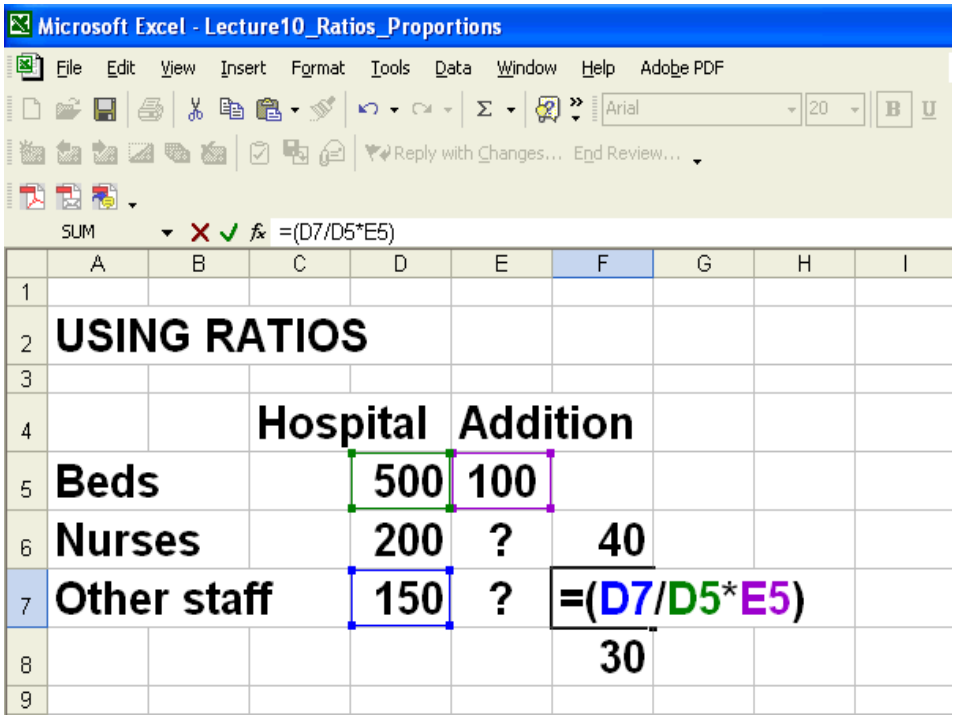
Other staff

Y = (150 x 100)/500 = 30

Calculation using EXCEL

The calculation using EXCEL was done in a similar fashion as the previous example.

The calculation is self-explanatory.



ESTIMATING USING RATIO-EXAMPLE 3

A Fruit Punch recipe requires mango juice, apple juice and orange juice in the ratio of 3:2:1. To make 2 liters of punch calculate quantity of other ingredients

Again we shall use the ratio of unknown to the unknown. The unknowns are mango and apple juice. Consider first ratio of required mango juice (3) to total quantity of punch (6). This was calculated from 3+2+1. Now the quantity of required mango for 2 liter would simply be (3/6)*2. Similarly the required quantity of apple juice is (2/6)*2.

Calculation

Mango juice : Apple juice : Orange juice

3 : 2 : 1

Total = 6

X? : Y? : Z?

Total = 2 litre

Mango juice (X) = (3 / 6) * 2 = 1 litre

Apple juice (Y) = (2 / 6) * 2 = 0.67 litre

Orange juice (Z) = (1 / 6) * 2 = 0.33 liter

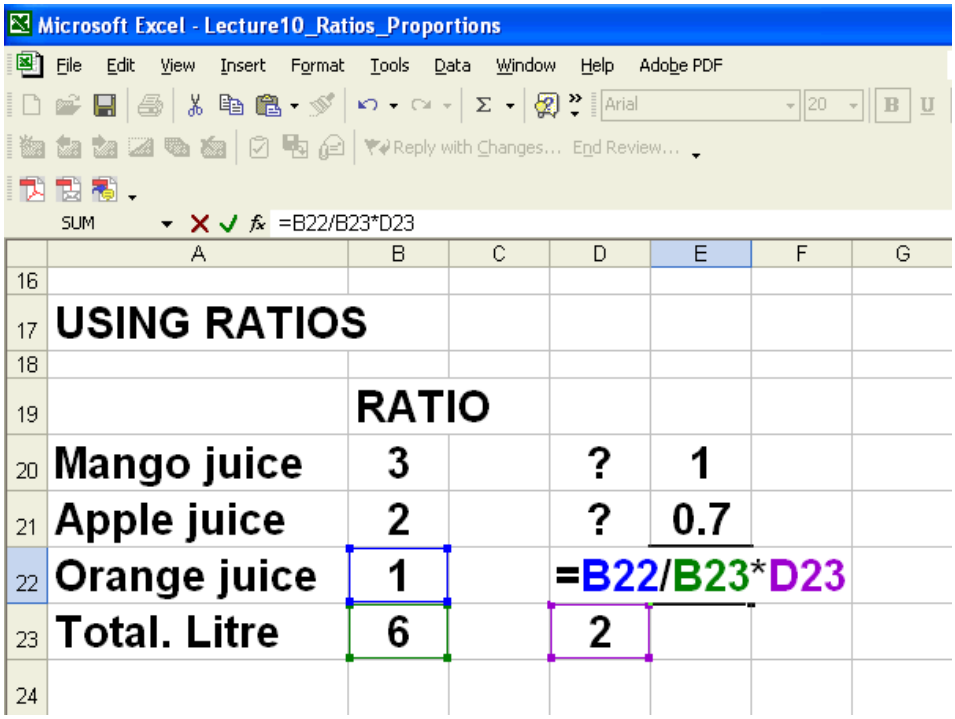
Calculation using EXCEL

Here also the similar ratios were used.

Mango = B20/B23*D23

Apple = B21/B23*D23

Orange = B22/B23*D23



LECTURE 12
RATIO AND PROPORTION
MERCHANDISING

OBJECTIVES

The objectives of the lecture are to learn about:

- Module 3
- Review Lecture 11
- Ratio and Proportions
- Merchandising
- Assignment 1A and 1B

MODULE 3

Module 3 has the following content:

- Ratio and Proportions
- Merchandising
  (Lectures 12)
- Mathematics of Merchandising
  (Lectures 13-16)

ESTIMATING USING RATIOS-EXAMPLE 1

In the previous lecture, we studied how ratios can be used to determine unknowns. Here is another example with a slightly different approach. Here, ratios between the quantities.and data of only one quantity is known. We will estimate the total quantity that can be made? It is the quantity of orange juice that will determine the total quantity that can be made. Again the method is to use the ratio of the unknown to the known.

Punch recipe

In a punch the ratio of mango juice, apple juice and orange juice is 3:2:1. If you have 1.5 liters of orange juice, how much punch can you make?

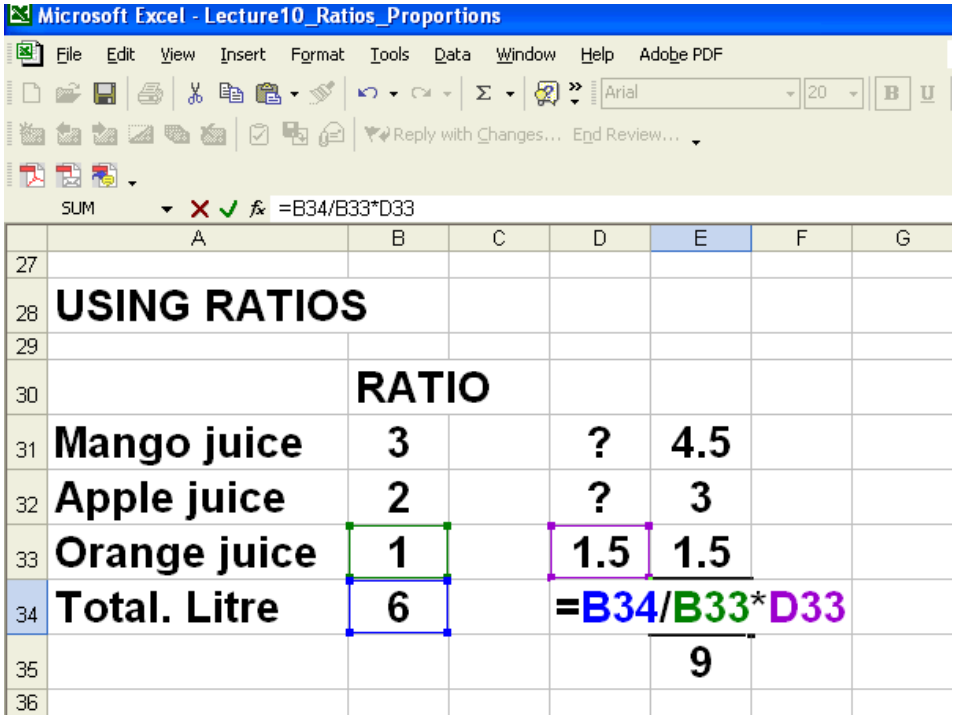
Calculation

Mango juice : Apple juice : Orange juice
3 : 2 : 1
Total = 3+2+1 = 6
X (say) : Y(say) : Z(say)
? : ? : 1.5litres
Total = ? litre

Mango juice (X) = (3/1)×1.5 = 4.5 litre
Apple juice (Y) = (2/1)×1.5 = 3.0 litre
Orange juice (Z) = 1.5 litre
Punch = 4.5 + 3.0 + 1.5 = 9 litres

EXCEL Calculation

The method used is the same as in previous examples.



ESTIMATING USING RATIOS-EXAMPLE 2

In a punch ratio of mango juice, apple juice and orange juice is 3 : 2 : 1.5. If you have 500 litres of orange juice find how much mango and apple juices are required to make the punch.

Punch recipe

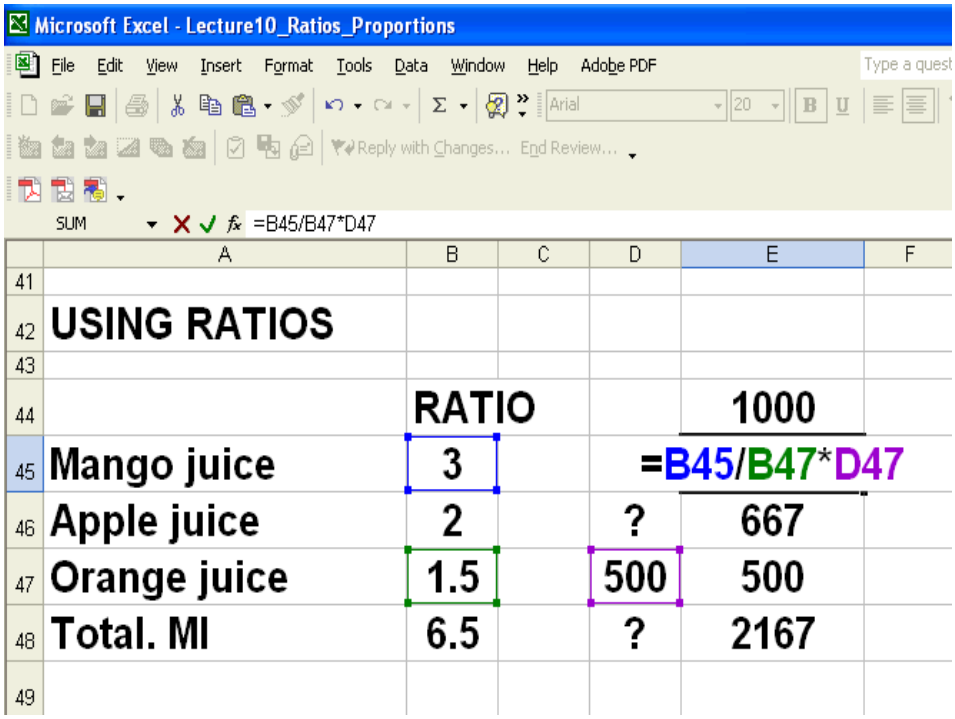
The ratio of mango juice, apple juice and orange juice is 3 : 2 : 1.5 If you have 500 milliliters of orange juice, how much mango juice and apple juice is needed?

Mango juice : Apple juice : Orange juice
3 : 2 : 1.5
Total = 6.5
X (say) : Y(say) : Z (say)
? : ? : 500 litres
Total = ? litres

Mango juice (X) = (3/1.5)*500 = 1000 litre
Apple juice (Y) = (2/1.5)*500 = 667 litre
Orange juice (Z) = 500 litre
Punch = 1000 + 667 + 500 = 2167 litre

EXCEL Calculation

Here also ratios were used.
Mango juice = B45/B47*D47
Apple juice = B46/B47*D47
Orange juice = D47



EXAMPLE

In a certain class, the ratio of passing grades to failing grades is 7 to 5. How many of the 36 students failed the course?

The ratio, "7 to 5" (or 7 : 5 or 7/5), tells you that, of every 7 + 5 = 12 students, five failed.

That is, 5/12 of the class failed.

Then (5/12)(36) = 15 students failed.

PROPORTION

a/b = c/d

...the values in the "b" and "c" positions are called the "means" of the proportion, while the values in the "a" and "d" positions are called the "extremes" of the proportion. A basic defining property of a proportion is that the product of the means is equal to the product of the extremes. In other words, given:

a/b = c/d

...it is a fact that ad = bc.

PROPORTION-EXAMPLES

Is 24/140 proportional to 30/176?

Check:

140×30 = 4200

24×176 = 4224

So the answer is that given ratios are not proportional.

PROPORTION EXAMPLE 1

Find the unknown value in the proportion: 2 : x = 3 : 9.

2 : x = 3 : 9

First, convert the colon-notation ratios to fractions:

2/x = 3/9

Cross multiply

18 = 3x

6 = x

PROPORTION EXAMPLE 2

Find the unknown value in the proportion: (2x + 1) : 2 = (x + 2) : 5

(2x + 1) : 2 = (x + 2) : 5

First, convert the colon-notation ratios to fractions:

(2x + 1)/2 = (x + 2)/5

Then solve:

5(2x + 1) = 2(x + 2)

10x + 5 = 2x + 4

8x = –1

x = –1/8

MERCHANDISING

What does merchandising cover?

- Understand the ordinary dating notation for the terms of payment of an invoice.
- Solve merchandise pricing problems involving mark ups and markdowns.
- Calculate the net price of an item after single or multiple trade discounts.
- Calculate a single discount rate that is equivalent to a series of multiple discounts.
- Calculate the amount of the cash discount for which a payment qualifies.

STAKEHOLDERS IN Merchandising

Who are the stakeholders in merchandising?

The main players are:

- **Manufacturer**
- **Middleman**
- **Retailer**
- **Consumer**

There are discounts at all levels in the above chain.

MIDDLEMAN

A middle man is a person who buys a product directly from the manufacturer, and then either sells the product at retail prices to the public, or sells the product at wholesale prices to a distributor. There can often be more than one middle man when the latter practice is adopted. A middle man can purchase from the manufacturer and then work with another middle man who buys for the distributor. The manufacturer often views the middle man as the alternative to direct distribution.

List price or Retail price

List price refers to the manufacturer's suggested retail pricing. It may or may not be the price asked of the consumer. Much depends on

1. the product itself,
2. the built-in profit margin,
3. Supply and demand.

A product that is in high demand with low availability will sometimes sell *higher* than the list price, though this is less common than the reverse.

Virtually all products have a suggested retail or list price. Resellers (middleman, retailer) buy products in bulk and get a substantial discount in order to be able to get profit from selling the product at or below list price.

Trade Discount

Let L is the list price, then amount of trade discount is some percentage of this price. List price less amount of discount is the net price. In mathematical terms, we can write:

Amount of discount = d × L
Where, d = Percentage of Discount
L = List Price
Net Price = L – Ld = L(1 – d)
Net Price = List Price – Amount of Discount

LECTURE 13
MATHEMATICS OF MERCHANDISING

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 12
- Solve merchandising pricing problems involving markup and markdown

MARKUP

Markup is an amount added to a cost price while calculating a selling price. Especially, an amount that takes into account of overhead and profit. Markup can be expressed as:

1. Percentage of cost.
2. Percentage of sale.
3. Rs Markup

Markup as Percentage of Cost (MUC):

Here markup is some percentage of cost price. For simplicity, it is also named as **%Markup on cost.** The relation between %markup on cost, cost price and selling price is:

Selling Price = Cost price + (Cost price × %Markup on cost)
= Cost price (1 + %Markup on cost)

Markup as Percentage of Sale price (MUS):

Here markup is some percentage of selling price. For simplicity, it is also named as **%Markup on sale.** The relation between %markup on sale, cost price and selling price is:

Selling Price = Cost price + (Selling price × %Markup on sale)

Cost price = Selling price – (Selling price × %Markup on sale)
= Selling price (1 – %Markup on sale)

Rs Markup:

Markup in terms of rupees is called Rs markup. The relations between Rs markup, cost price and selling price are:

1. Selling Price = Cost price + Rs Markup
2. Rs Markup = %Markup on cost × Cost price
3. Rs Markup = %Markup on sale × Selling price

Any of the above formula can be used to find Rs Markup.

For example:
The cost price of certain item is 80Rs and its selling price is 100Rs. Then
Rs Markup = Selling price – Cost price
= 100 – 80
= 20 Rs

Remember: If some percentage is given as markup, without mentioning that whether it is markup on cost or markup on sale, it is evident that %markup on cost is under consideration

EAMPLE 1

A golf shop pays its wholesaler 2,400Rs. for a certain club, and then sells it for 4,500Rs. What is the markup rate?

Calculation of Markup

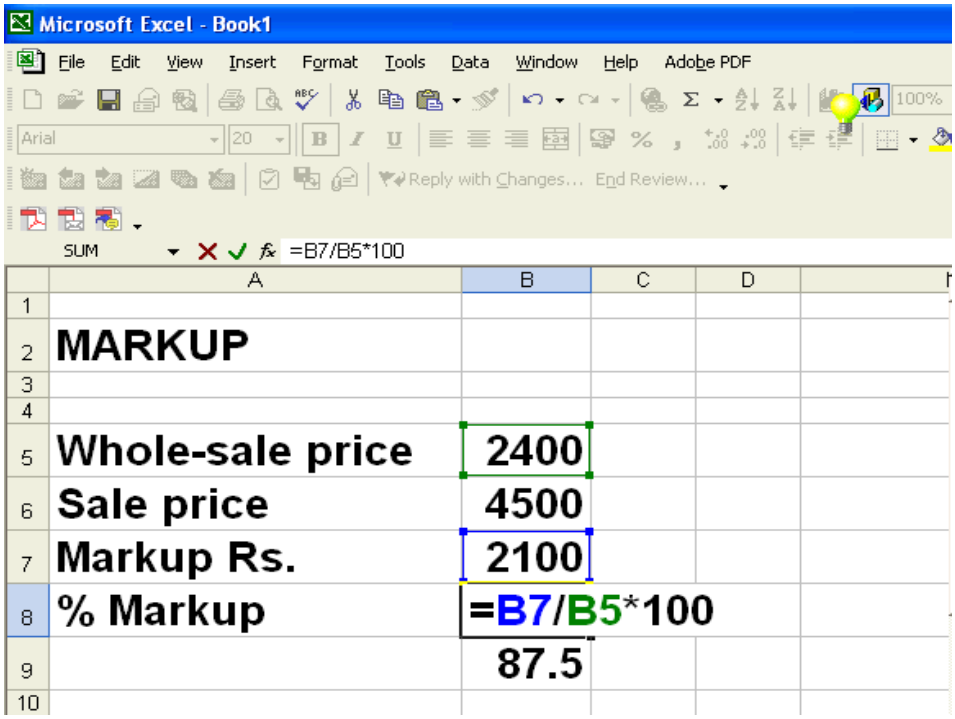
Cost price = 2400Rs
Selling price = 4500Rs

Selling Price = Cost price + (Cost price × %Markup on cost)
%Markup on cost = Selling price – Cost price ×100%
Cost price
Since
Rs Markup = Selling Price - Cost price
%Markup on cost = Rs markup ×100%
Cost price

1. First, calculate Rs markup :
Rs markup = 4500 – 2400 = 2100Rs
Then %markup on cost
%Markup on cost = Rs markup ×100%
Cost price

= 2100/2400 ×100% = 87.5%
Remember to convert this fraction value to a percent. The markup rate is 87.5%.

Calculation using EXCEL
Enter whole-sale price 2400 in cell B5
Enter sale price 4500 in cell B6
Enter formula for Rs. Markup =B6-B5 in cell B7and press enter. The answer is 2100.
Enter formula for % markup =B7/B5*100 in cell B8 and press Enter. The answer is 87.5% shown in cell B9.



MARKUP-EXAMPLE 2
A computer software retailer used a markup rate of 40%.
Find the selling price of a computer game that cost the retailer Rs. 1,500.

Markup
The markup is 40% on the cost, so as
Rs Markup = %Markup on cost × Cost price

Rs Markup = (0.40) (1,500) = Rs. 600

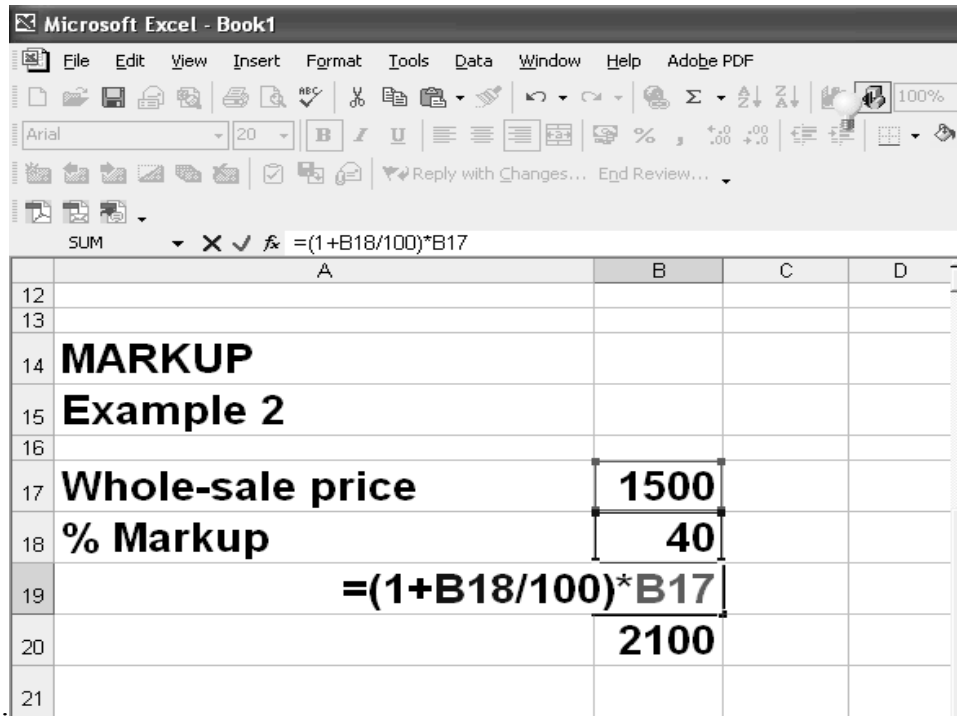
Selling Price

Then the selling price, being the cost plus markup, that is:
Selling Price = Cost price + Rs Markup
=1,500 + 600 = Rs. 2,100
The item sold for Rs. 2,100.

Calculation using EXCEL

Here we use the following formula to show an alternate method of solving the above problem.
Selling Price = Cost price (1 + %Markup on cost)

Enter whole-sale price 1500 in cell B17.
Enter % Markup in cell B18.
Enter formula =(1+B18/100)*B17 in cell B19. Here the term 1+B18/100 is the **multiplication factor**. B18/100 is the %markup in fraction. The answer 2100 is shown in cell B20.
We could have calculated the multiplication factor separately. But as you see it is not necessary as the entire calculation can be done in one line.



MARKDOWN

Markdown means a reduction from the original sale price to

1. stimulate demand or
2. take advantage of reduced costs or
3. force competitors out of the market.

Markdown can be expressed as

- Percentage of current selling price
- Rs markdown

Markdown as Percentage of current selling price

Here markdown is some percentage of current selling price. For simplicity, it is also named as percent markdown (%markdown). The relation between current selling price, %markdown and new selling price is:

New selling price = Current selling price – (Current selling price × %markdown)
= Current selling price (1 - %markdown)

Rs Markdown
Markdown in terms of rupees is called Rs markdown.

1. New selling price = Current selling price – Rs Markdown
2. Rs markdown = Current selling price × %markdown

Let us look at an example to understand how markdown is calculated.
MARKDOWN-EXAMPLE 1
An item originally priced at 3,300 Rs. is marked 25% off.
What is the sale price?
Markdown
First, find the Rs markdown.
The markdown is 25% of the original price or current selling price, as
Rs markdown = original price × %markdown

= (0.25)(3300) = 825Rs

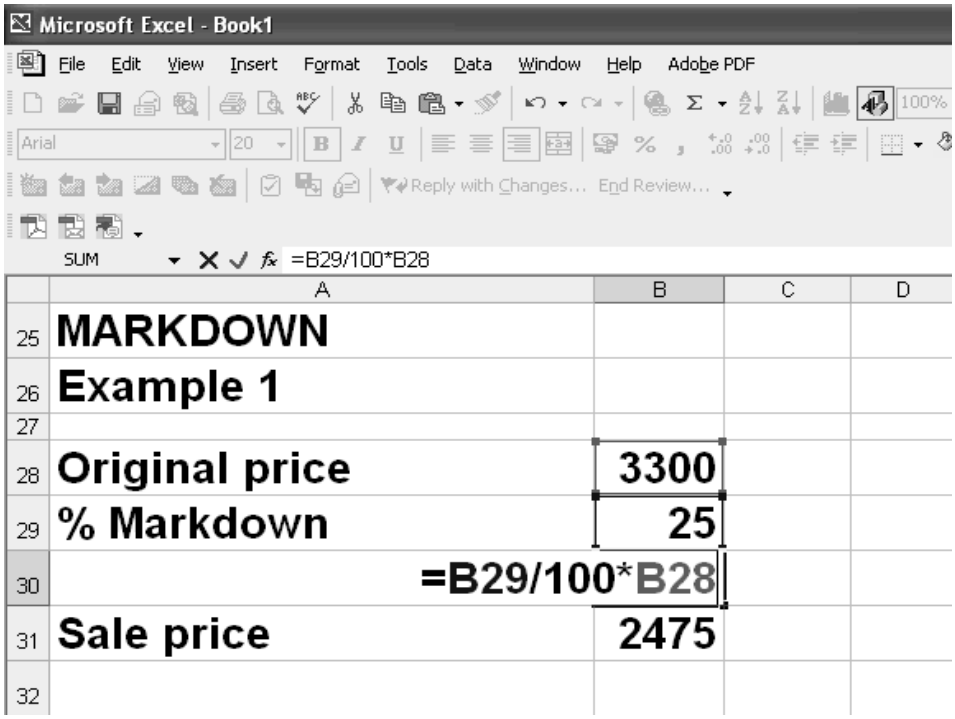
Selling Price
Then calculate the sale price, by subtracting the markdown from the original price:
New Selling price = 3,300 – 825 = 2,475Rs
The sale price is 2,475 Rs.

Calculation using EXCEL
Enter original price 3300 in cell B28.
Enter % Markdown 25 in cell B29.
Enter formula for Rs. Markdown (=B29/100*B28) in cell B30. Here the term B29/100 is the markdown in fraction. The result of this part of the calculation is 825.
Enter formula for new sale price (=B28-B30) in cell B31. This formula is not shown in the slide.

We could have calculated the new sale price directly also by writing just one formula (=(1-B29/100)*B28) by using following formula

New selling price = Current selling price (1 - %markdown)

In other words the multiplication factor is calculated as 1-0.25 = 0.75 and multiplied with the original price 3300. The answer would be the same. By breaking the calculation in parts you can check the intermediate result and avoid errors. But if you become very conversant with formulas then you may wish to reduce the number of unnecessary steps in the calculations.



DISCOUNT

Discount is a reduction in price which the seller offers to the buyer. There are different types of discount.

1. Trade discount.
2. Cash discount
3. Seasonal discount, etc

TRADE DISCOUNT

When a manufacturer or wholesaler offers goods for sale, a list price or retail price is set for each item. This is the suggestion price to be charged from the ultimate consumer. A discount on the list price granted by a manufacturer or wholesaler to buyers in the same trade is called trade discount. Trade discount represents a reduction in list price in return for quantity purchases. Thus

Rs Trade discount = list price × discount rate
Net price = list price – Rs trade discount

There are two main types of trade discounts

1. Single trade discount.
2. Multiple or series trade discount

SINGLE TRADE DISCOUNT-EXAMPLE 1

The price of office equipment is 3000 Rs. The manufacturer offers a 30% trade discount. Find the net price and the trade discount amount.

~~**Discount**~~

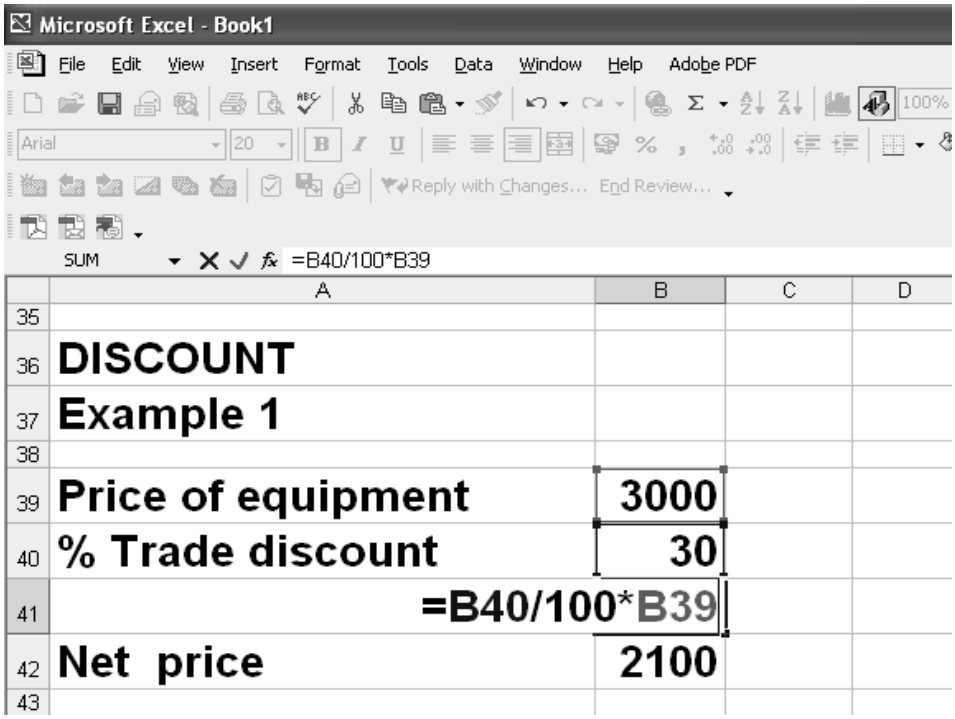
Net Price = L(1 – d)
= 3000(1 – 0.3)
= 3000(0.7)
= 2100 Rs.

Amount of discount = dL
= 0.3 × 3000 = 900 Rs.

Calculation using EXCEL

Enter price of equipment 3000 in cell B39.

Enter % trade Discount 30 in cell B40.
Enter formula for Rs. Discount =B40/100*B3 in cell B41. Here the term B40/100 is the discount in fraction. The result of this part of the calculation is 900.
Enter formula for net price =B39-B41 in cell B42. This formula is not shown in the slide.
The result is 2100 as shown in cell B42.



LECTURE 14
MATHEMATICS OF MERCHANDISING
PART 2

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 13
- Financial Mathematics Part *1*

SERIES TRADE DISCOUNT

This refers to the giving of further discounts as incentives for more sales. Usually such discount is offered for selling product in bulk.

If series discount of 15%, 10%, 5% are offered on list price, say L, of an item then net price is calculated as follows:

Subtract 15% of L from L. Let the new price is L_1.

$L_1 = L - (L \times 15\%)$

Then subtract 10% of L_1 from L_1. Let the new price is L_2.

$L_2 = L_1 - (L_1 \times 10\%)$

Then subtract 5% of L_2 from L_2. The new price is net price of an item.

$N = L_2 - (L_2 \times 5\%)$

Or alternatively

$N = L\ (1 - 15\%)\ (1 - 10\%)\ (1 - 5\%)$

Let $d_1 = 15\%$, $d_2 = 10\%$, $d_3 = 5\%$, then above formula becomes

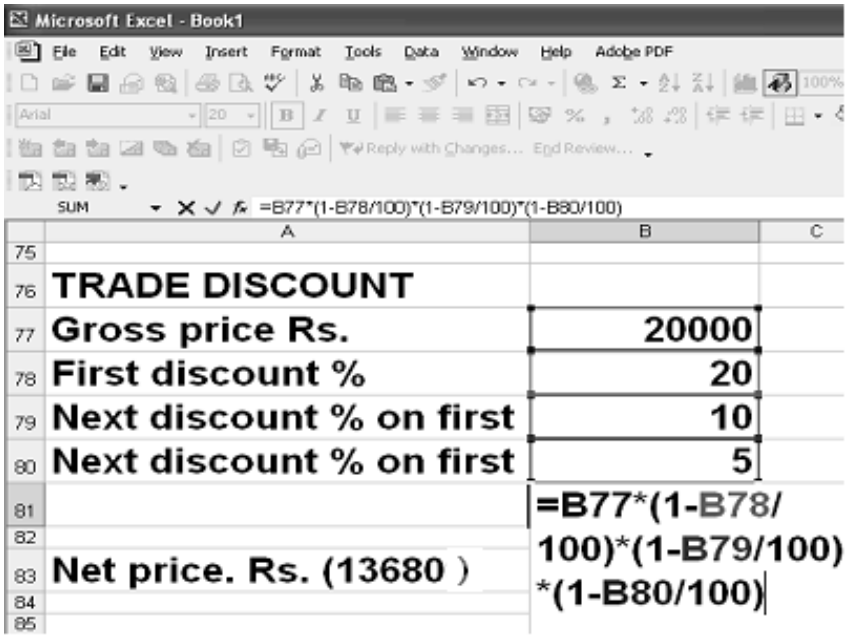
$N = L\ (1 - d_1)(1 - d_2)(1 - d_3)$

Remember: **total discount is not 15% + 10% + 5% = 30%**

SERIES TRADE DISCOUNT-EXAMPLE

The price of office furniture is Rs. 20,000
The series discounts are:
20%,10%, 5%
What is the net price?
For series trade Discount
Net price = $(1-d_1)\ (1-d_2)\ (1-d_3)$
Here $d_1 = 20\%$, $d_2 = 10\%$, $d_3 = 5\%$. So

Net price = 20,000(1-0.2)(1-0.10)(1-0.05)
= 20,000(0.8)(0.9)(0.95)
= 20,000(0.6840)
= **13,680 Rs.**



LIST PRICE

An order for power tools has a Rs. 2100 net price after a 30% trade discount. What is the list price?

Net Price

Net Price = L(1 – d)

L = N / (1 – d)

= 2100/(1 – 0.3)

= 2100 / (0.7)

= 3000 Rs.

EXCEL Calculation

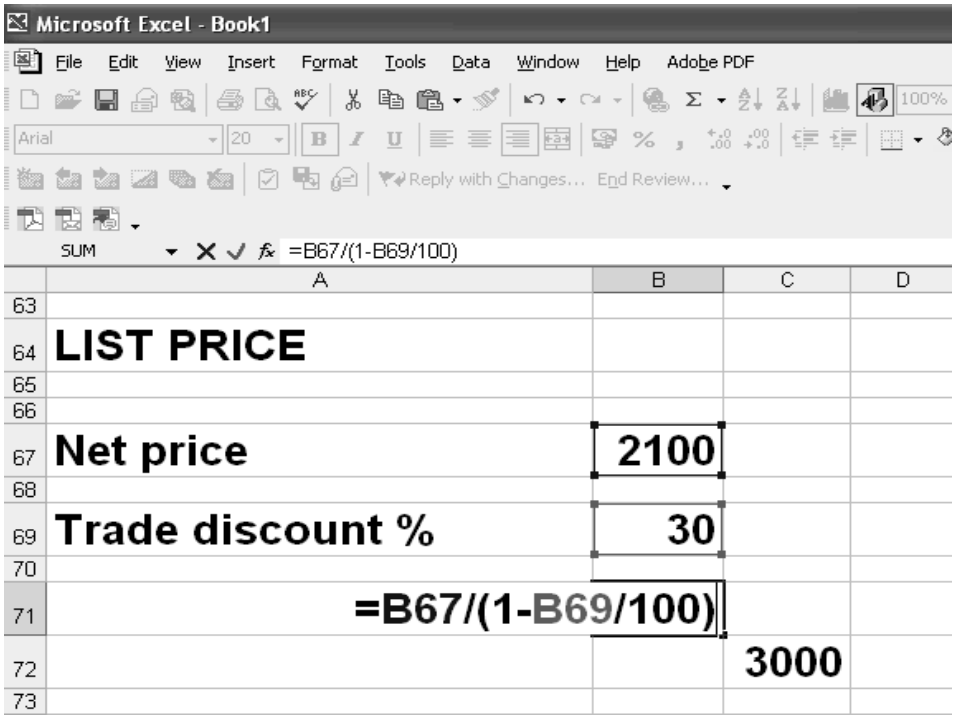
EXCEL formula for list price was based on the calculation = 2100/(1 – 0.3).

The net price was entered in cell B67.

% Trade discount was entered in B69.

The formula for list price was entered in cell B71 as =B67/ (1-B69/100).

The answer is shown in cell C72 as 3000.



TRADE DISCOUNT-EXAMPLE 2

Find the single discount rate that is equivalent to the series 15%, 10% and 5%.

Trade Discount

Apply the multiple discount to a list price of Rs. 100.

Net price = $(1-d_1)(1-d_2)(1-d_3)$

= 100(1 -15%) (1 - 10%) (1 - 5%)

=100(0.85) (0.9) (0.95)

= 100(0.7268)

= 72.68

% Discount = 100 - 72.68

= 27.62%

EXCEL Calculation

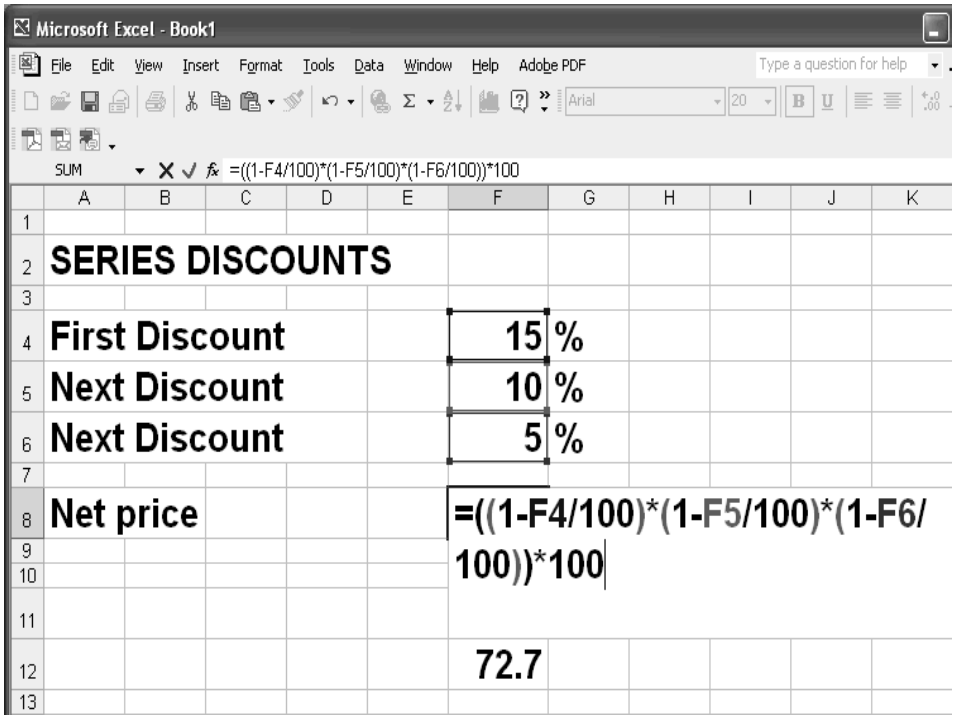
EXCEL formula for net price was based on the calculation

=100*(1-0.15)*(1-0.1)*(1-0.05)

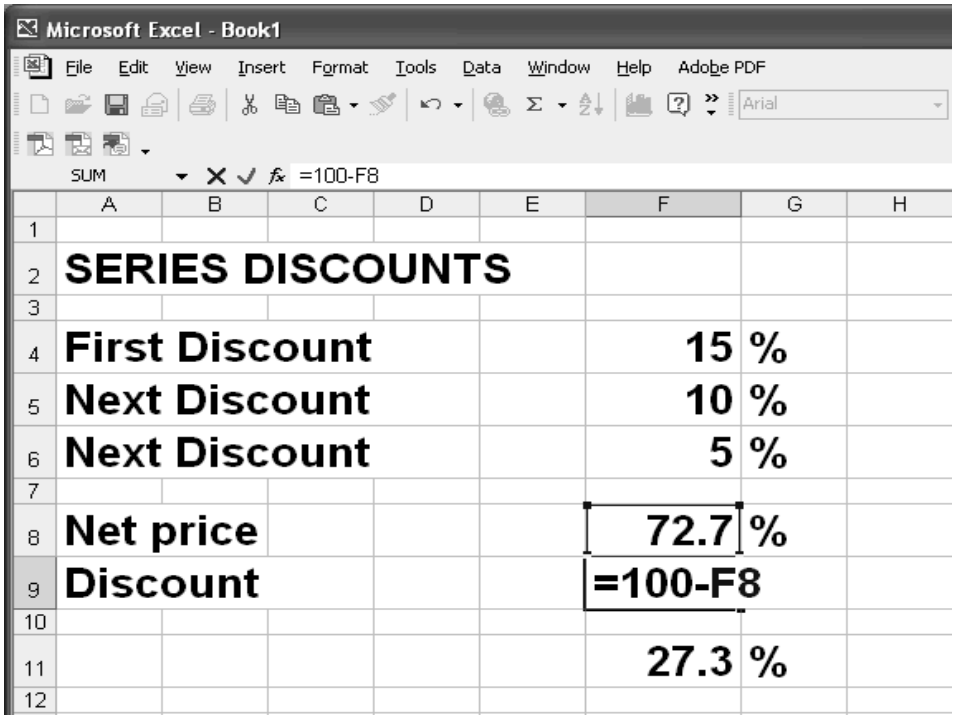
The formula for net price was entered in cell F8.

The formula is shown in cell F8.

The answer is shown in cell F12.



In the following slide, the net price was calculated in cell F8. Then, the discount was calculated assuming the list price was 100. This is a common method to assume 100 as the list price when no price is given but you are required to calculate the net discount.



TRADE DISCOUNT-EXAMPLE 3

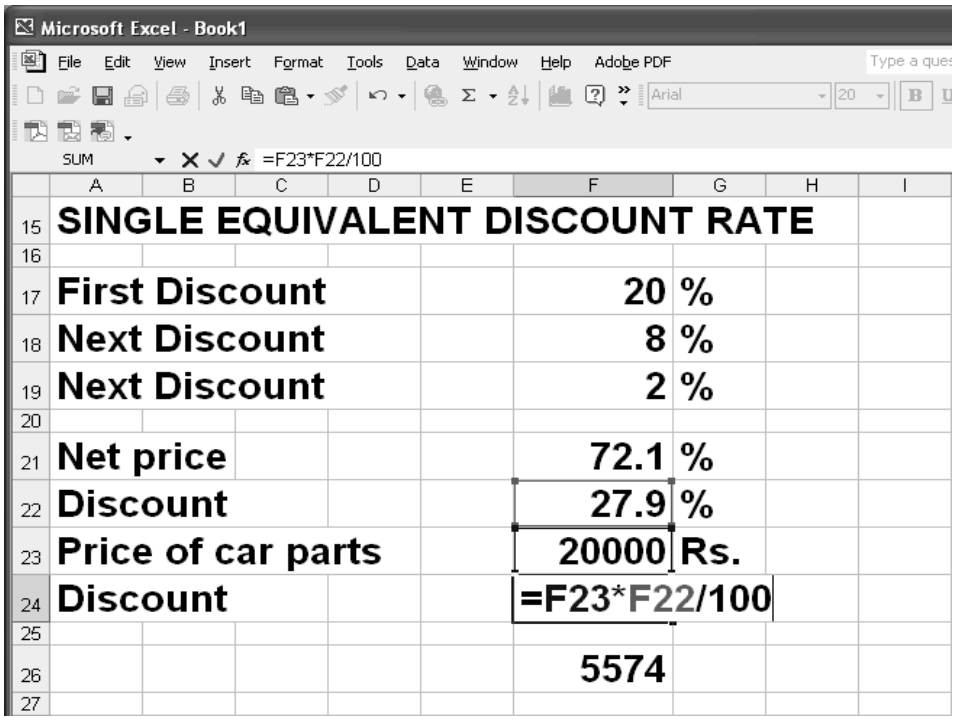
The price of car parts is Rs. 20,000.
The series discounts are 20%, 8%, 2%.
What is the single equivalent discount rate. Also find Rs discount?

Let Rs 100 is the list price then
Net price = 100(1 – 0.2)(1 - 0.08)(1 - 0.02)
=100(0.8)(0.92)(0.98)

= 100(0.7213)
= 72.13%
Single equivalent discount rate = 100 - 72.13 = 27.87%
Rs. Discount = (0.2787)(20000)
= 5,574 Rs.

EXCEL Calculation

EXCEL formula for net price was based on the calculation
=100*(1-0.2)*(1-0.08)*(1-0.02)
The formula for net price was entered in cell F21.
The formula is not shown.
Price of car parts was entered in cell F23.
Formula for discount was based on (0.2787)(20000) and is shown in cell F24.
The answer is shown in cell F26 as 5574.



CASH DISCOUNT

A seller always desires to be paid by the buyer as soon as possible. A discount given for the prompt payment of the dues is called Cash Discount. Such a discount is an advantage to both the seller and the buyer. The buyer has a saving of money while the seller has funds at his disposal.

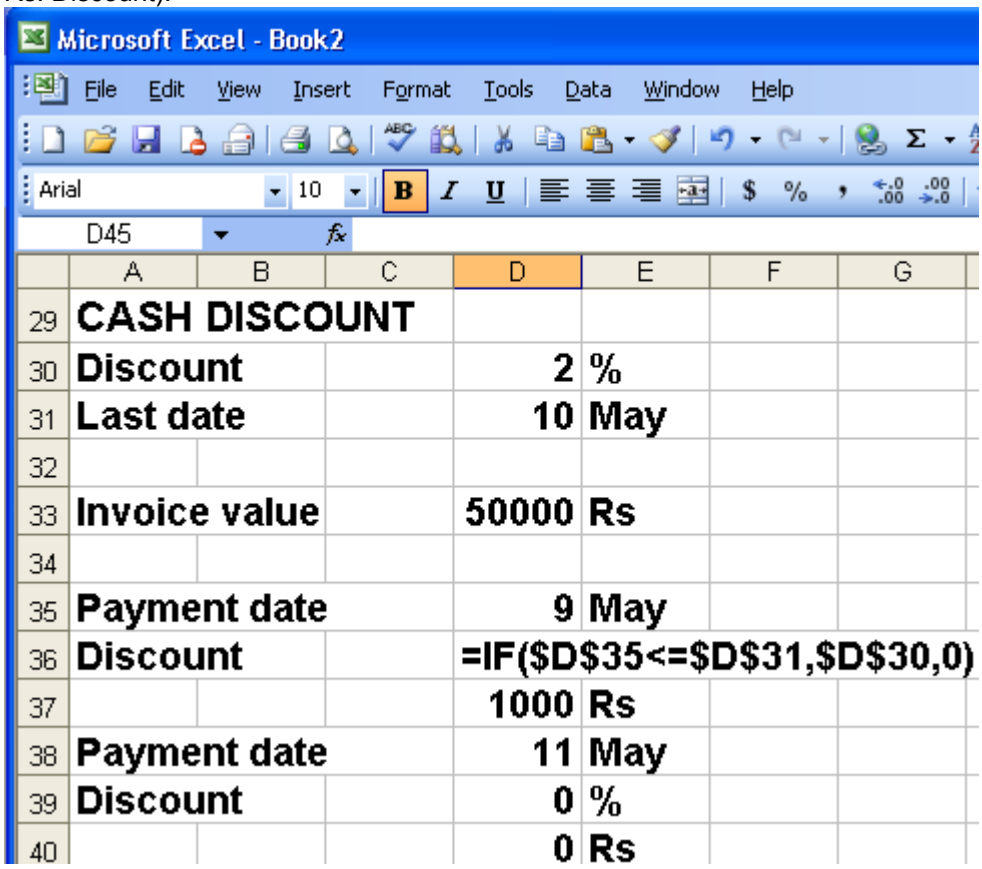
Cash Discount is allowed on Invoices, Returned Goods, Freight, Sales Tax and

A common business phrase for a cash discount is "3/10, net/30," meaning that a 3% discount is offered if the amount due is paid within 10 days; otherwise 100% of the amount due is payable in 30 days. For example, if the amount due is 100Rs, the buyer may pay 97Rs within 10 days or 100 Rs within 30 days.

DISCOUNT PERIODS
Discount Periods are periods for the buyer to take advantage of Discount Terms.
CREDIT PERIODS
Credit Periods are periods for the buyers to pay invoices within specified times.
CASH DISCOUNT-EXAMPLE
Invoice was dated May 1st. The terms 2/10 mean that 2% discount is offered if invoice is paid up to 10thMay.
What is the net payment for invoice value of Rs. 50,000 if paid up to 10th May?

Cash Discount
N = L(1 – d)
= 50,000(1-0.02)
= 50,000(0.98)
= 49,000 Rs.

EXCEL Calculation
EXCEL formula for net price was based on the calculation = 50000*(1-0.02). However, here an IF condition was applied, that means that if the payment date in cell D31 ($ sign is put in front of row and column to fix its location) is less than or equal to 10 May then the discount will be as given in cell D30. Here also $ sign was used to fix the location of the cell.
In cell D38, the date was changed to 11 May and the same formula was applied again. The result as shown in cell D39 and D40 are 0% (for % discount) and 0 Rs(for Rs. Discount).



LECTURE 15
MATHEMATICS OF MERCHANDISING
PART 3

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 14
- Financial Mathematics Part 2

PARTIAL PAYMENTS

When you buy on credit and have cash discount terms, part of the invoice may be paid within the specified time. These part payments are called **Partial Payments**.
Let us look at an example:
You owe Rs. 40,000.
Your terms were 3/10 (3% discount by 10^{th} day).
Within 10 days you sent in a payment of Rs. 10,000.
Rs. 10,000 was a part payment.
How much is your new balance?

First we will find the amount that if 3% discount is given on it, the net amount is 10000Rs.
Let that amount is t.Then
$10000 = t(1 - 0.03)$

This implies, $t = \frac{10000}{(1 - 0.03)}$

Thus, t = 10309Rs
This means that although you pay 10,000Rs, due to 3% cash discount 10309Rs among 40,000Rs is paid.
Hence the new balance = 40000 – 10309 = 29691Rs.

MARKETING TERMS

There are a number of marketing terms.
First of these is the **Manufacturer Cost**. This is the cost of manufacturing.
Next is the price charged to middlemen in 'The Distribution Chain'.
The Distributor>Wholesaler>Retailer is a chain.
The next term is the **Selling Price**. This is the price charged to Consumers by Retailers. It may or may not be the same as list price.

MARKETING, OPERATING EXPENSES AND SELLING PRICE

Gross Sales less Cost of Goods sold gives the Gross Profit. The gross Profit less the Operating Expenses gives the Net Profit.

Operating Expenses

Expenses the company incurs in operating the business, e.g. rent, wages and utilities is called operating Expenses.

Selling Price

Selling Price is composed of Cost and Rs Markup.
Selling Price (S) = Cost (C) + Rs Markup (M)

MARGIN

While determining Sale Price, a company includes the operating expenses and profit to their own cost. This amount is called the margin of the company. It is usually calculated as percentage but can also be expressed as rupees. It is also named as **markup on sale.**

Margin or markup on sale = $\frac{\text{Selling price - Cost price}}{\text{Selling Price}} \times 100\%$

Selling price = Cost price + Rs Margin

Margin and markup confuse many. By margin, company evaluates that for every rupee generated in sales, how much is left over to cover basic operating costs and profit. Markup represents the amount added to a cost to arrive at a selling price

Markup on cost = $\frac{\text{Selling price – Cost price}}{\text{Cost price}} \times 100\%$

For example, an item costs 50Rs, and is sold for 100Rs,

Markup = $\frac{100 - 50}{50} \times 100\% = 100\%$

Margin = $\frac{100 - 50}{100} \times 100\% = 50\%$

Note: Remember unless it is mentioned that markup is on sale, simple markup means markup on cost.

Example

A computer's cost is 9000Rs. An amount of Rs. 3,000 was added to this cost by the retailer to determine the sale price for the consumer.
Thus, the selling price
= 9,000 + 3,000 Rs.
= 12,000 Rs.
Rs. 3,000 is Margin available to meet Expenses and make a Profit.

MARKUP

If the Markup on cost 33% then…
Selling Price (S) = Cost (C) + {Cost (C) × Markup on cost (MUC)}
S = C + (C × MUC)

MARKUP-EXAMPLE

You buy candles for Rs. 10.
You plan to sell them for Rs.15.
What is your Rs. Markup?
What is your percent Markup on cost?
Rs Markup = Selling price – cost price
Selling price – Cost = 15 – 10
= Rs Markup = Rs. 5
Markup on cost = $\frac{\text{Selling price – Cost price}}{\text{Cost price}} \times 100\%$

%Markup = 5/10*100% = 50%

SELLING PRICE

Fawad's Appliances bought a sewing machine for Rs. 1,500.
To make the desired profit, he needs a 60% Markup on Cost.
What is Fawad's Rs. Markup?
What is his Selling price?

Selling Price

Rs Markup = Cost price × %Markup on cost

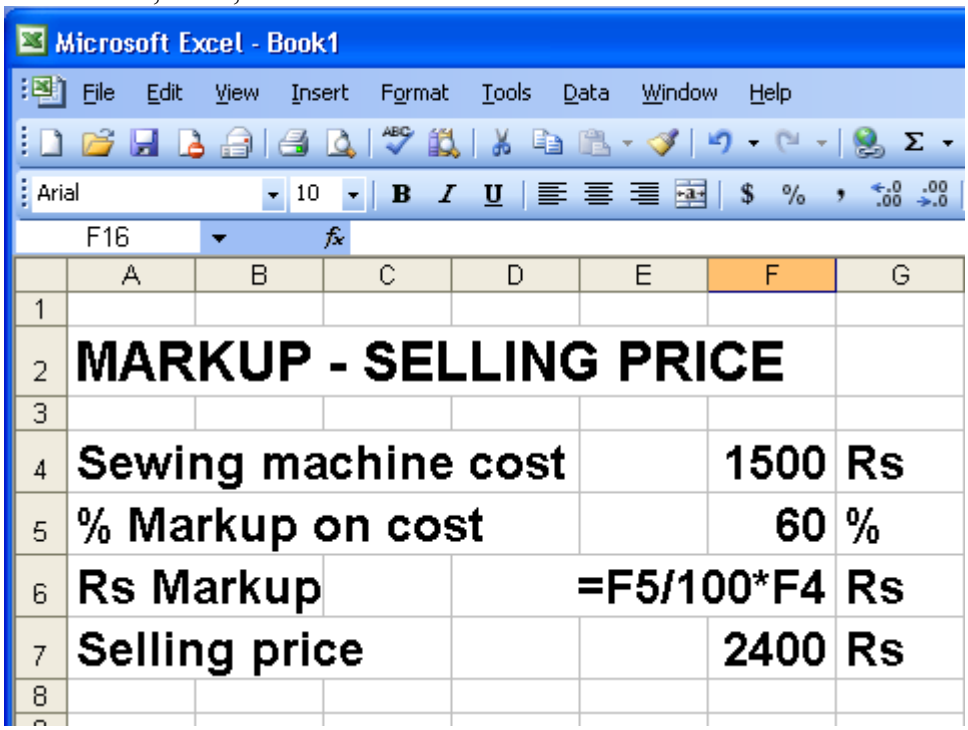
Rs. Markup = 1,500 x 0.6 = 900 Rs.

Selling Price (S) = Cost (C) + Rs Markup (M)
Selling Price = 1,500 + 900 = 2,400 Rs.
~~Or~~
Alternatively, since
Selling Price (S) = Cost (C) + {Cost (C) × Markup on cost (MUC)}

$S = C + (C \times MUC)$
$= C (1 + MUC)$

Selling price
= 1,500 x (1+0.6)
= 1,500 x 1.6
= 2,400 Rs.

EXCEL Calculation
Here 1,500 is the Sewing machine cost in cell F4 and 60 is the Percent Markup on cost in cell F5.
EXCEL formula in cell F6 for Rs. Markup was based on the calculation =60/100*1500.
The Selling price was calculated in cell F7 by using the formula =F4+F6.
The answer, 2400, is shown in cell F7.



RS. MARKUP AND PERCENT ON COST

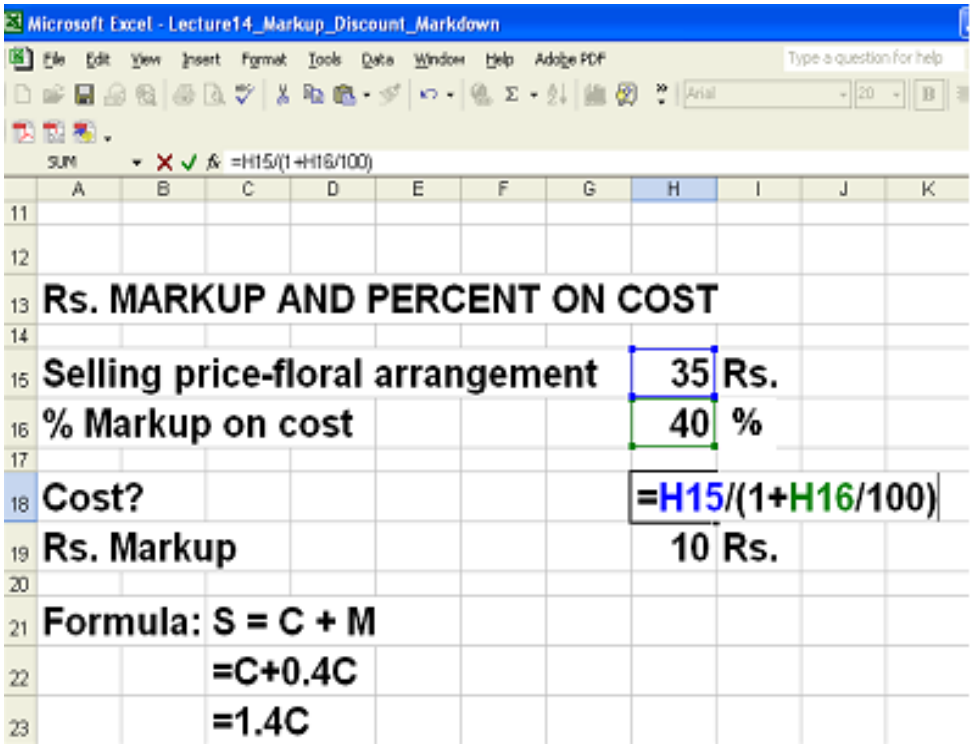
Tanveer's flower business sells floral arrangements for Rs. 35.
To make his desired profit, Tanveer needs a 40% Markup on cost.
What do the flower arrangements cost Tanveer?
What is the Rs. Markup?

Rs. Markup and Percent Markup on Cost

Sale price S = Cost C + {C ×Markup on cost (MUC)}
S = C + 0.40(C)
35 = 1 .40(C)
C = 35/1,4 = 25 Rs.
Rs Markup = 25 x 0.4
= 10 Rs.

EXCEL Calculation

Here 35 is the Selling price-floral arrangement in cell H15.
% Markup on cost is in cell H16.
EXCEL formula in cell H18 for Cost was based on the calculation = 35/1,4.
The Rs. Markup was calculated in cell H19 by using the formula =H18*H16/100.
The answer, as shown in cell H19, was 10.



MARKUP

You buy candles for 2 Rs.
You plan to sell them for 2.50 Rs
What is your Rs. Markup?
What is your Percent Markup on Selling Price?

Rs. Markup

Rs. Markup = 2.5 – 2 = 0.5 Rs.

Percent Markup on Selling Price

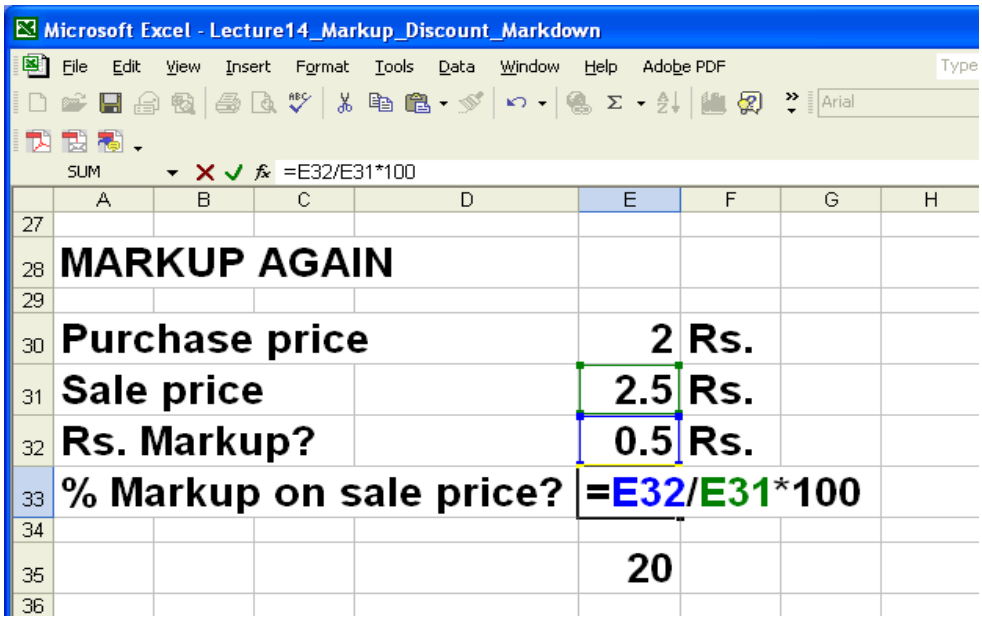
As explained in lecture 13
Cost price = Selling price (1 – %Markup on sale)

Markup on selling price = Selling price - Cost price ×100%
Selling price

Markup on Selling Price = (0.5/2.5) ×100%
= 20%

EXCEL Calculation

Here 2 is the Purchase price in cell E30.
Sale price is entered in cell E31.
Rs. Markup on Purchase Price was calculated by using the formula =E31-E30 in cell E32.
The % Markup on sale price was calculated in cell E33 by using the formula =E32/E31*100.
The answer, as shown in cell E35, was ~~10~~ 20.



SELLING PRICE

Fawad's Appliances bought a sewing machine for Rs. 1,500.
To make the desired profit, he needs a 60% Markup on Selling price.
What is Fawad's Rs. Markup?
What is his Selling Price?

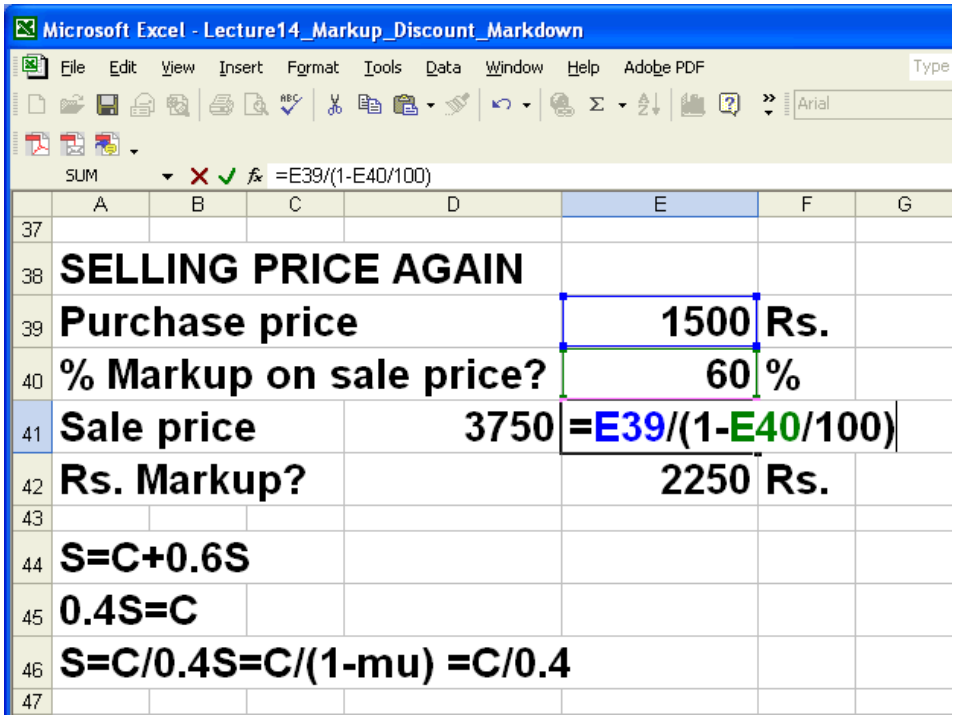
Selling Price

As explained in lecture 13
Selling Price = Cost price + (Selling price × %Markup on sale)

Selling Price S = 1,500 + 0.6S
S - 0.6S = 1,500 Rs.
Or
0.4S = 1,500 = 3,750 Rs

Rs. Markup
Rs. Markup = 3,750 x 0.6
= 2,250 Rs.

EXCEL Calculation
Here 1500 is the Purchase price in cell E39.
% Markup on Sale Price is entered as 60 in cell E40.
Sale Price was calculated by using the formula =E39/(1-E40/100). The result 3750 is shown in cell D41.
EXCEL formula in cell E42 for Rs. Markup was = E41-E39. The result 2250 is shown in cell E42.
Basic formula S=C+0.6S is shown in cell A44. In cell A45 it was simplified to 0.4=C. In cell A46, it is rewritten as S=C/0.4S=C/(1-muc)=C/0.4. Here muc is the %Markup on cost.



RS. MARKUP AND PERCENT MARKUP ON COST
Tanveer's flower business sells floral arrangements for Rs. 35.
To make his desired profit, Tanveer needs a 40% Markup on Selling Price.
What do the flower arrangements Cost Tanveer?
What is the Rs. Markup?

Selling Price

Selling Price = Cost price + (Selling price × %Markup on sale)

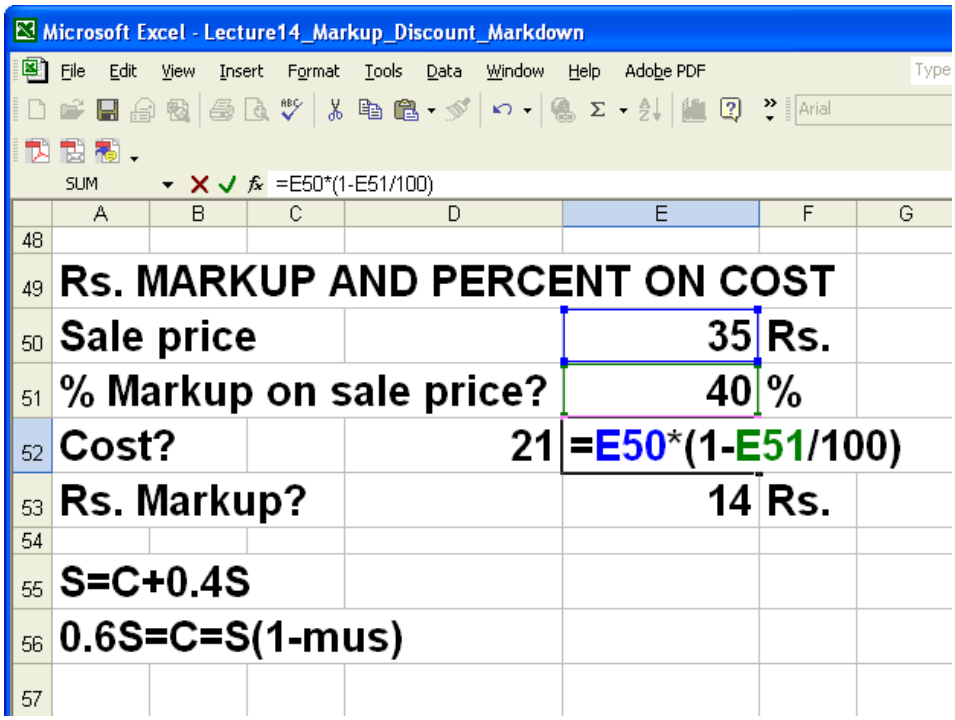
Selling Price =
35 = C + (0.4 x 35)
35 = C + 14
C = 35 – 14
= 21 Rs.
Or, alternatively
C = S- 0.4 S
= 0.6 S = 0.6 x 35= 21 Rs.

Rs. Markup

Rs. Markup = 35 x 0.4 = 14 Rs.

EXCEL Calculation

Here 35 is the Sale price in cell E50.
% Markup on Sale Price is entered as 40 in cell E51.
Cost was calculated by using the formula =E50*(1-E51/100). The result 21 is shown in cell D52.
EXCEL formula in cell E53 for Rs. Markup was = E50-E52. The result 14 is shown in cell E53.
Basic formula S=C+0.4S is shown in cell A55. In cell A56 it was simplified to 0.6S=C=S(1-mus).



CONVERTING MARKUPS

Convert 50% Markup (MU) on Cost to %MU on Sale

Formula for converting %Markup on Sale (mus) to %Markup on Cost Price (muc) is:

% Markup on Selling Price (mus) =
%Markup on Cost / (1 + %Markup on Cost)

mus = muc/(1+muc)

Solution

% Markup on Sale (mus) = 0.5 / (1+0.5) = 0.5/1.5
mus= 0.3333 = 33.33%

Converting Markups
Converting 33.33% MU on Sale to %MU on C

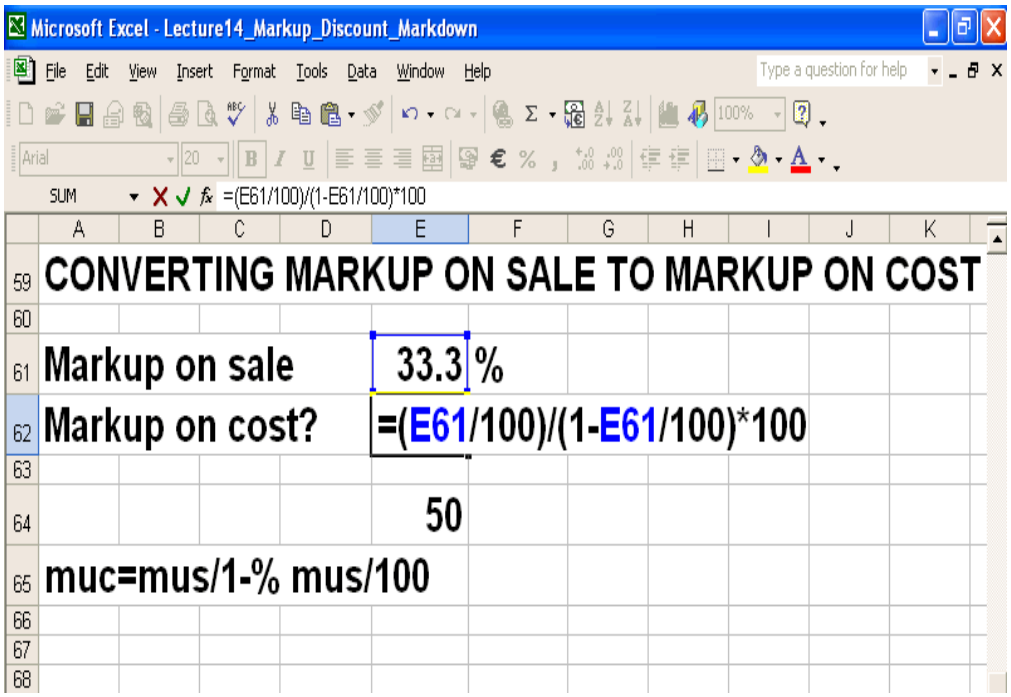
Formula
Convert % Markup on Cost (muc) to % Markup on selling price (mus):

% Markup on cost = % Markup on S / (1 - % Markup on S)

muc = mus / (1-mus)
Markup on cost = 0.3333/(1 – 0.333)
= 0.3333/0.6666 = 0.5
= 50%

EXCEL Calculation

Here 33.3 is the Markup on sale in cell E61.
EXCEL formula in cell E62 for Markup on cost was = (E61/100)/(1-E61/100)*100.
The result 50 is shown in cell E64.
Basic formula muc=mus / (1-%mus) is shown in cell A65.



LECTURE 16
MATHEMATICS OF MERCHANDISING
PART 4

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 15
- Markup and Markdown

Financial Mathematics Part 3

MARKDOWN

Reduction from original selling Price is called Markdown.

Formula

%Markdown = (Rs. Markdown / Selling Price (original)) ×100%

MARKDOWN-EXAMPLE 1

Store A marked down a Rs. 500 shirt to Rs. 360.
What is the Rs. Markdown?
What is the %markdown?

Rs. Markdown

Let S = Sale price
Rs. Markdown = Old S – New S
= Rs. 500 – Rs. 360
= Rs. 140 Markdown

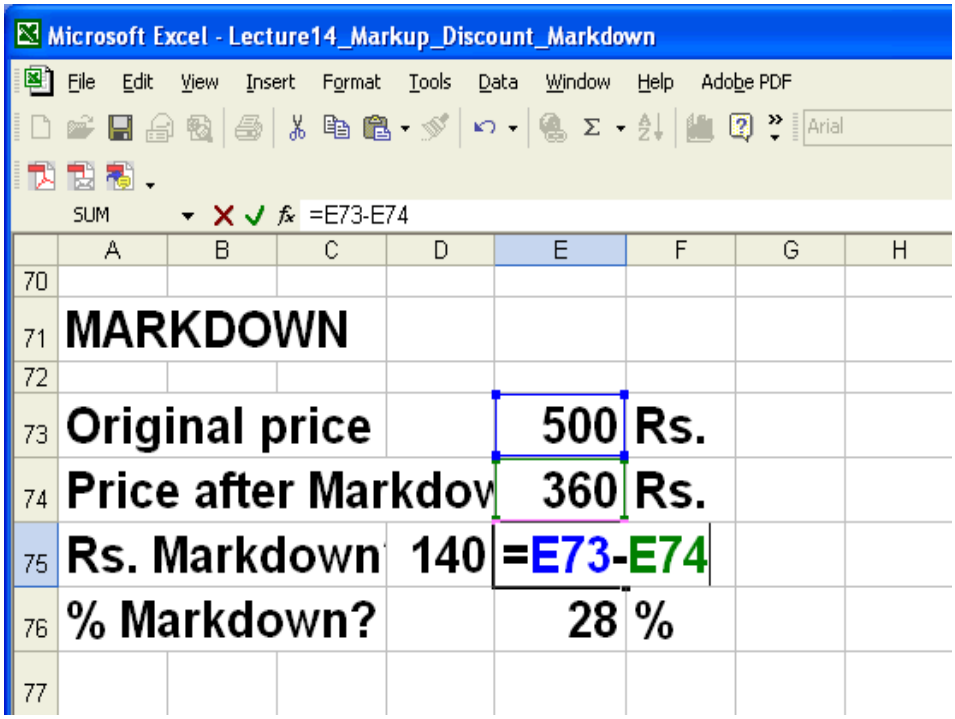
% Markdown

$$\% \text{ Markdown} = \frac{\text{Markdown}}{\text{Old S}} \times 100\%$$

$$\% \text{ Markdown} = \frac{140}{500} \times 100\%$$
$$= 0.28 \times 100\%$$
$$= 28\ \%$$

EXCEL Calculation

Here 500 is the Original price in cell E73.
Price after Markdown is entered as 360 in cell E74.
Rs. Markdown was calculated in cell E75 by using the formula =E73-E74. The result 140 is shown in cell D75.
EXCEL formula in cell E76 for % Markdown was = E75/E73*100. The result 28 is



MARKDOWN-EXAMPLE 2

A variety of plastic jugs that was bought for Rs. 57.75 was marked up 45% of the SellingPrice.
When the jugs went out of production, they were marked down 40%
What was the Sale Price after the 40% markdown?
Here, there are two parts to this problem. First we must find the original sale price so that markdown can be calculated on that price.

Original Sale Price

Let Selling price = 100
%Markup on selling price = 45%
Cost = 100 – 45 = 55
Thus Original Sale price = (100/55) x 57.75 = 105Rs

Rs. Markdown

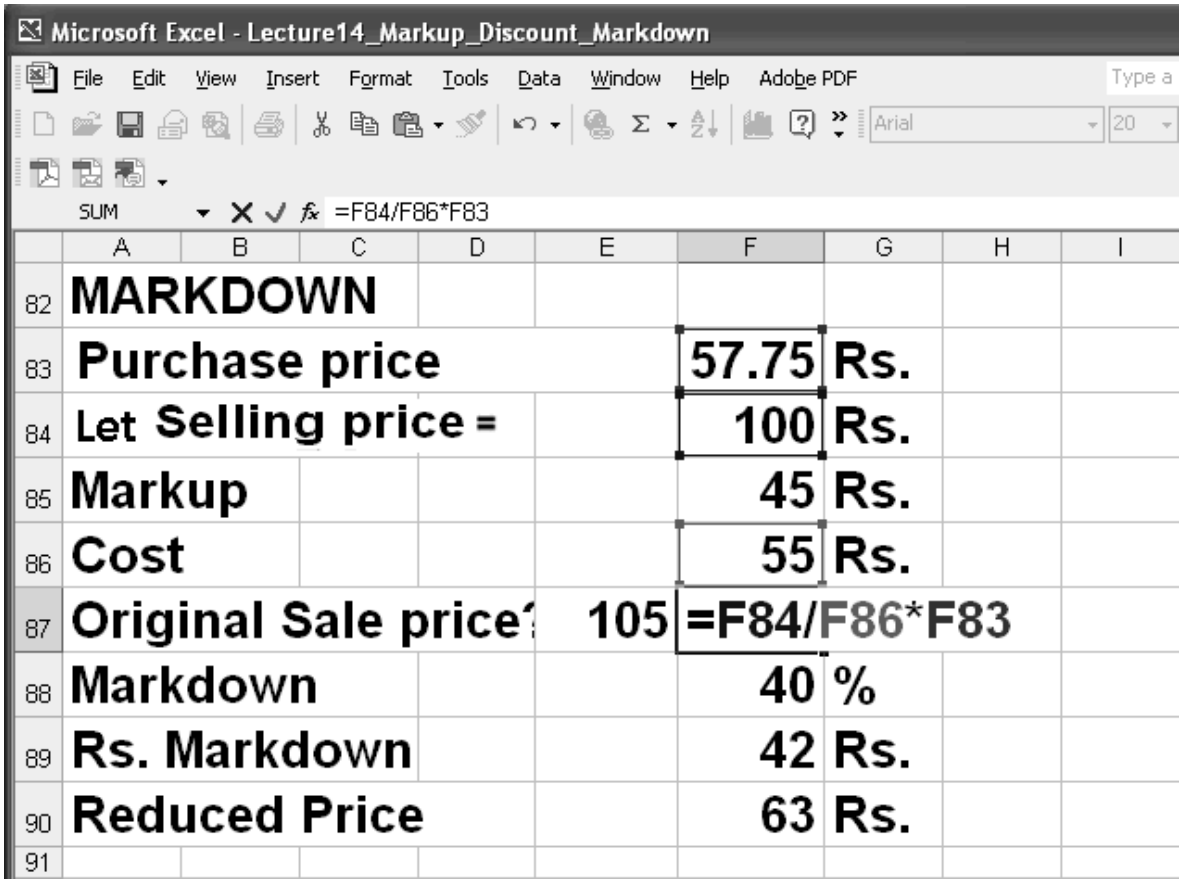
%Markdown = 40% = 0.4
Rs. Markdown = 105 x 0.4 = 42

Sale price after markdown

Sale price after markdown = 105 – 42 = 63 Rs.

EXCEL Calculation

Here 57.75 is the purchase price in cell F83.
Selling price is entered as 100in cell F84.
Rs. Markup was calculated in cell F85 using the formula =F84-F83. The result is shown as 45 in cell F85.
Original Sale Price was calculated in cell F87 by using the formula =F84/F86*F83. The result 105 is shown in cell E87.
% Markdown was entered as 40 in cell F88.
The Rs. Markdown was calculated using the formula =F87*F88/100 in cell F89. The result 42 is shown in cell F89.
The reduced price was calculated by using the formula =F87-F89 in cell F90. The result is shown as 63 in cell F90.



PROJECT FINANCIAL ANALYSIS

Financial analysis is the analysis of the accounts and the economic prospects of a firm, which can be used to monitor and evaluate the firm's financial position, to plan future financing, and to designate the size of the firm and its rate of growth.

When you carry out Project Financial analysis, a number of Financial Calculations are required. The important ones are summarized below:

- Cost estimates
- Revenue estimates
- Forecasts of costs
- Forecasts of revenues
- Net cash flows
- Benefit cost analysis
- Internal Rate of Return
- Break-Even Analysis

COST ESTIMATES

In every project you will be required to prepare a cost estimate. Generally, such cost estimates cover calculations based on quantities and unit rates. Such calculations are done in the form of tabular worksheets. In large projects there may be a number of separate calculations for part projects. Such component costs are then combined to calculate total cost. These are simple worksheet calculations unless conditional processing is required. Such conditional processing is useful if unit prices are to be found for a specific model from a large database.

REVENUE ESTIMATES

Along with costs even revenues are calculated. These calculations are similar to component costs.

FORECASTS OF COSTS

Forecasting requires a technique for projections. One of such technique, Time Series Analysis, will be covered later in this course. Forecasting techniques vary from case

to case. The applicable method should be determined first. Calculation of future forecasts can then be done through worksheets.

FORECASTS OF REVENUES

These will be done similar to the forecast of costs. Here also the method must be determined first. Once the methodology is clear, the worksheets can be prepared easily.

NET CASH FLOWS

The difference between Revenue and Cost is called the Net Cash flow. This is an important calculation as the entire Project Operation and Performance is based on its cash flows.

BENEFIT COST ANALYSIS

This is the end result of the Project Analysis. The ratio between Present Worth of Benefits and Costs is called the Benefit Cost (BC) ratio. For a project to be viable without profit or loss, the BC Ratio must be 1 or more. Generally a BC Ratio of 1.2 is considered acceptable. For Public projects even lesser BC ratio may be accepted for social reasons.

INTERNAL RATE OF RETURN

Internal Rate of Return or IRR is that Discount Rate at which the Present Worth of Costs is equal to the Present Worth of Benefits. IRR is the most important parameter in Financial and Economic Analysis. There are a number of functions in EXCEL for calculation of IRR.

BREAK-EVEN ANALYSIS

In every project where investment is made it is important to know how long it takes to recover the investment. It is also important to find the breakeven point where the Cash Inflow becomes equal to Cash Outflow. After that point the company has a positive cash flow (i.e. there is surplus cash after meeting expenses).

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LECTURE 17
MATHEMATICS FINANCIAL MATHEMATICS
INTRODUCTION TO SIMULTANEOUS EQUATIONS

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 16
- Financial Mathematics
- Introduction to Linear Equations

Module 4

Module 4 covers the following:

- Financial Mathematics (Lecture 17)
- Applications of Linear Equations
- (Lecture 17-18)
- Break-even Analysis
- (Lectures 19-22)
- Mid-Term Examination

PROJECT FINANCIAL ANALYSIS

Project Financial Analysis covers the following:

- Cost estimates
- Revenue estimates
- Forecasts of costs
- Forecasts of revenues
- Net cash flows
- Benefit cost analysis
- Internal Rate of Return
- Break-Even Analysis

EXCEL FUNCTIONS FOR FINANCIAL ANALYSIS

List of Excel Financial functions is as under. The name and utility of each function is given below:

AMORDEGRC

Returns the depreciation for each accounting period. If an asset is purchased in the middle of the accounting period, the prorated depreciation is taken into account. The function is similar to AMORLINC, except that a depreciation coefficient is applied in the calculation depending on the life of the assets.

Syntax

AMORDEGRC(cost,date_purchased,first_period,salvage,period,rate,basis)

Important Dates should be entered by using the DATE function, or as results of other formulas or functions. For example, use DATE(2008,5,23) for the 23rd day of May, 2008. Problems can occur if dates are entered as text.

Cost cost of the asset.
Date_purchased date of the purchase of the asset.
First_period date of the end of the first period.
Salvage salvage value at the end of the life of the asset.
Period period.
Rate rate of depreciation.
Basis year basis to be used.

Basis	Date system
0 or omitted	360 days (NASD method)
1	Actual
3	365 days in a year
4	360 days in a year (European method)

Remarks

- Microsoft Excel stores dates as sequential serial numbers so they can be used in calculations. By default, January 1, 1900 is serial number 1, and January 1, 2008 is serial number 39448 because it is 39,448 days after January 1, 1900.
- This function will return the depreciation until the last period of the life of the assets or until the cumulated value of depreciation is greater than the cost of the assets minus the salvage value.
- The life of the asset is calculated by (1 / "rate"). The depreciation coefficient depends on the life of the asset.
- If the life of the asset is between 3 and 4 years, the coefficient is 1.5.
- If the life of the asset is between 5 and 6 years then the coefficient is 2.
- If the life is the asset is greater than 6 years then the coefficient is 2.5.
- The depreciation rate will grow to 50 percent for the period preceding the last period and will grow to 100 percent for the last period.
- If the life of assets is between 0 (zero) and 1, 1 and 2, 2 and 3, or 4 and 5, the #NUM! error value is returned.

See its example in next lecture.

AMORLINC

Returns the depreciation for each accounting period. If an asset is purchased in the middle of the accounting period, the prorated depreciation is taken into account.

Syntax

AMORLINC(cost,date_purchased,first_period,salvage,period,rate,basis)
Cost cost of the asset
Date_purchased date of the purchase of the asset.
First_period date of the end of the first period.
Salvage salvage value at the end of the life of the asset.
Period period.
Rate rate of depreciation.
Basis year basis to be used.

Basis	Date system
0 or omitted	360 days (NASD method)
1	Actual
3	365 days in a year
4	360 days in a year (European method)

AMORLINC-EXAMPLE

	Data	Description
A2	2400	Cost
A3	8/19/2008	Date purchased
A4	12/31/2008	End of the first period
A5	300	Salvage value
A6	1	Period
A7	15%	Depreciation rate
A8	1	Actual basis (see above)

Formula	***Result Description***
=AMORLINC(A2,A3,A4,A5,A6,A7,A8)	*First period depreciation (360)*

CUMIPMT
Returns the cumulative interest paid between two periods.
For description see lecture 8.

CUMPRINC
Returns the cumulative principal paid on a loan between two periods
For description see lecture 8.

DB
Returns the depreciation of an asset for a specified period using the fixed-declining balance method.

Syntax

DB(**cost**,**salvage**,**life**,**period**,month)
Cost is the initial cost of the asset.
Salvage is the value at the end of the depreciation (sometimes called the salvage value of the asset).
Life is the number of periods over which the asset is being depreciated (sometimes called the useful life of the asset).
Period is the period for which you want to calculate the depreciation. Period must use the same units as life.
Month is the number of months in the first year. If month is omitted, it is assumed to be 12.
Remarks

- The fixed-declining balance method computes depreciation at a fixed rate. DB uses the following formulas to calculate depreciation for a period:
  (cost - total depreciation from prior periods) * rate
  where:
  rate = 1 - ((salvage / cost) ^ (1 / life)), rounded to three decimal places
- Depreciation for the first and last periods is a special case. For the first period, DB uses this formula:
  cost * rate * month / 12
- For the last period, DB uses this formula:
  ((cost - total depreciation from prior periods) * rate * (12 - month)) / 12

DDB
Returns the depreciation of an asset for a specified period using the double-declining balance method or some other method you specify

Syntax

DDB(**cost**,**salvage**,**life**,**period**,factor)
Cost is the initial cost of the asset.
Salvage is the value at the end of the depreciation (sometimes called the salvage value of the asset). This value can be 0.
Life is the number of periods over which the asset is being depreciated (sometimes called the useful life of the asset).
Period is the period for which you want to calculate the depreciation. Period must use the same units as life.
Factor is the rate at which the balance declines. If factor is omitted, it is assumed to be 2 (the double-declining balance method).

Important All five arguments must be positive numbers.

Remarks

- The double-declining balance method computes depreciation at an accelerated rate. Depreciation is highest in the first period and decreases in successive periods. DDB uses the following formula to calculate depreciation for a period:
  Min((cost - total depreciation from prior periods) * (factor/life), (cost - salvage - total depreciation from prior periods))

- Change factor if you do not want to use the double-declining balance method.

MIRR

Returns the modified internal rate of return for a series of periodic cash flows. MIRR considers both the cost of the investment and the interest received on reinvestment of cash.

Syntax

MIRR(**values**,**finance_rate**,**reinvest_rate**)

Values is an array or a reference to cells that contain numbers. These numbers represent a series of payments (negative values) and income (positive values) occurring at regular periods.

Values must contain at least one positive value and one negative value to calculate the modified internal rate of return. Otherwise, MIRR returns the #DIV/0! error value.

If an array or reference argument contains text, logical values, or empty cells, those values are ignored; however, cells with the value zero are included.

Finance_rate is the interest rate you pay on the money used in the cash flows.

Reinvest_rate is the interest rate you receive on the cash flows as you reinvest them.

INTERNAL RATE OF RETURN IRR

Returns the internal rate of return for a series of cash flows

Syntax

IRR(values,guess)

Values is an array or a reference to cells that contain numbers for which you want to calculate the internal rate of return.

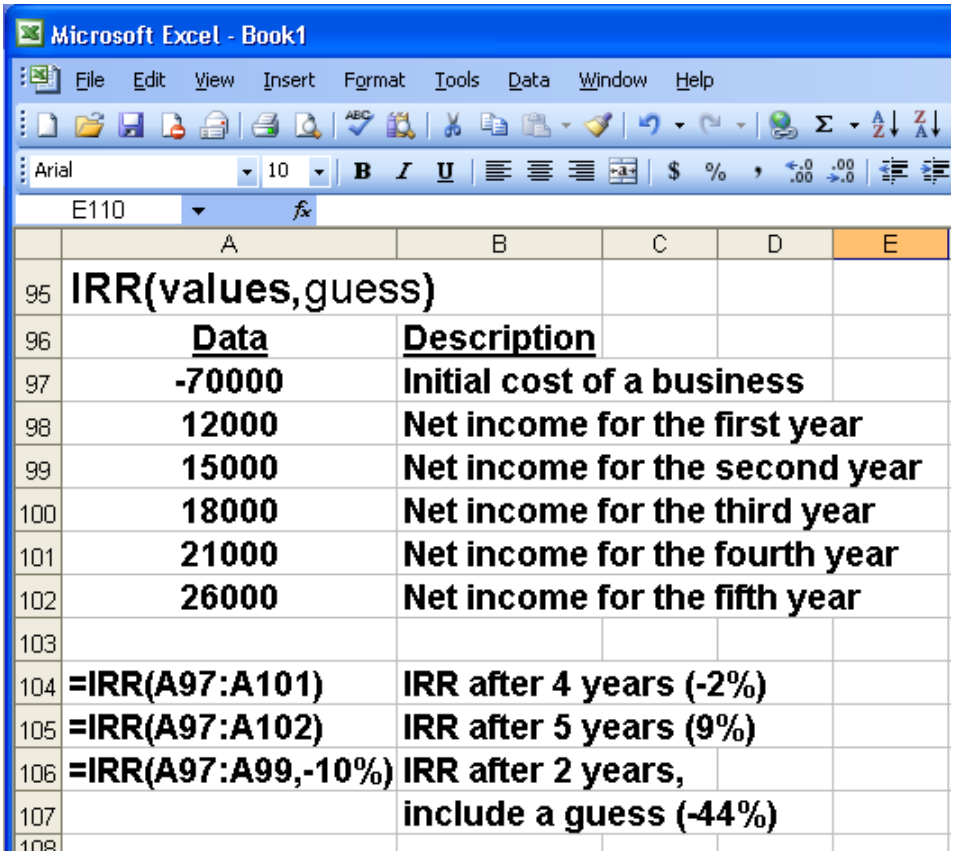
- Values must contain at least one positive value and one negative value to calculate the internal rate of return.
- IRR uses the order of values to interpret the order of cash flows. Be sure to enter your payment and income values in the sequence you want.
- If an array or reference argument contains text, logical values, or empty cells, those values are ignored.

Guess is a number that you guess is close to the result of IRR.

- Microsoft Excel uses an iterative technique for calculating IRR. Starting with guess, IRR cycles through the calculation until the result is accurate within 0.00001 percent. If IRR can't find a result that works after 20 tries, the #NUM! error value is returned.
- In most cases you do not need to provide guess for the IRR calculation. If guess is omitted, it is assumed to be 0.1 (10 percent).
- If IRR gives the #NUM! error value, or if the result is not close to what you expected, try again with a different value for guess.

IRR-EXAMPLE

In the slide the Excel worksheet is shown.
In cell A97, the investment of 70,000 is entered with minus sign to denote negative cash flow.
In cell A98 to A102, revenue per year ~~(1 to 5)~~ is entered.
=IRR(A97:A101) formula in cell A103, only years 1 to 4 were selected from the revenue stream. The IRR is -2% in this case.
In the next formula in cell A105, the entire revenue stream was considered. The IRR improved to 9%.
Next only first 2 years of revenue stream were considered with an initial guess of -10%. The result was -44%.


Microsoft Excel - Book1
File Edit View Insert Format Tools Data Window Help
Arial 10
E110
A B C D E
95 IRR(values,guess)
96 Data Description
97 -70000 Initial cost of a business
98 12000 Net income for the first year
99 15000 Net income for the second year
100 18000 Net income for the third year
101 21000 Net income for the fourth year
102 26000 Net income for the fifth year
103
104 =IRR(A97:A101) IRR after 4 years (-2%)
105 =IRR(A97:A102) IRR after 5 years (9%)
106 =IRR(A97:A99,-10%) IRR after 2 years,
107 include a guess (-44%)

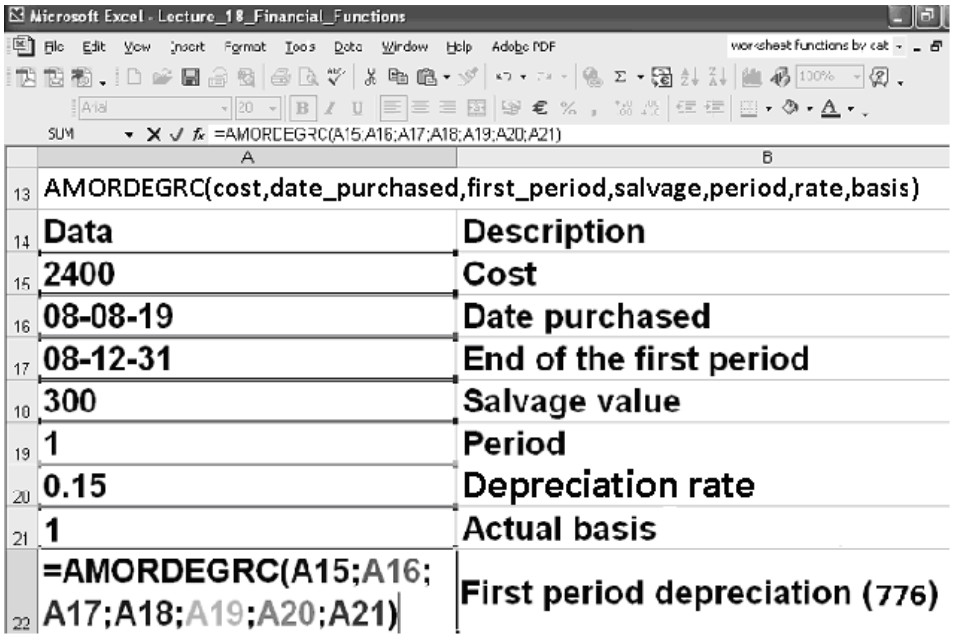
LECTURE 18
MATHEMATICS FINANCIAL MATHEMATICS
SOLVE TWO LINEAR EQUATIONS WITH TWO UNKNOWNS

OBJECTIVES

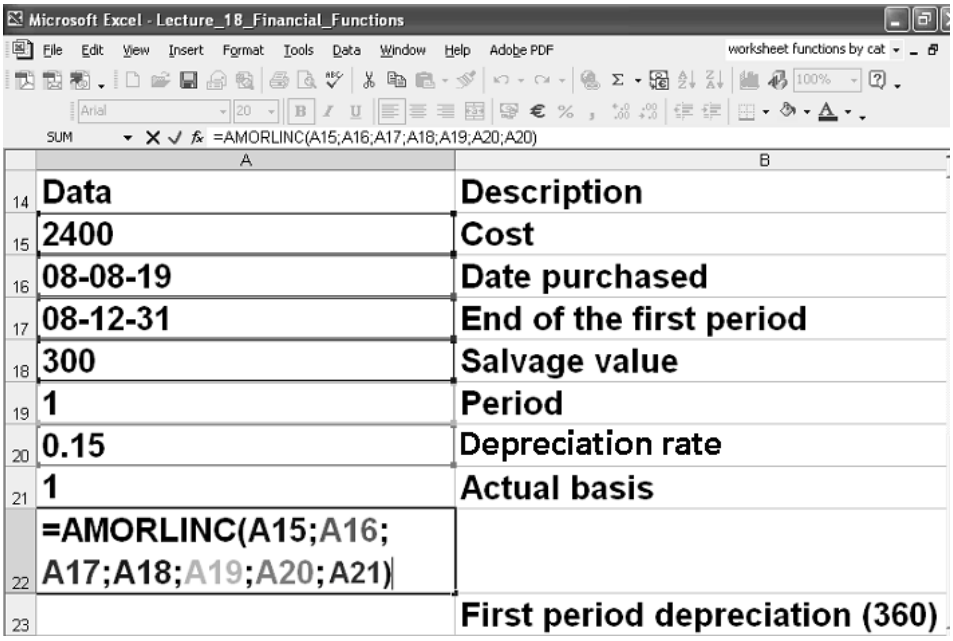
The objectives of the lecture are to learn about:

- Review Lecture 17
- Solve two linear equations with two unknowns

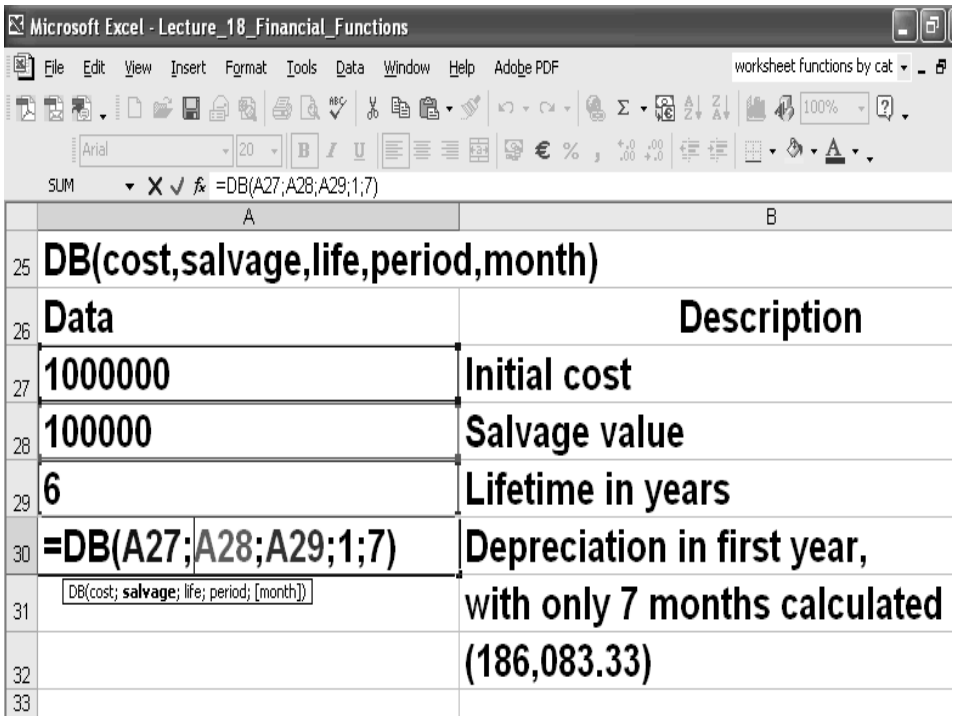
AMORDEGRC-EXAMPLE



AMORLINC-EXAMPLE



DB-EXAMPLE



ADDITIONAL DB_EXAMPLES

Look at the following examples to see how the DB function can be used in different ways.

=DB(A27,A28,A29,1,7) Depreciation in first year, with only 7 months calculated (186,083.33)

- =DB(A27,A28,A29,2,7) Depreciation in second year (259,639.42)
- =DB(A27,A28,A29,3,7) Depreciation in third year (176,814.44)
- =DB(A27,A28,A29,4,7) Depreciation in fourth year (120,410.64)
- =DB(A27,A28,A29,5,7) Depreciation in fifth year (81,999.64)
- =DB(A27,A28,A29,6,7) Depreciation in sixth year (55,841.76)
- =DB(A27,A28,A29,7,5) Depreciation in seventh year, with only 5 months calculated (15,845.10)

PV

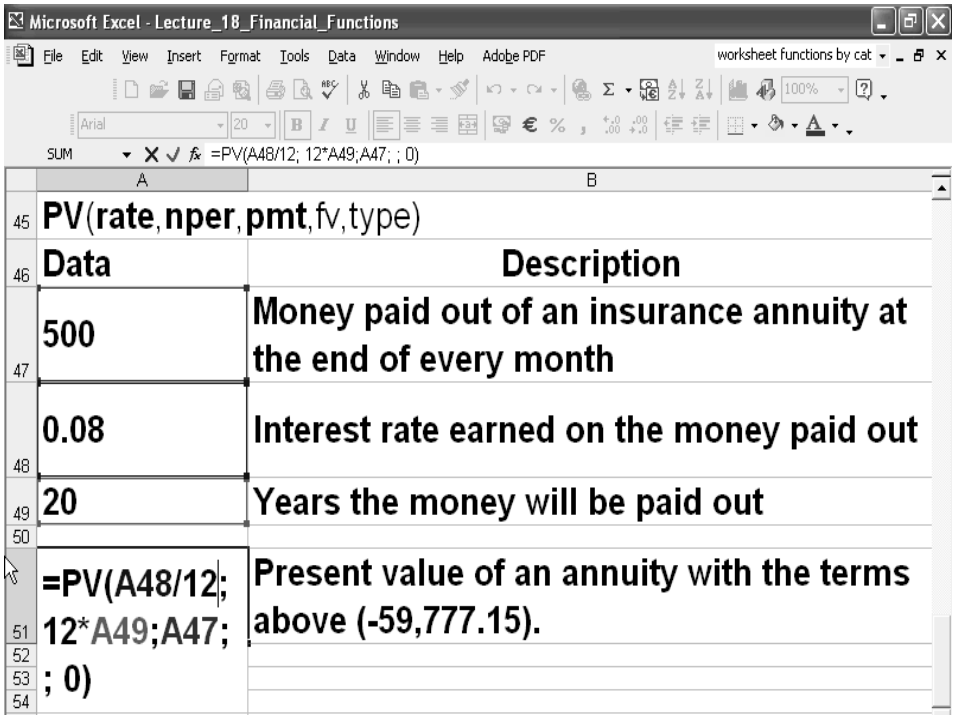
Returns the present value of an investment

Syntax

PV(rate,nper,pmt,fv,type)

Rate interest rate per period

Nper total number of payment periods in an annuity
Pmt payment made each period and cannot change over the life of the annuity
Fv future value, or a cash balance you want to attain after the last payment is made
Type number 0 or 1 and indicates when payments are due.

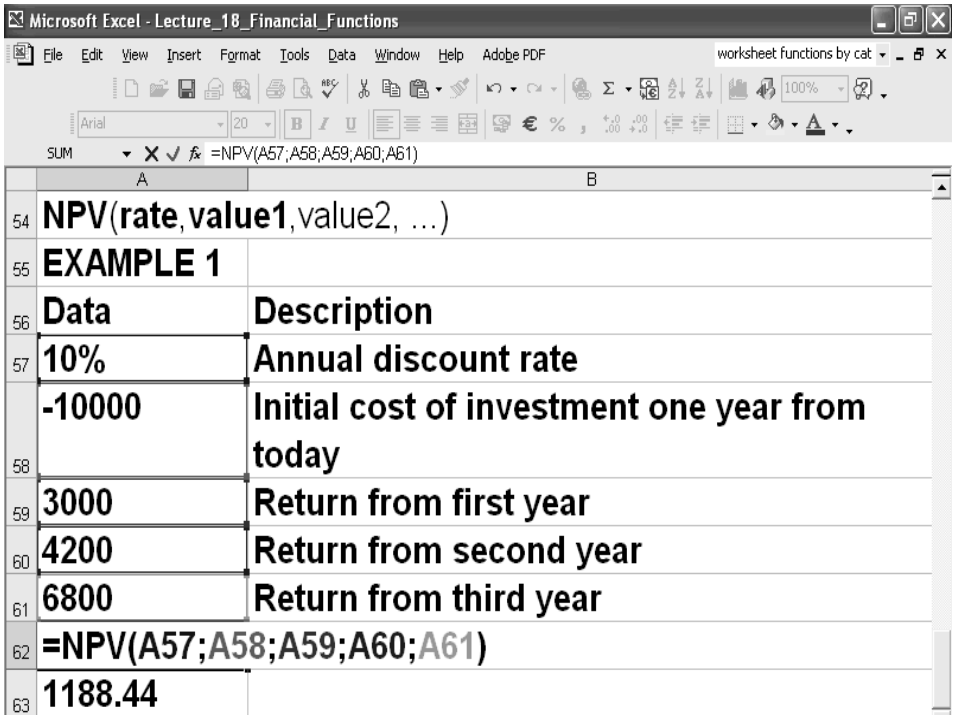


NPV

Returns the net present value of an investment based on a series of periodic cash flows and a discount rate

Syntax

NPV(rate,value1,value2, ...)
Rate rate of discount over the length of one period
Value1, value2, ...
1 to 29 arguments representing the payments and income



XNPV

Returns the net present value for a schedule of cash flows that is not necessarily periodic

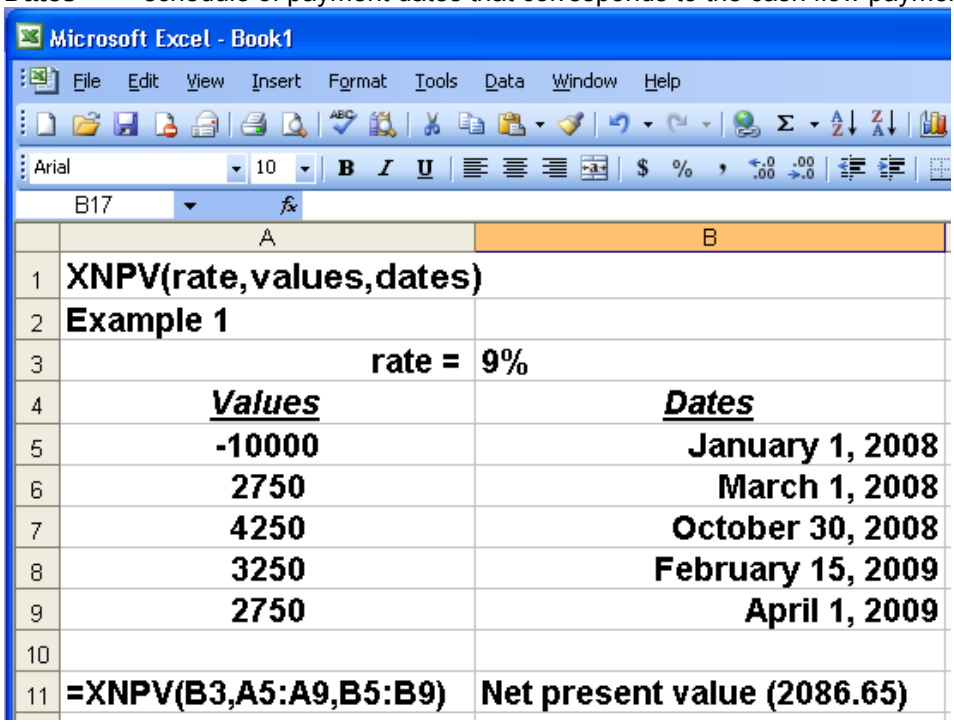
Syntax

XNPV(rate,values,dates)

Rate discount rate to apply to the cash flows

Values series of cash flows that corresponds to a schedule of payments in dates

Dates schedule of payment dates that corresponds to the cash flow payments



LECTURE 19
PERFORM BREAK-EVEN ANALYSIS
EXCEL FUNCTIONS FINANCIAL ANALYSIS

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 18
- Perform break-even analysis
- MS EXCEL Financial Functions

SLN

Returns the straight-line depreciation of an asset for one period

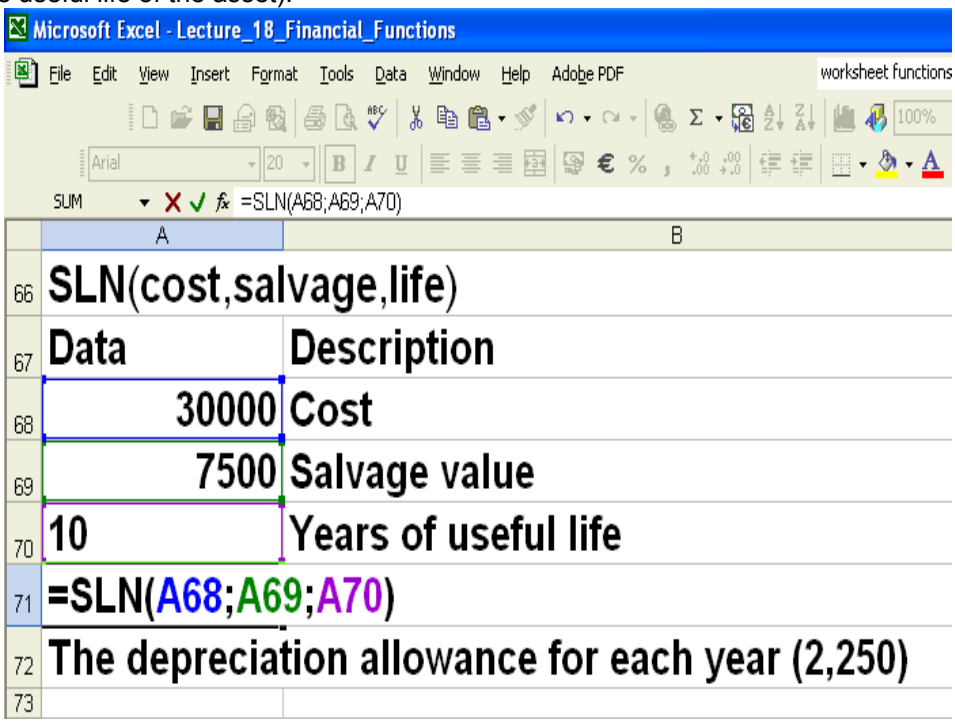
Syntax:

SLN(cost,salvage,life)

Cost is the initial cost of the asset.

Salvage is the value at the end of the depreciation (sometimes called the salvage value of the asset).

Life is the number of periods over which the asset is depreciated (sometimes called the useful life of the asset).



SYD

Returns the sum-of-years' digits depreciation of an asset for a specified period

Syntax

SYD(**cost**,**salvage**,**life**,**per**)

Cost is the initial cost of the asset.

Salvage is the value at the end of the depreciation (sometimes called the salvage value of the asset).

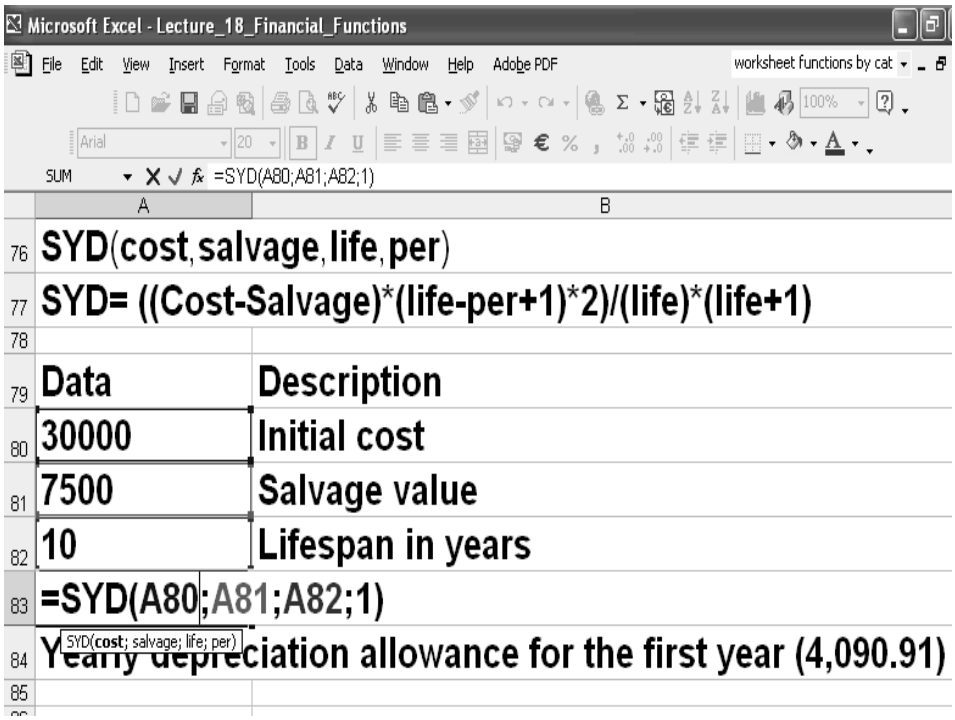
Life is the number of periods over which the asset is depreciated (sometimes called the useful life of the asset).

Per is the period and must use the same units as life.

Remark

- SYD is calculated as follows:

$$SYD = \frac{(cost - salvage) * (life - per + 1) * 2}{(life)(life + 1)}$$



VDB

Returns the depreciation of an asset for any period you specify, including partial periods, using the double-declining balance method or some other method you specify. VDB stands for variable declining balance.

Syntax

VDB(cost,salvage,life,start_period,end_period,factor,no_switch)

Cost is the initial cost of the asset.

Salvage is the value at the end of the depreciation (sometimes called the salvage value of the asset).

Life is the number of periods over which the asset is depreciated (sometimes called the useful life of the asset).

Start_period is the starting period for which you want to calculate the depreciation. Start_period must use the same units as life.

End_period is the ending period for which you want to calculate the depreciation. End_period must use the same units as life.

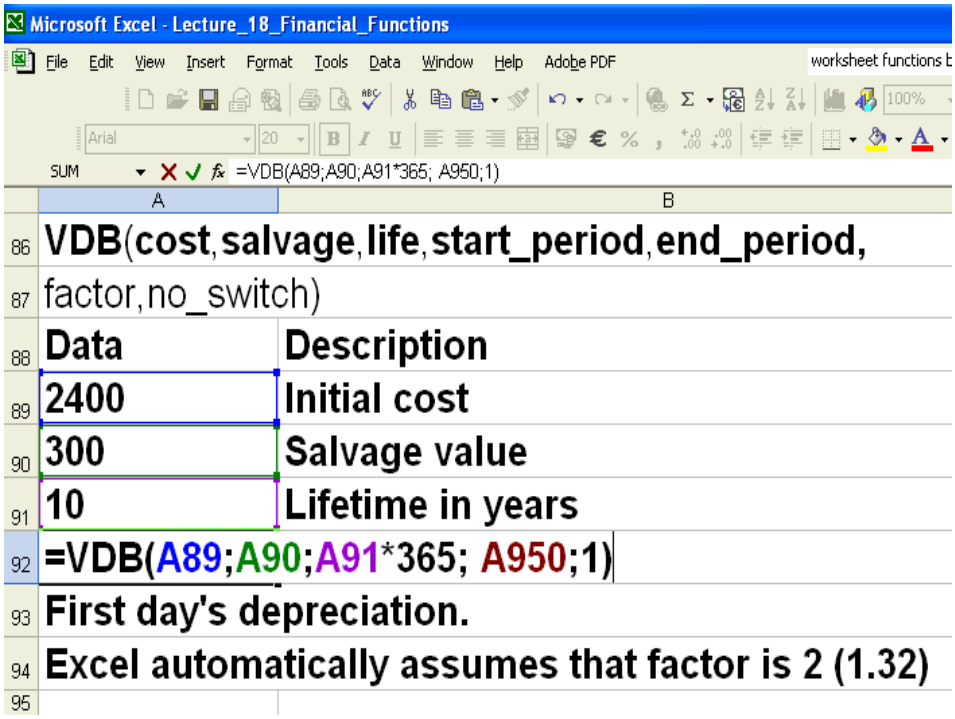
Factor is the rate at which the balance declines. If factor is omitted, it is assumed to be 2 (the double-declining balance method). Change factor if you do not want to use the double-declining balance method. For a description of the double-declining balance method, see DDB.

No_switch is a logical value specifying whether to switch to straight-line depreciation when depreciation is greater than the declining balance calculation.

- If **no_switch** is **TRUE**, Microsoft Excel does not switch to straight-line depreciation even when the depreciation is greater than the declining balance calculation.
- If **no_switch** is **FALSE** or omitted, Excel switches to straight-line depreciation when depreciation is greater than the declining balance calculation.

  All arguments except no_switch must be positive numbers.

All arguments except no_switch must be positive numbers.



IRR

Returns the internal rate of return for a series of cash flows represented by the numbers in values. These cash flows do not have to be even, as they would be for an annuity. However, the cash flows must occur at regular intervals, such as monthly or annually. The internal rate of return is the interest rate received for an investment consisting of payments (negative values) and income (positive values) that occur at regular periods.

Syntax

IRR(**values**,guess)

Values is an array or a reference to cells that contain numbers for which you want to calculate the internal rate of return.

- **Values** must contain at least one positive value and one negative value to calculate the internal rate of return.
- **IRR** uses the order of values to interpret the order of cash flows. Be sure to enter your payment and income values in the sequence you want.
- If an array or reference argument contains text, logical values, or empty cells, those values are ignored.

Guess is a number that you guess is close to the result of IRR.

- Microsoft Excel uses an iterative technique for calculating IRR. Starting with guess, IRR cycles through the calculation until the result is accurate within 0.00001 percent. If IRR can't find a result that works after 20 tries, the #NUM! error value is returned.
- In most cases you do not need to provide guess for the IRR calculation. If guess is omitted, it is assumed to be 0.1 (10 percent).
- If IRR gives the #NUM! error value, or if the result is not close to what you expected, try again with a different value for guess.

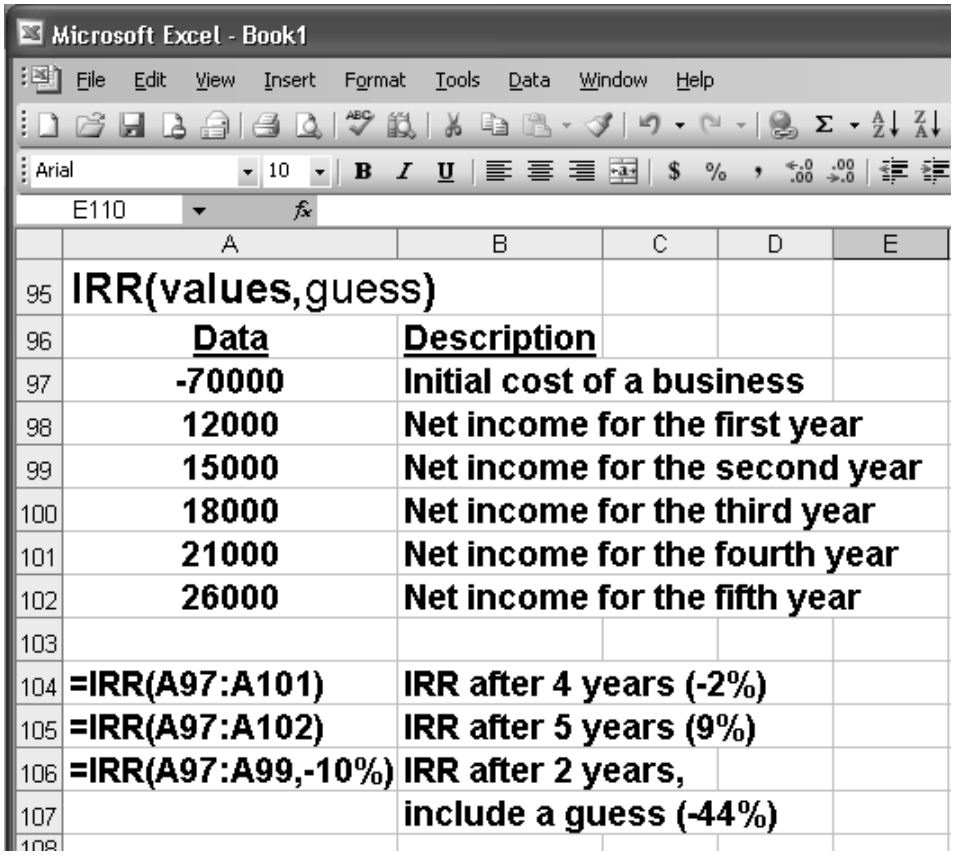
Remarks

IRR is closely related to **NPV**, the net present value function. The rate of return calculated by IRR is the interest rate corresponding to a 0 (zero) net present value. The following formula demonstrates how NPV and IRR are related: NPV(IRR(B1:B6),B1:B6) equals 3.60E-08 [Within the accuracy of the IRR calculation, the value 3.60E-08 is effectively 0 (zero).]

IRR-EXAMPLE

In the slide the Excel worksheet is shown.

In cell A97, the investment of 70,000 is entered with minus sign to denote negative cash flow.
In cell A98 to A102, revenue per year (1 to 5) is entered.
In the first formula in cell A103 (=IRR(A97:A101)), only years 1 to 4 were selected for the revenue stream. The IIR is -2% in this case.
In the next formula in cell A105, the entire revenue stream was considered. The IRR improved to 9%.
Next only first 2 years of revenue stream were considered with an initial guess of 10% (not shown in slide). The result was -44%.



XIRR
Returns the internal rate of return for a schedule of cash flows that is not necessarily periodic. To calculate the internal rate of return for a series of periodic cash flows, use the IRR function.
If this function is not available, and returns the #NAME? error, install and load the Analysis ToolPak add-in. To do that:

1. On the **Tools** menu, click **Add-Ins**.
2. In the **Add-Ins available** list, select the **Analysis ToolPak** box, and then click **OK**.
3. If necessary, follow the instructions in the setup program.

Syntax
XIRR(**values**,**dates**,**guess**)
Values is a series of cash flows that corresponds to a schedule of payments in dates. The first payment is optional and corresponds to a cost or payment that occurs at the beginning of the investment. If the first value is a cost or payment, it must be a negative value. All succeeding payments are discounted based on a 365-day year. The series of values must contain at least one positive and one negative value.

Dates is a schedule of payment dates that corresponds to the cash flow payments. The first payment date indicates the beginning of the schedule of payments. All other dates must be later than this date, but they may occur in any order. Dates should be entered by using the DATE function, or as results of other formulas or functions. For example, use DATE(2008,5,23) for the 23rd day of May, 2008. Problems can occur if dates are entered as text.

Guess is a number that you guess is close to the result of XIRR.

Remarks

- Microsoft Excel stores dates as sequential serial numbers so they can be used in calculations. By default, January 1, 1900 is serial number 1, and January 1, 2008 is serial number 39448 because it is 39,448 days after January 1, 1900. Microsoft Excel for the Macintosh uses a different date system as its default.
- Numbers in dates are truncated to integers.
- XIRR expects at least one positive cash flow and one negative cash flow; otherwise, XIRR returns the #NUM! error value.
- If any number in dates is not a valid date, XIRR returns the #VALUE! error value.
- If any number in dates precedes the starting date, XIRR returns the #NUM! error value.
- If values and dates contain a different number of values, XIRR returns the #NUM! error value.
- In most cases you do not need to provide guess for the XIRR calculation. If omitted, guess is assumed to be 0.1 (10 percent).
- XIRR is closely related to XNPV, the net present value function. The rate of return calculated by XIRR is the interest rate corresponding to XNPV = 0.
- Excel uses an iterative technique for calculating XIRR. Using a changing rate (starting with guess), XIRR cycles through the calculation until the result is accurate within 0.000001 percent. If XIRR can't find a result that works after 100 tries, the #NUM! error value is returned. The rate is changed until:

where:

$$0 = \sum_{i=1}^{N} \frac{P_i}{(1+rate)^{\frac{(d_i - d_1)}{365}}}$$

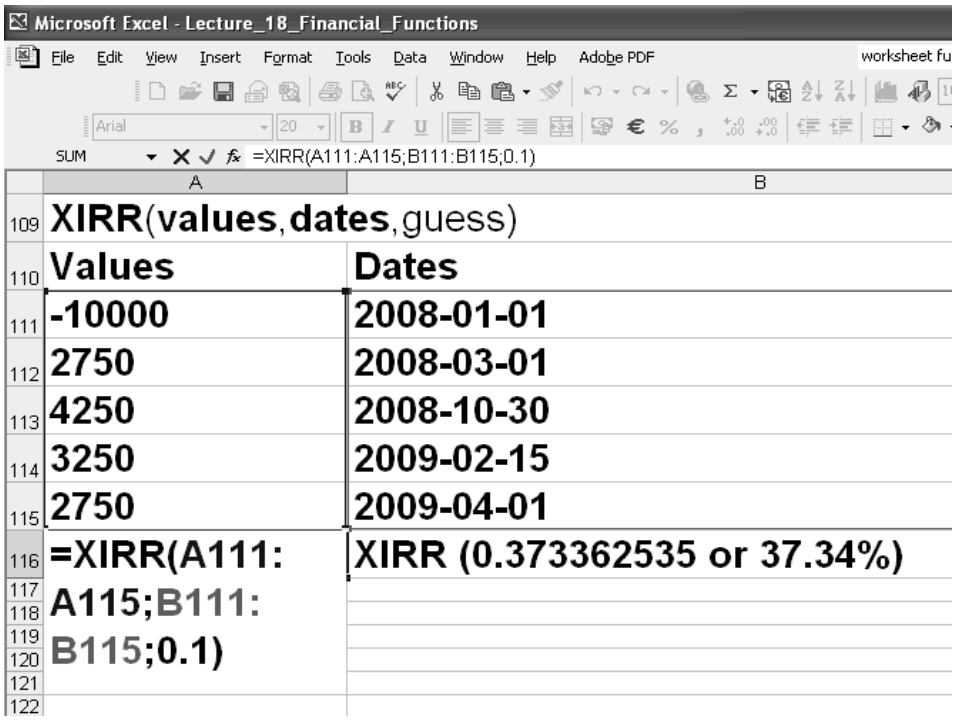
di = the ith, or last, payment date.

d1 = the 0th payment date.

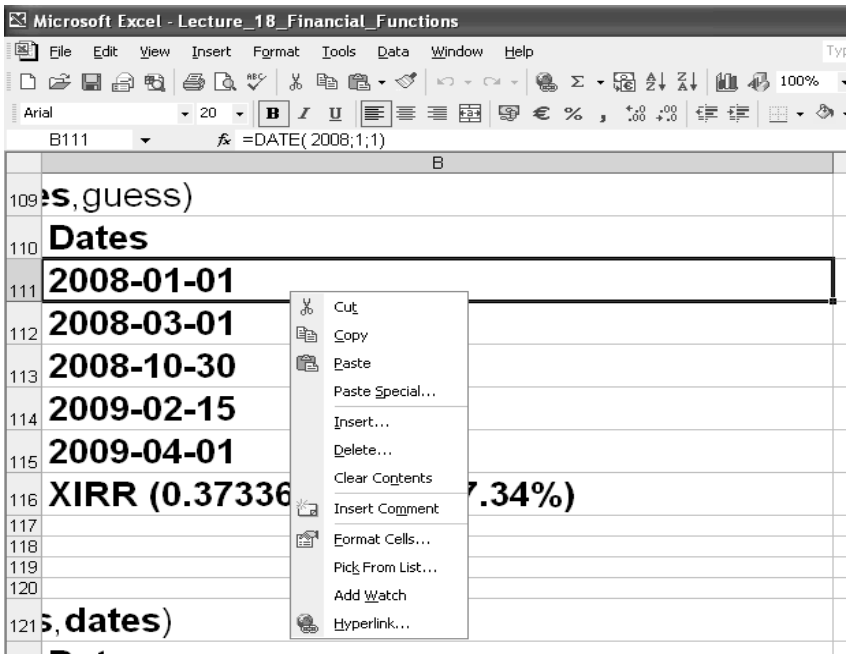
Pi = the ith, or last, payment.

XIRR EXAMPLE

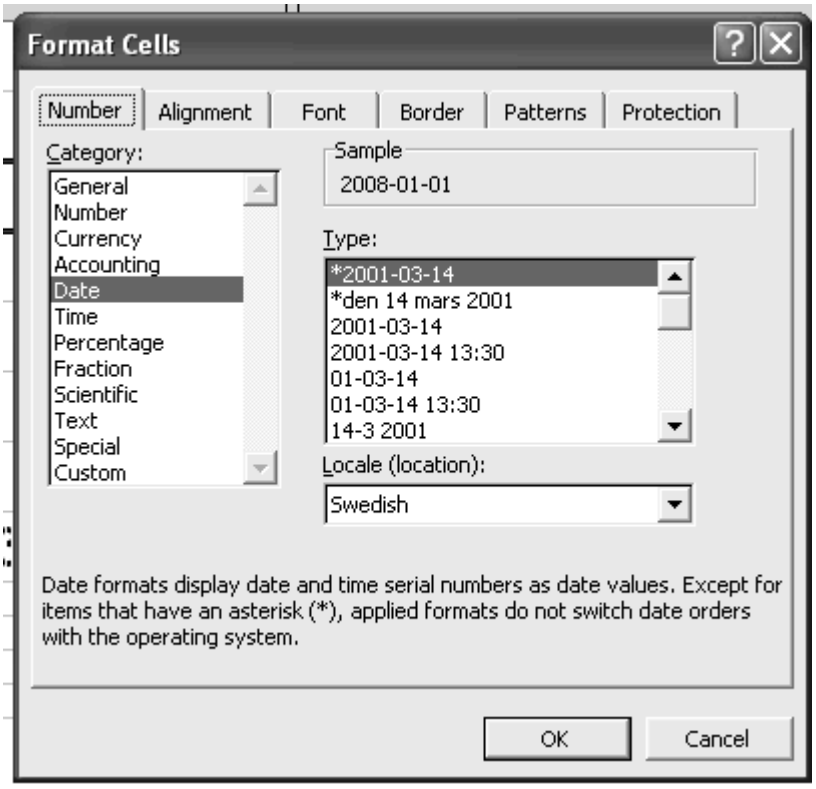
Here, the investment is in cell A111. The revenue stream is in cells A112 to a115. The dates for each investment or revenue are given in cells B111 to B115. Please note that the dates are in European format year-month-day. On your computer, you may not have this format.



After entering these days in Excel, you can right click on the cell. You see a short cut menu as shown below.



When you will select **Format Cells**, the Format Cells Dialog Box appears as shown below. You can then choose the desired format for the date.



In cell A116, the formula =XIRR(A111:A115;B111:B115;0.1), the range A111:A115 is the cost and revenue stream. The range B111:B115 is the stream for dates. The third term 0.1 is the initial guess for XIRR.
The answer in fraction or % is given in cell B116(37.34%).

LINEAR EQUATIONS

Linear equations have following applications in Merchandising Mathematics:

- Solve two linear equations with two variables
- Solve problems that require setting up linear equations with two variables
- Perform linear Cost-Volume-Profit and break-even analysis employing:
- The contribution margin approach
- The algebraic approach of solving the cost and revenue functions

SOLVING LINEAR EQUATIONS –

Here is an example of solving simultaneous linear equations.

$2x - 3y = -6$
$x + y = 2$

Solve for y

$2x - 3y = -6$
$2x + 2y = 4$
$-5y = -10$
$y = 10/5$
$y = 2$

Let us look at the same equations again.

$2x - 3y = -6$
$x + y = 2$

We solved for x.
Now let us substitute y by 2

$2x - 3(2) = -6$
$2x - 6 = -6$
$2x = 0$
$x = 0$

Check your answer
By substituting the values into *each of the equations*
Equation 1:
2x – 3y = -6
x = 0 y = 2
LHS = *2x – 3y = 2(0)-3(2)*
= *-6* **= RHS**
Equation 2 :
x + y = 2
LHS = = x + y = 0 +2 = 2 = RHS
The right side is equal to left hand side. Hence the answer is correct.

LECTURE 20
PERFORM BREAK-EVEN ANALYSIS
EXCEL FUNCTIONS FOR FINANCIAL ANALYSIS

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 18
- MS EXCEL Financial Functions
- Perform Break-Even Analysis.

LINEAR EQUATIONS

Zain purchases the same amount of commodity 1 and 2 each week.
After price increases from Rs. 1.10 to Rs. 1.15 per item of commodity 1 , and from Rs. 0.98 to Rs. 1.14 per item of commodity 2, the weekly bill rose from Rs. 84.40 to Rs. 91.70.
How many items of commodity 1 and 2 are purchased each week?

Let x = # of commodity 1
Let y = # of commodity 2

Setting up Linear Equations

Equation 1

1.10x +0.98y= 84.40 (1)
Eliminate x in (1) by Dividing both sides by 1.10.
(1.10x + 0.98y)/1.10 = 84.40/1.10
x + 0.8909y = 76.73

Equation 2

1.15x+1.14y=91.7 (2)
Eliminate x in (2) by Dividing both sides by 1.15
(1.15x + 1.14y)/1.15 = 91.70/1.15
x + 0.9913y = 79.74

Result 1:

x + 0.8909y = 76.73 (3)
x + 0.9913y = 79.74 (4)
Next:
Subtract (4) from (3):

Result 2:

0.1004y = 3.01
y = 3.01/0.1004
Or
y=29.98 approximantely 30
30 items of commodity 2 are purchased each week

1.10x + 0.98y = 84.40

Substitution

Substitute value of y in (1).
Result:
1.10x + 0.98(29.98) = 84.40
Solve:
1.10x + 29.38 = 84.40
1.10x = 84.40 - 29.38
1.10x = 55.02

Result:

x = 50.02 approximately 50
50 items of commodity 1 are purchased each week

Check your answer
New weekly cost
Commodity 1:

50 x 1.15 = 57.50Rs
Commodity 2:
30 x 1.14 = 34.20
Total cost = 91.70Rs

TERMINOLOGY
There are either Business Costs or Expenses.

Break Even Analysis
Break Even Analysis refers to the calculation to determine how much product a company must sell in order to get break even point (The point at which no profit is made and no losses are incurred) on that product. Break-even analysis provides insight into whether or not revenue from a product or service has the ability to cover the relevant costs of production of that product or service. Managers can use this information in making a wide range of business decisions, including setting prices, preparing competitive bids, and applying for loans.

Cost-Volume-Profit Analysis
Cost-volume-profit (CVP) analysis expands the use of information provided by breakeven analysis. It deals with how profits and costs change, with a change in volume. More specifically, it looks at the effects on profits by changes in such factors as variable costs, fixed costs, selling prices, volume. By studying the relationships of costs, sales, and net income, management is better able to cope with many planning decisions. For example, CVP analysis attempts to answer the following questions: (1) What sales volume is required to break even? (2) What sales volume is necessary in order to earn a desired (target) profit? (3) What profit can be expected on a given sales volume? (4) How would changes in selling price, variable costs, fixed costs, and output affect profits?

Fixed Costs (FC)
Fixed Costs are such costs that do not change if sales increase or decrease e.g. rent, property taxes, some forms of depreciation.

Variable Costs (VC)
Variable costs do change in direct proportion to sales volume e.g. material costs and direct labor costs.

Production Capacity (PC)
It is the number of units a firm can make in a given period.

Break Even Point
Break Even point is a point at which neither a profit nor loss is made. Revenue is exactly equal to costs. Break even point can be expressed as

1. units
2. Sales or Rupees (Rs)
3. Percent of capacity

BEP in units calculates how many units should be sold to break even. If the product is sold in a quantity greater than this, the firm will makes a profit; below this point, a loss.

$$\text{BEP in units} = \frac{\text{Fixed Costs}}{\text{Contribution Margin per unit}}$$

BEP in Rs calculates the revenue that must be obtained to reach break even point.

$$\text{BEP in Rs} = \frac{\text{Fixed Costs}}{\text{Contribution Margin}} \times \text{Net Sales}$$

$$\text{BEP in Rs} = \frac{\text{Fixed costs}}{\text{Contribution Margin per unit}} \times \text{Selling Price per unit}$$

BEP as percent of capacity calculates what percent of production capacity will be utilized to produce the number of units required to reach break even point.

$$\text{BEP as \% of capacity} = \frac{\text{BEP in units}}{\text{Production capacity}} \times 100\ \%$$

Contribution Margin

Contribution Margin is the Rs. amount that is found by deducting **Variable Costs** from **Sales or revenues** and **'contributes'** to meeting **Fixed Costs** and making a **'Net Profit'**. It can be calculated on a total or per unit basis.

Contribution Margin = Net Sales – Variable Cost = S – VC

Contribution margin per unit =CM =Sale price per unit – Variable cost per unit

Contribution Rate (CR)

$$\text{Contribution rate} = \frac{\text{Contribution Margin}}{\text{Net sales}} \times 100\% = \frac{\text{CM}}{\text{S}} \times 100\%$$

$$\text{Contribution rate} = \frac{\text{Contribution Margin per unit}}{\text{Sale price per unit}} \times 100\% = \frac{\text{CM}}{\text{S}} \times 100\%$$

A CONTRIBUTION MARGIN STATEMENT

	Rs.	%
Net Sales (Price × # Units Sold)	x	100
Less: Variable Costs	x	x
Contribution Margin	x	x
Less: Fixed Costs	x	x
Net Income	x	x

The **net sales** are calculated by multiplying price per unit with number of units sold.

Net Sales = Sale price per unit × number of units sold

This figure is treated as 100%.
Next, variable costs are specified and deducted from the Net sales to obtain the Contribution Margin.
Next, Fixed costs are deducted from the contribution Margin. The result is **Net Income**.

Net Income = Contribution Margin – Fixed Costs

Under the % column, percentage of each item is calculated with respect to the Net Sales.

Here's an example.

Net Sales	462,452 Rs	100%
Less: Variable Costs		

Cost of goods sold	230,934 Rs	50%
Sales Commissions	58,852 Rs	12.7%
Delivery Charges	13,984 Rs	3%
Total Variable Costs	(230934 + 58852 + 13984) 303,770 Rs	65.7%
Contribution Margin	(462,452 – 303770) 158,682 Rs	34.3%
Less: Fixed Costs		
Advertising	1,850 Rs	0.4%
Depreciation	13,250 Rs	2.8%
Insurance	5,400 Rs	1.2%
Payroll Taxes	8,200 Rs	1.8%
Rent	9,600 Rs	2.1%
Utilities	17,801 Rs	3.8%
Wages	40,000 Rs	8.6%
Total Fixed Costs	**(**1850+13250+5400+8200 +9600+17801+40000**)** 96,101 Rs	20.8%
Net Income	**(**158682 – 96101**)** 62,581 Rs	13.5%

SCENARIO 1

A firm is planning to add a new item in its product line**.** Market research indicates that the new product can be sold at Rs. 50 per unit. Cost analysis provides the following information:

Fixed Costs (FC) **per period** = Rs. 8640

Variable Costs (VC) = Rs. 30 **per unit.**

Production Capacity **per period** = 900 units

How much does the sale of an additional unit of a firm's new product contribute towards increasing its net income?

Formula

Contribution Margin per unit = CM = S – VC

Contribution Rate = CR = CM/S × 100%

***Break Even Point (BEP):**

...in Units (x): ~~Rs~~. x = (FC / CM)

...in Sales Rs. : ~~Rs~~. x = (FC / CM)* S

...in % of Capacity : BEP in Units/PC × 100%

*** At Break Even**, Net Profit or Loss = 0

Scenario 1 Summary

Selling price per unit = S = 50 Rs

Fixed Costs per period = FC = Rs. 8640

Variable Costs = VC = Rs. 30 per unit.

Production Capacity per period = PC = 900 units

Solution of this problem is in the next lecture.

LECTURE 21

PERFORM LINEAR COST-VOLUME PROFIT AND BREAK-EVEN ANALYSIS USING THE CONTRIBUTION MARGIN APPROACH

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 18
- Perform Break-Even Analysis
- MS EXCEL Financial Functions
- .

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 20
- Perform linear cost-volume profit and break-even analysis.
- Using the contribution margin approach

SCENARIO 1

Contribution Margin per unit = CM = S – VC = 50 - 30 = 20 Rs.
Contribution rate = CR = CM/S × 100% = Rs. 20/50×100% = 40%

Break Even Point:

In Units, x = FC / CM = 8640/20 = 432 Units
In Rs, x = (FC / CM)* S =
(Rs. 8640/Rs.20)* Rs.50 = Rs.21,600
% of Capacity = BEP in units/ PC ×100%
= 432/ 900×100%
= 48%
Thus by selling more than 432 units of its new product, a firm can make profit.

SCENARIO 2

The Lighting Division of A Lighting Fitting Manufacturer plans to introduce a new street light based on the following accounting information:
FC per period = Rs. 3136
VC = Rs.157 per unit
S= Rs.185 per unit
Production Capacity per period = 320 units

Calculate the break even point (BEP)
…in units
…in rupees
…as a percent of capacity

Break Even Point

…in units = FC / CM
CM = S – VC
= Rs.185 – 157 = Rs.28

BEP in units = 3136/28 = 112 Units

Break Even Point
…in Rupees = (FC / CM)* S
= (3136/28) * 185 = 20720 Rs.

Break Even Point
…as a percent of capacity = BEP in units/PC×100%

= 112/320×100%
= 35% Capacity

SCENARIO 2-1
FC = Rs.3136
VC = Rs.157per unit
S= Rs.185 per unit
Production Capacity = 320 units
Determine the BEP as a % of capacity if FC are reduced to Rs.2688.
Formula:
BEP as a % of capacity = BEP in units/PC×100%
Step 1… Find CM
Step 2… Find BEP in units
Step 3… Find % of Capacity
Step 1… Find CM per unit
S = 185Rs per unit
VC = 157Rs per unit
CM = S – VC = 185 – 157 = Rs. 28
Step 2… Find BEP in units
BEP in units = FC/CM
= Rs. 2688/ Rs.28
= 96 Units
Step 3… Find % of Capacity
BEP as a % of capacity = BEP in units / PC×100%
= 96/320×100%
= 30% of Capacity
SCENARIO 2-2
FC = Rs.3136
VC = Rs.157 per unit
S = Rs.185 per unit
Production Capacity = 320 units per period
Determine the BEP as a % of capacity if FC are increased to Rs.4588, and VC reduced to 80% of S.
BEP as a % of capacity = BEP in units/PC×100%
New VC = S × 80% = 185 × 0.8 = Rs.148
New FC = 4588 Rs

Step 1… Find CM per unit
S = 185 Rs per unit
VC = 148Rs per unit
CM = S – VC = Rs. 37
Step 2… Find BEP in units
BEP in units = FC/CM
= Rs. 4588 / Rs. 37
= 124 Units
Step 3… Find % of Capacity
BEP as a % of capacity = BEP in units /PC×100%
= 124/320×100%
= 39% of Capacity

SCENARIO 2 -3
FC = Rs. 3136
VC = Rs.157 per unit
S= Rs.185 per unit
Production Capacity = 320 units per period
Determine the BEP as a % of capacity if S is reduced to Rs.171.
BEP as a % of capacity = BEP in units/PC×100%
Step 1… Find CM per unit
S = 171Rs per unit
VC = 157Rs per unit
CM = S – VC = Rs. 14
Step 2… Find BEP in units
BEP in units = FC/CM
= Rs. 3136/ Rs. 14
= 224 Units
Step 3… Find BEP as a % of Capacity
BEP as a % of capacity = BEP in units /PC×100%
= 224/320×100%
= 70 % of Capacity

LECTURE 22
PERFORM LINEAR COST-VOLUME PROFIT AND BREAK-EVEN ANALYSIS

OBJECTIVES

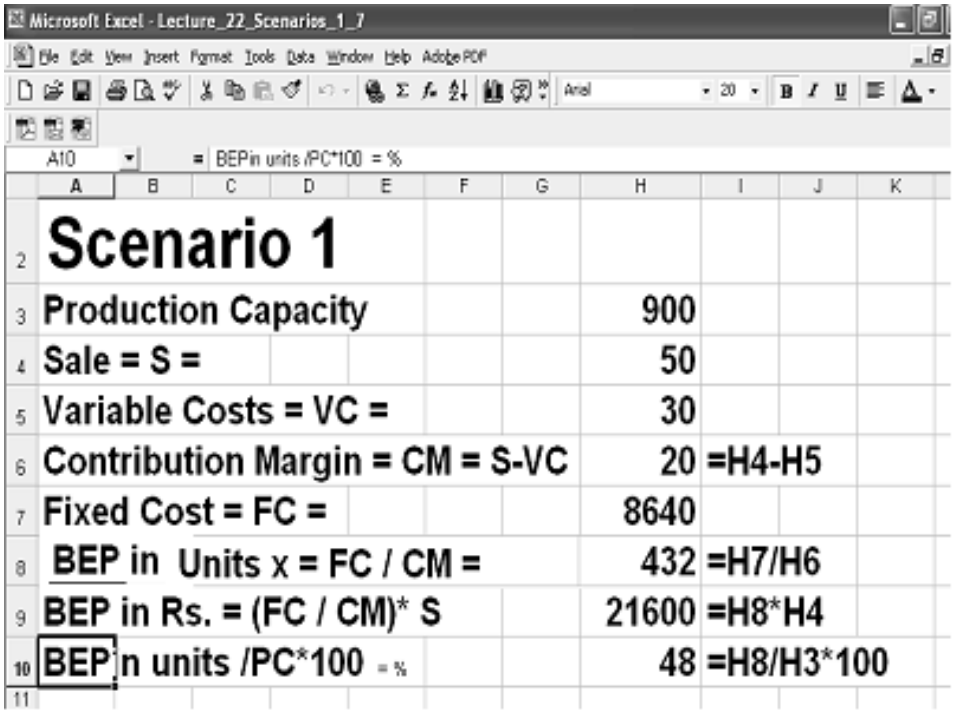
The objectives of the lecture are to learn about:

- Review Lecture 21
- Perform Linear Cost-Volume Profit and Break-Even analysis.
- Using Microsoft Excel

SCENARIO 1

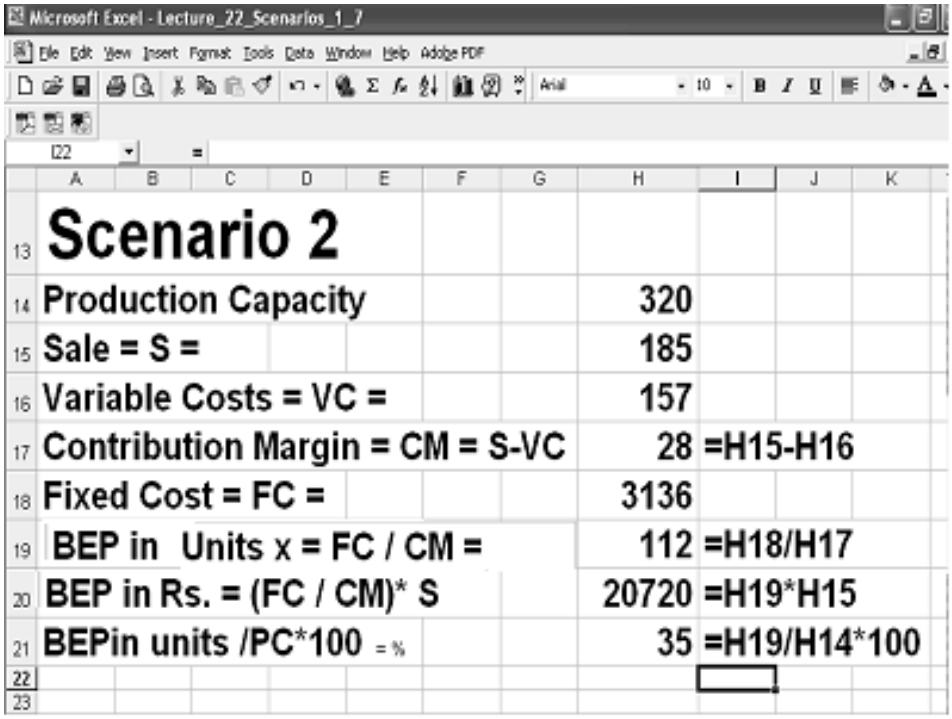
Let us look at different scenarios for calculation of contribution margin and net profit. The explanations are given in the slides.

The Break Even Point in Rs. is 21,600. The break Even point as a % of capacity is 48%.



SCENARIO 2

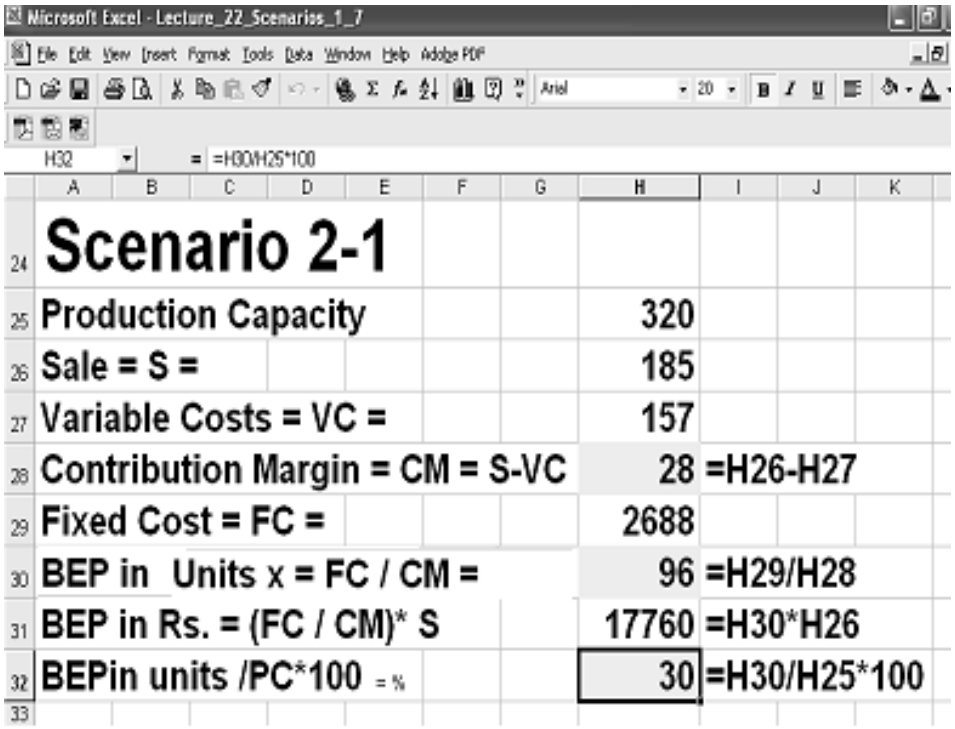
The Break Even Point in Rs.is 20,720. The break Even Point as a % of capacity is 35%.



13	**Scenario 2**			
14	Production Capacity		320	
15	Sale = S =		185	
16	Variable Costs = VC =		157	
17	Contribution Margin = CM = S-VC		28	=H15-H16
18	Fixed Cost = FC =		3136	
19	BEP in Units x = FC / CM =		112	=H18/H17
20	BEP in Rs. = (FC / CM)* S		20720	=H19*H15
21	BEPin units /PC*100 = %		35	=H19/H14*100

SCENARIO 2-1

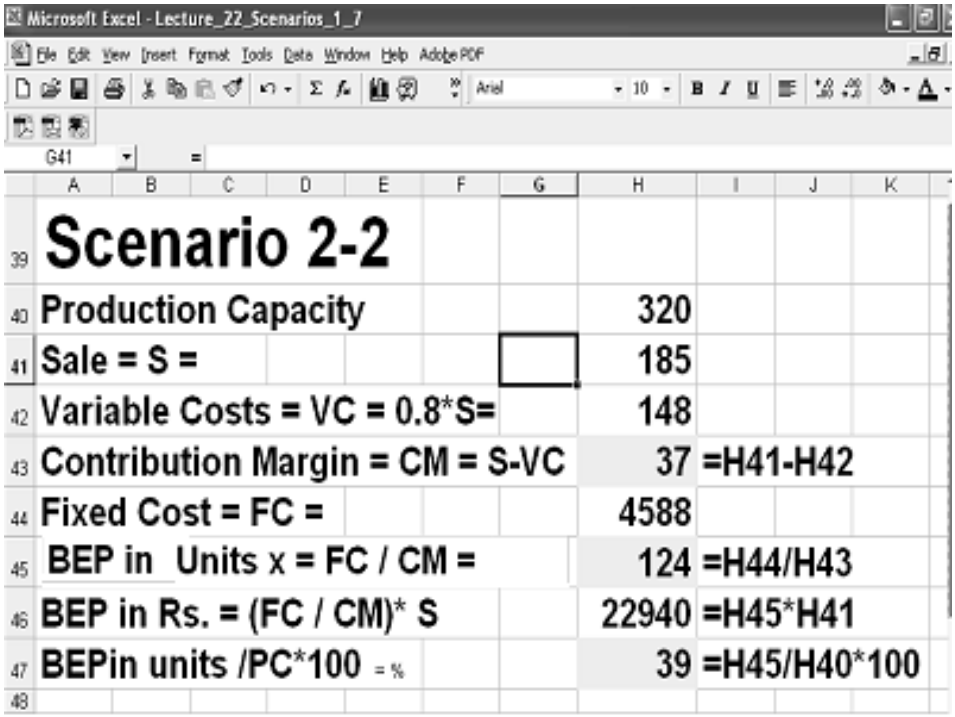
The Break Even Point in Rs. is 17,760. The break Even Point as a % of capacity is 30%.



24	**Scenario 2-1**			
25	Production Capacity		320	
26	Sale = S =		185	
27	Variable Costs = VC =		157	
28	Contribution Margin = CM = S-VC		28	=H26-H27
29	Fixed Cost = FC =		2688	
30	BEP in Units x = FC / CM =		96	=H29/H28
31	BEP in Rs. = (FC / CM)* S		17760	=H30*H26
32	BEPin units /PC*100 = %		30	=H30/H25*100

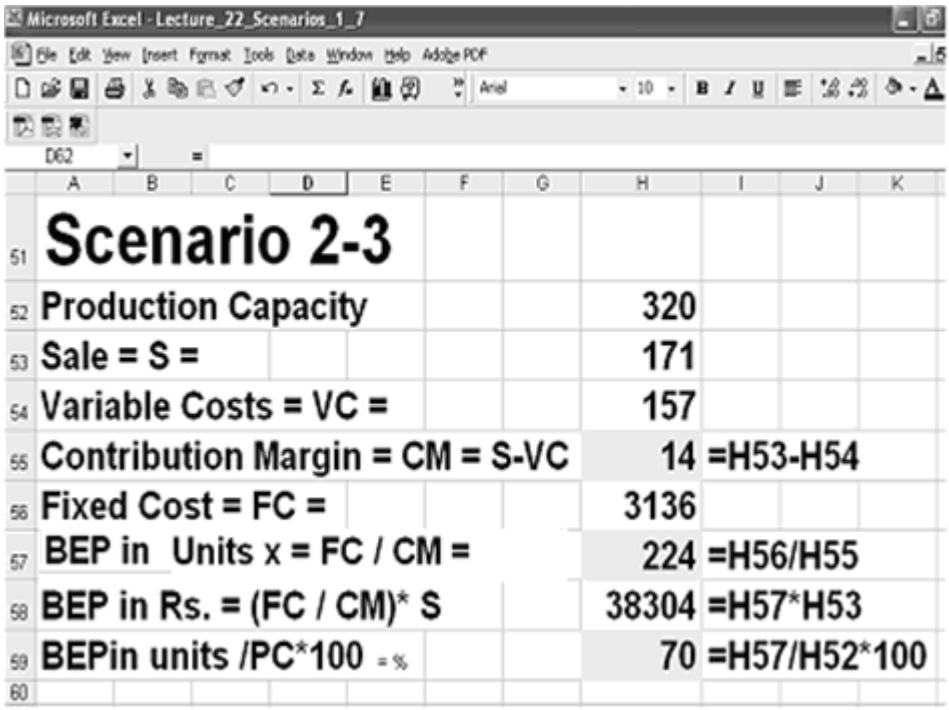
SCENARIO 2-2

The Break Even Point in Rs. is 22,940. The break Even Point as % of capacity is 39%.



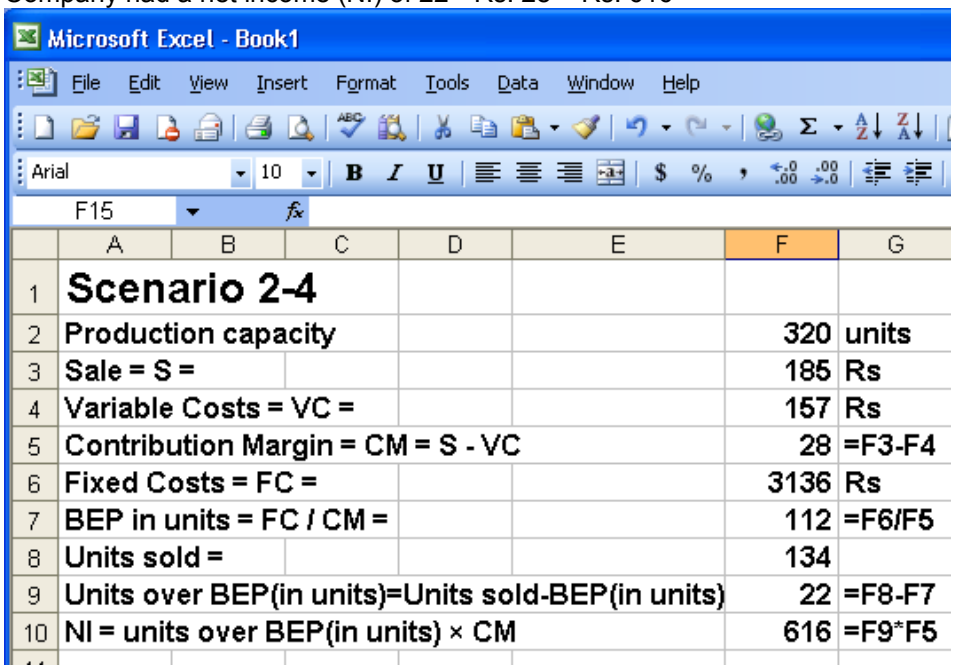
SCENARIO 2-3

The Break Even Point in Rs.is 38,304. The break Even Point as % of capacity is 70%.



Net Income (NI) or Profit
Net income=NI=Number of units sale above BEP in units×Contribution Margin per unit

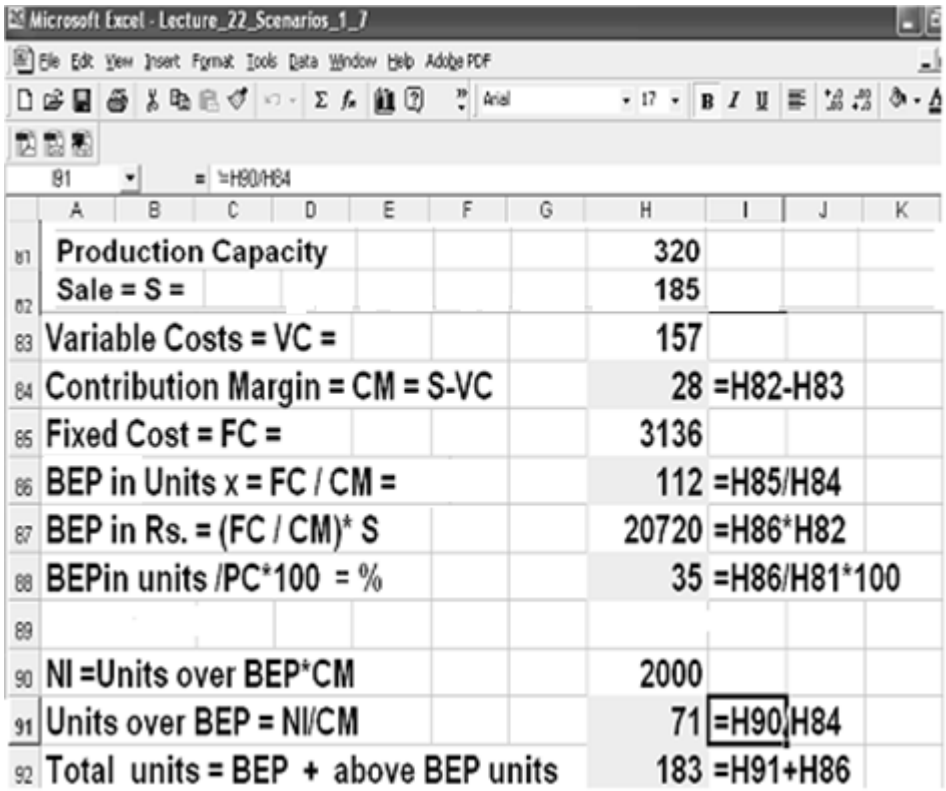
SCENARIO 2-4
FC = Rs. 3136, VC = Rs. 157, S= Rs. 185, Capacity = 320units
Determine the Net income (NI), if 134 units are sold!
Formula for Net Income (NI)
NI = Number of units sold above BEP × CM
Step 1… Find CM per unit
S = 185Rs per unit
VC = 157Rs per unit
CM = S – VC = 185 – 157 = Rs. 28
(CM of Rs.28 per unit)
Step 2… Find BEP in units
BEP in units = FC/CM
= Rs. 3136/ Rs. 28
= 112 Units
Step 3… Find units sold over BEP Units
Sold = 134 units
BEP in units = 112 units
Number of units sold above BEP in units = 134 – 112 = 22
Hence:
Company had a net income (NI) of 22 * Rs. 28 = Rs. 616



	A	B	C	D	E	F	G
1	Scenario 2-4						
2	Production capacity					320	units
3	Sale = S =					185	Rs
4	Variable Costs = VC =					157	Rs
5	Contribution Margin = CM = S - VC					28	=F3-F4
6	Fixed Costs = FC =					3136	Rs
7	BEP in units = FC / CM =					112	=F6/F5
8	Units sold =					134	
9	Units over BEP(in units)=Units sold-BEP(in units)					22	=F8-F7
10	NI = units over BEP(in units) × CM					616	=F9*F5

SCENARIO 2-5
FC = Rs. 3136
VC = Rs.157
S= Rs.185
Capacity = 320 units
How many units must be sold to generate NI of Rs. 2000?
Formula for Net Income
Number of Units sold above BEP in units = NI / CM
Step 1… Find CM per unit
S = 185Rs per unit
VC = -157 Rs per unit
CM = Rs. 28
(CM of Rs.28 per unit)

Step 2… Find BEP in units
BEP in units = FC/CM
= Rs. 3136/ Rs. 28
= 112 Units
Step 3… Find units over BEP
Number of units above BEP = NI / CM
= Rs. 2000/Rs. 28
= 71 Units
Total units sold = 71 Units above BEP + 112 BEP Units = 183 units



Net loss
Net loss=NL=Number of units sale below BEP in units × Contribution Margin per unit

Alternatively,
Negative sign with Net Income means Net loss. That is
Net loss = - Net Income.
Number of units sale below BEP in units = - Number of units sale above BEP in units
Thus
Net loss= - Net income= - Number of units sale above BEP in units×Contribution Margin per unit

SCENARIO 2-6
FC = Rs. 3136
VC = Rs.157
S= Rs.185
Capacity = 320 units
Find the number of units sold if there is a Net Loss (NL) of Rs. 336?

Formula
Net loss = Number of units sale below BEP in units × Contribution Margin per unit
Number of Units below BEP in units = (NL)/CM

Step 1… Find CM per unit
S = 185 Rs per unit
VC = -157 Rs per unit
CM = 28 Rs
(CM of Rs.28 per unit)

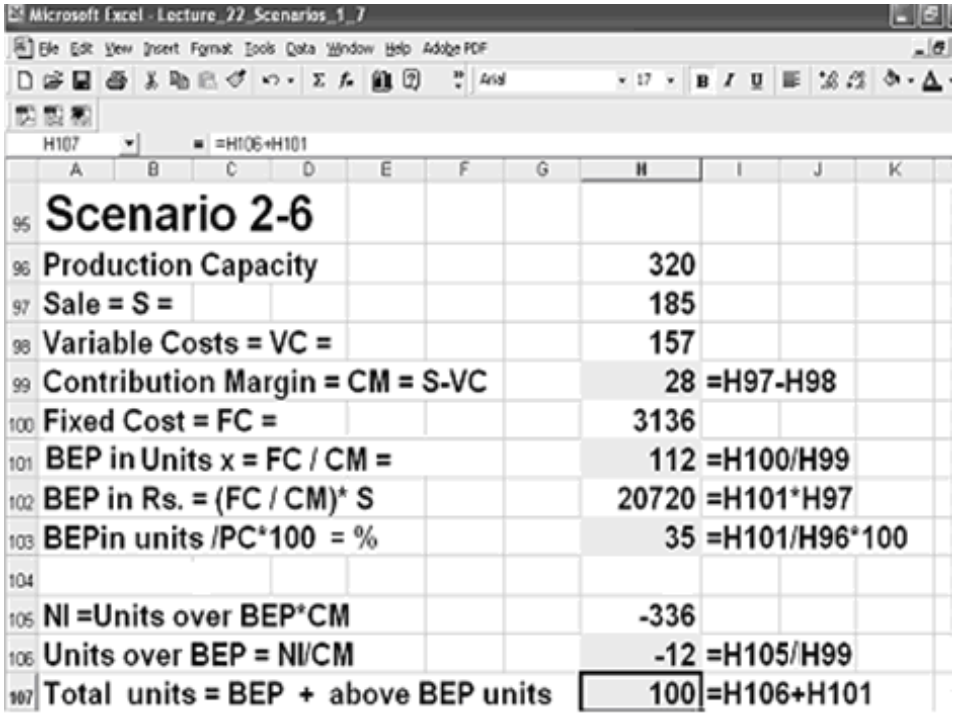
Step 2… Find BEP in units
BEP in units = FC/CM
= Rs. 3136/ Rs. 28
= 112 Units

Step 3… Find units below BEP=NL/CM
Number of unit sales below BEP = Rs. 336/Rs. 28 per Unit
= 12 Units
Hence:
Total Sold Units = BEP in units – Number of units sale below BEP
=112 - 12 = 100

Alternate Method
Net loss= - Net income= - Number of units sale above BEP in units×Contribution Margin per unit

$$\text{Number of units sale above BEP in units} = \frac{\text{- Net loss}}{\text{Contribution Margin per unit}}$$

= - 336 / 28 = -12
Total sold units = BEP in units + Number of units sale above BEP in units
= 112 + (-12) = 100



SCENARIO 2-7
FC = Rs. 3136
VC = Rs.157per unit
S= Rs.185 per unit
Production Capacity = 320 units
The company operates at 85% of its capacity.

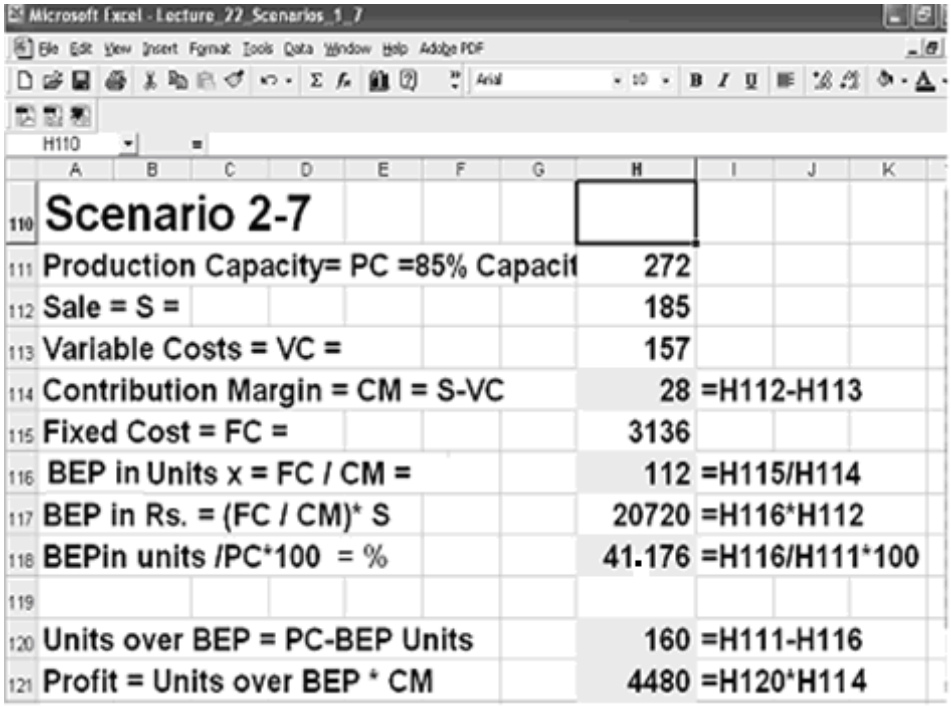
Find the Profit or Loss.

Formula
Number of units above BEP in units × CM = NI

Step 1… Find CM per unit
S = 185 Rs per unit
VC = -157 Rs per unit
CM = Rs. 28
(CM of Rs.28 per unit)

Step 2… Find BEP in units
BEP in units = FC/CM
= Rs. 3136/ Rs. 28
= 112 Units

Step 3… Find units over BEP
Units produced = 320*0.85 = 272 Units
BEP in units = 112 units
Number of units over BEP in units = 272 – 112 = 160
Hence:
Net income =NI = 160 Units * 28 = 4480 Rs.



CASE
Company A's year end operating results were as follows:
Total Sales of Rs. 375,000
Operated at 75% of capacity
Total Variable Costs were Rs. 150,000
Total Fixed Costs were Rs. 180,000
What was Company A's BEP expressed in rupees of sales?

LECTURE 23
STATISTICAL DATA REPRESENTATION

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 22
- Statistical Data Representation.

MODULE 5

Statistical data representation
(Lecture 23)
Measures of central tendency
(Lectures 24-25)
Measures of dispersion and skewness
(Lectures 26-27)

MODULE 6

Correlation
(Lecture 28-29)
Line Fitting
(Lectures 30-31)
Time Series and Exponential Smoothing
(Lectures 32-33)

MODULE 7

Factorials
Permutations and Combinations
(Lecture 34)
Elementary Probability
(Lectures 35-36)
Chi-Square
(Lectures 37)
Binomial Distribution
(Lectures 38)

MODULE 8

Patterns of probability: Binomial, Poisson and Normal Distributions
(Lecture 39-41)
Estimating from Samples: Inference
(Lectures 42-43)
Hypothesis testing: Chi-Square Distribution
(Lectures 44-45)
End-Term Examination

STATISTICAL DATA

Information is collected by government departments, market researchers, opinion pollsters and others.
Information then has to be organized and presented in a way that is easy to understand

BASIS FOR CLASSIFICATION

1. Qualitative: Attributes: sex, religion
2. Quantitative Characteristics: Heights, weights, incomes etc.
3. Geographical: Regions: Provinces, divisions etc.
4. Chronological or Temporal
5. By time of occurrence: Time series

TYPES OF CLASSIFICATION

There are different types of classifications.

- One-way

One characteristic: Population
- Two-way
  Two characteristics at a time
- Three-way
  Three characteristics at a time

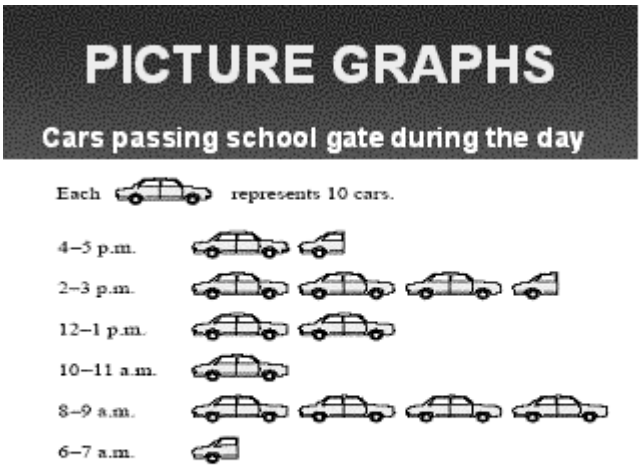
METHODS OF PRESENTATION

Different methods of representation are:
- Text
  "The majority of population of Punjab is located in rural areas."
- Semi tabular
  Data in rows
- Tabular
  Tables with rows and columns
- Graphic
- Charts and graphs

TYPES OF GRAPHS
- Picture graph
- Column Graphs
- Line Graphs
- Circle Graphs (Sector Graphs)
- Conversion Graphs
- Travel Graphs
- Histograms
- Frequency Polygon
- Cumulative -Polygon or Ogive

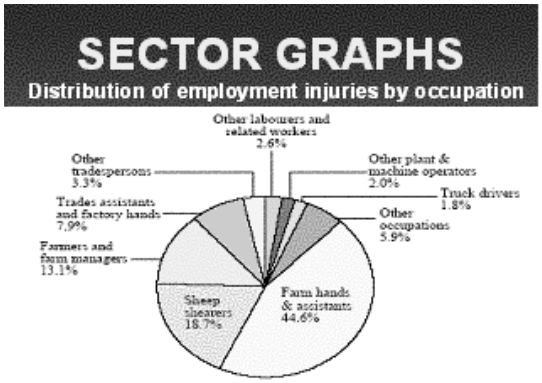
PICTURE GRAPH or PICTOGRAPH

In picture graph or pictograph, each value is represented by a proportional number of pictures.. In the example below, one car represents 10 cars

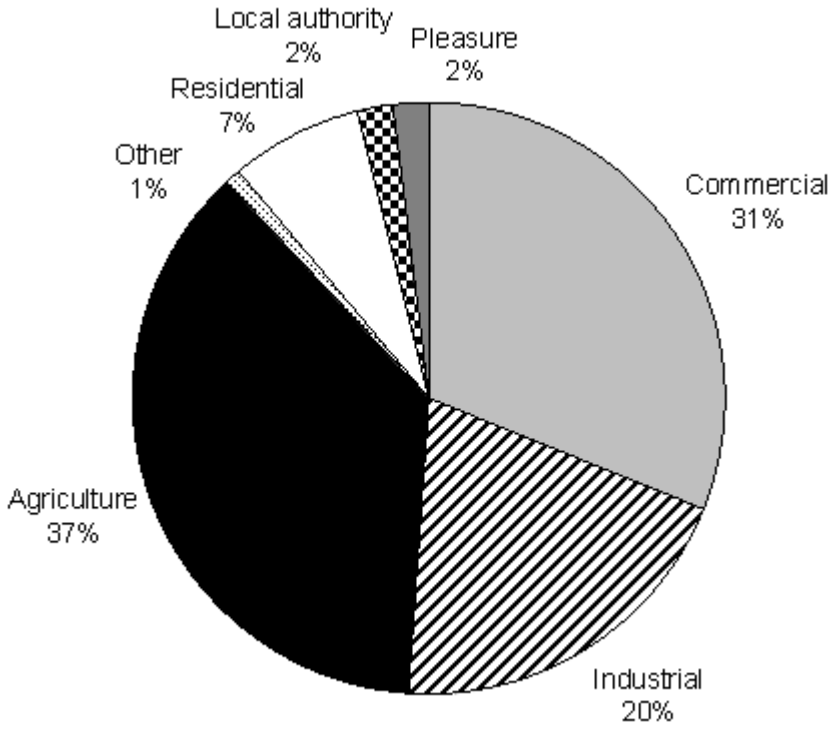


SECTOR GRAPHS

Sector graphs use the division of a circle into different sectors. The full circle is 360 degrees. For each percentage, degees are calculated and sectors plotted.



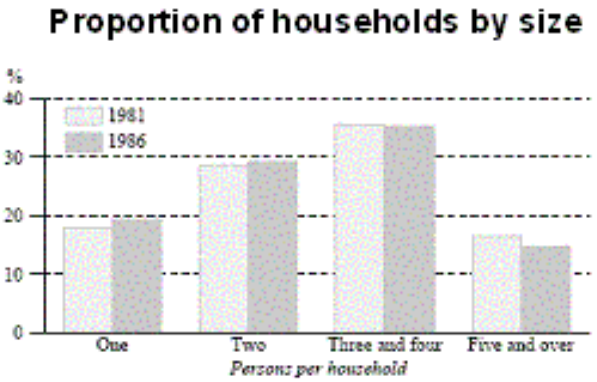
Example of Sector Graph



COLUMN AND BAR GRAPHS

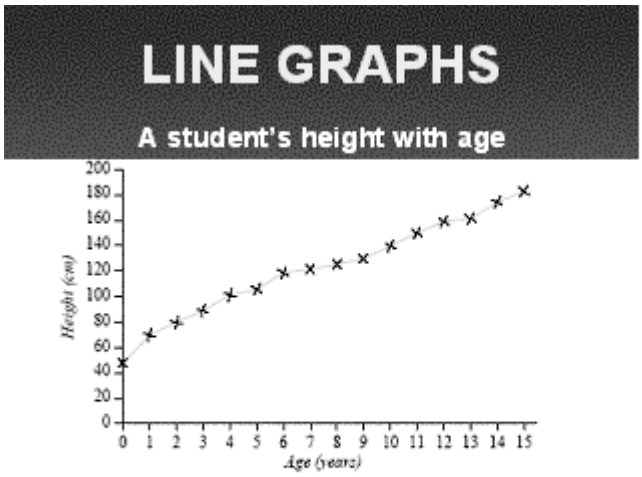
The following slide gives the Proportion of households by size in the form of a Column and Bar graph.

COLUMN AND BAR

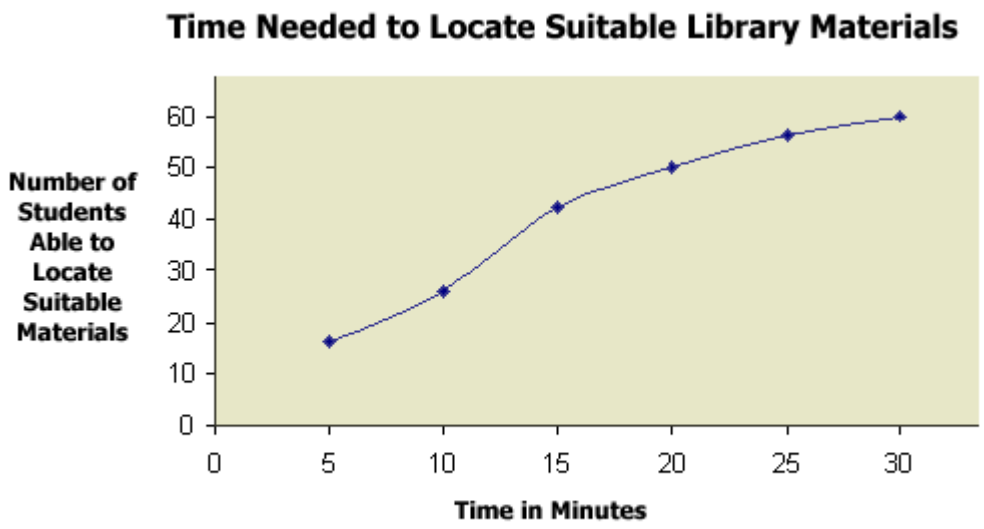


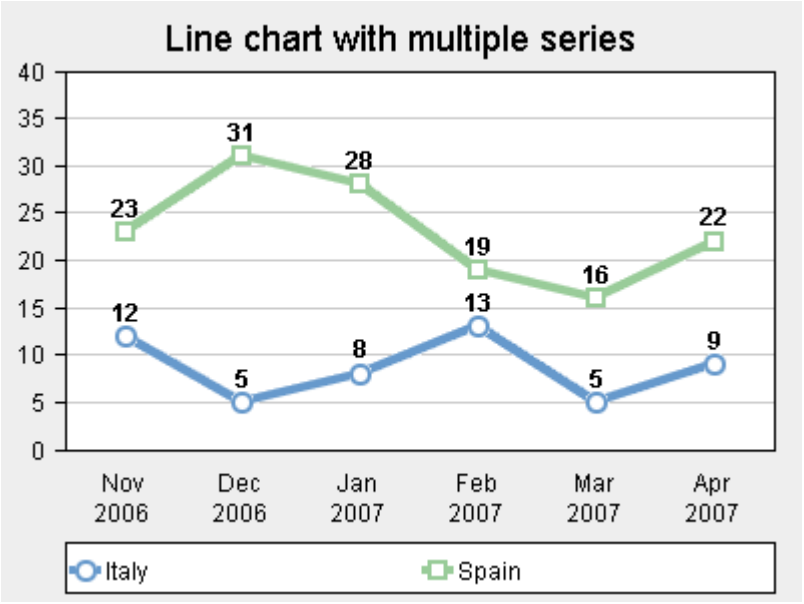
LINE GRAPHS

Line graphs are the most commonly used graphs. A line graph plots data as points and then joins them with a line.



Example


Time Needed to Locate Suitable Library Materials
Number of Students Able to Locate Suitable Materials
60
50
40
30
20
10
0
0
5
10
15
20
25
30
Time in Minutes


Line chart with multiple series
40
35
30
25
20
15
10
5
0
23
31
28
19
16
22
12
5
8
13
5
9
Nov 2006
Dec 2006
Jan 2007
Feb 2007
Mar 2007
Apr 2007
Italy
Spain

LECTURE 24
STATISTICAL REPRESENTATION
MEASURES OF CENTRAL TENDENCY
PART 1

OBJECTIVES

The objectives of the lecture are to learn about:

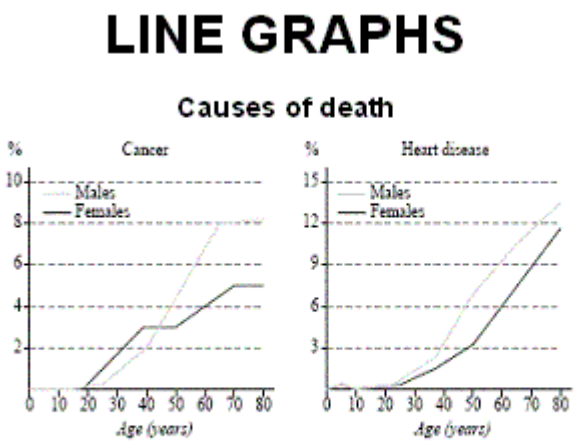
- Review Lecture 18
- Statistical Representation
- Measures of Central Tendency

LINE GRAPHS

Line graphs are the most commonly used graphs. In the following graph, you can see the occurrence of causes of death due to cancer in males and females. You can see that after the age of 40, the occurrence of cancer is much greater in the case of males.

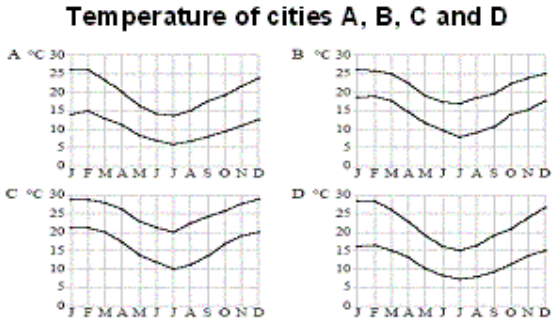
The line graph of heart diseases also shows that the disease is more prominent in the case of males.

As you see line graphs help us to understand the trends in data very clearly.



Another line graph of temperature in 4 cities A, B, C and D shows that although the general pattern is similar, the temperature in city A is lowest followed by D, B and C. In city C the highest temperature is close to 30 whereas in city A and B it is about 25. The highest temperature in city D is about 28 degrees.

LINE GRAPHS



Central Tendency

The term central tendency refers to the middle value (sometime a typical value) of the data.
Measures of central tendency are measures of the location of the middle or the center of a distribution. The "Mean" is the most commonly used measure of central tendency.

MEAN

Also known as the arithmetic mean, the mean is typically what is meant by the word average. The mean is perhaps the most common measure of central tendency. The mean of a variable is given by
(the sum of all its values)/(the number of values).
For example, the mean of 4, 8, and 9 is
$(4 + 8 + 9)/3 = 7$

Example:
58 69 73 67 76 88 91 and 74 (8 marks).
Sum = 596
Mean = 596/8 = 74.5
Please note that the mean is affected by extreme values.

MEDIAN

Another typical value is the median. To find the median of a number of values first arrange the data in ascending or descending order then locate the middle value, , If there are odd number of data points then median is the middle values. If there are even number of data points then median is mean of the two middle values.

Median is easier to find than the mean, and unlike the mean it is not affected by values that are unusually high or low

Example:
3 6 11 14 19 19 21 24 31 (9 values)
In the above data there are 9 values. So, median is The middle value i-e 19.

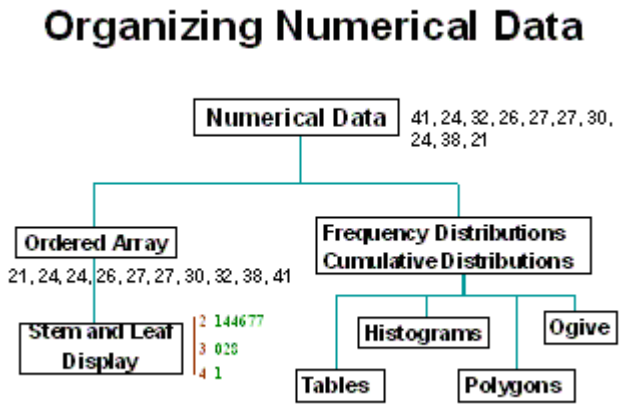
MODE

The most common score in a set of scores is called the mode.
There may be more than one mode, or no mode at all
2 2 1 2 0 3 2 1 1 4 1 1 1 2 2 0 3 2 1
The mode, or most common value, is 1.

	Description/Explanation	Advantages	Disadvantages
Mean	the sum of all the results included in the sample divided by the number of observations	Quick and easy to calculate	May not be representative of the whole sample
Median	the middle value of all the numbers in the sample.	Takes all numbers into account equally	More tedious to calculate than the other two Can be affected by a few very large (or very small) numbers
Mode	the most frequently observed value of the measurements in the sample. There can be more than one mode or no mode. • for an even number of values, the median is the average of the middle two values • for an odd number of values, the median is the middle of the all of the values.	Fairly easy to calculate Half of the sample (normally) lies above the median	Tedious to find for a large sample which is not in order

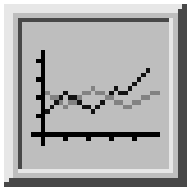
ORGANISING DATA
There are many different ways of organizing data.

Organizing Numerical Data



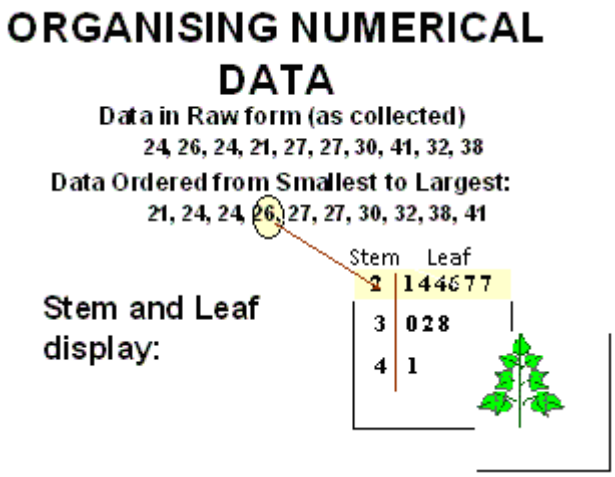
Numerical data can be organized in any of the following forms:

- The Ordered Array and Stem-leaf Display
- Tabulating and Graphing Numerical Data
- Frequency Distributions: Tables, Histograms, Polygons
- Cumulative Distributions: Tables, the Ogive



Stem and Leaf Display

A stem and leaf display (also called a stem and leaf plot) is particularly useful when the data is not too numerous.



Since 21 = 20 + 1 = (10 × 2) + 1

This is represented in the plot as a stem of 2 and a leaf of 1. The digit at the tenth place is taken as stem and the digit at units place is taken as leaf. Similarly, 26 is represented in the plot as a stem of 2 and a leaf of 6.

Remember, a stem is displayed once and the leaf can take on the values from 0 to 9.

Example

Consider Figure 1. It shows the number of touchdown (TD) passes thrown by each of the 31 teams in the National Football League in the 2000 season.

37, 33, 33, 32, 29, 28, 28, 23, 22, 22, 22, 21,
21, 21, 20, 20, 19, 19, 18, 18, 18, 18, 16, 15,
14, 14, 14, 12, 12, 9, 6

Figure 1: Number of touchdown passes.

A stem and leaf display of the data is shown in the Table 1 below. The left portion of the table contains the stems. They are the numbers 3, 2, 1, and 0, arranged as a column to the left of the bars. (As in 34, 3 is stem and 4 is leaf. In 16, 1 is stem and 6 is leaf)

Stem and leaf display showing the number of passing touchdowns.

3|2337
2|001112223889
1|2244456888899
0|69

Table 1

To make this clear, let us examine this Table 1 more closely. In the top row, the four leaves to the right of stem 3 are 2, 3, 3, and 7. Combined with the stem, these leaves represent the numbers 32, 33, 33, and 37, which are the numbers of TD passes for the first four teams in the table. The next row has a stem of 2 and 12 leaves. Together, they represent 12 data points. We leave it to you to figure out what the third row represents. The fourth row has a stem of 0 and two leaves.
One purpose of a stem and leaf display is to clarify the shape of the distribution. You can see many facts about TD passes more easily in Figure 1 than in the Table 1. For example, by looking at the stems and the shape of the plot, you can tell that most of the teams had between 10 and 29 passing TDs, with a few having more and a few having less. The precise numbers of TD passes can be determined by examining the leaves.

Tabulating and Graphing Univariate Categorical Data

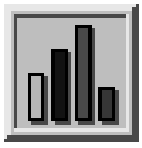
There are different ways of organizing univariate categorical data:

- The Summary Table
- Bar and Pie Charts, the Pareto Diagram

Tabulating and Graphing Bivariate Categorical Data

Bivariate categorical data can be organized as :

- Contingency Tables
- Side by Side Bar charts



GRAPHICAL EXCELLENCE AND COMMON ERRORS IN PRESENTING DATA

It is important that data is organized in a professional manner and graphical excellence is achieved in its presentation. High quality and attractive graphs can be used to explain and highlight facts which otherwise may go unnoticed in descriptive presentations. That is why all companies in their annual reports use different types of graphs to present data.

TABULATING NUMERICAL DATA:

Group data into classes
In some cases it is necessary to group the values of the data to summarize the data properly.

The process is described below.
Step 1: Sort Raw Data in Ascending Order
Data: 12, 13, 17, 21, 24, 24, 26, 27, 27, 30, 32, 35, 37, 38, 41, 43, 44, 46, 53, 58

Step 2: Find Range
Range = Maximum value – Minimum Value
Thus, Range = 58 - 12 = 46

Step 3: Select Number of Classes
Select the number of classes. (The classes are usually selected between 5 and 15)
In our example let us make 5 classes.

Step 4: Compute Class (width)
Find the class width by dividing the range by the number of classes and rounding up. Be careful of two things.

a) You must *round up*, not off. Normally 3.2 would round to be 3, but in rounding up, it becomes 4.
b) If the range divided by the number of classes gives an integer value (no remainder), then you can either add one to the number of classes or add one to the class width.

In our example

$$\text{Class width} = \frac{\text{Range}}{\text{Number of classes}} = \frac{46}{5} = 9.2$$

Round up 9.2 to 10

Step 5: Determine Class Boundaries (limits)
Pick a suitable starting point less than or equal to the minimum value. You will be able to cover, "the class width times the number of classes", values. Your starting point is the lower limit of the first class. Continue to add the class width to this lower limit to get the lower limit of other classes.
In this example if we start with 10 we will cover 10 × 5 = 50 values, which is close to our range. So let 10 be the lower limit of the first class. Continue to add 10 to this lower limit to get the lower limit of other classes.
: 10, 20(=10+10), 30(=20+10), 40(=30+10), 50(=40+10)
To find the upper limit of the first class, subtract one from the lower limit of the second class. Then continue to add the class width to this upper limit to find the rest of the upper limits
Upper limit of first class is 20 – 1 = 19.
Rest upper limits are; 29 (=19+10), 39 (=29+10), 49 (=39+10)

Step 6: Compute Class Midpoints

$$\text{Class Midpoint} = \frac{(\text{Lower limit} + \text{Upper limit})}{2}$$

First midpoint is (10+19)/2 = 14.5

Other midpoints are:
(20+29)/2 = 24.5
(30+39)/2 = 34.5
(40+49)/2) = 44.5
(50+59)/2) = 54.5

Depending on what you're trying to accomplish, it may not be necessary to find the midpoint.

Step 7: Compute Class Intervals
First class : Lower limit is 10. Higher limit is 19.
We can write first class interval as **10 to 19** or **10 – 19** or "**10 but under 20**".
In "**10 but under 20**" a value greater than 19.5 will be treated as above 20.

Similarly other 4 class intervals are
20 – 29
30 – 39
40 – 49
50 – 59

Important points to remember

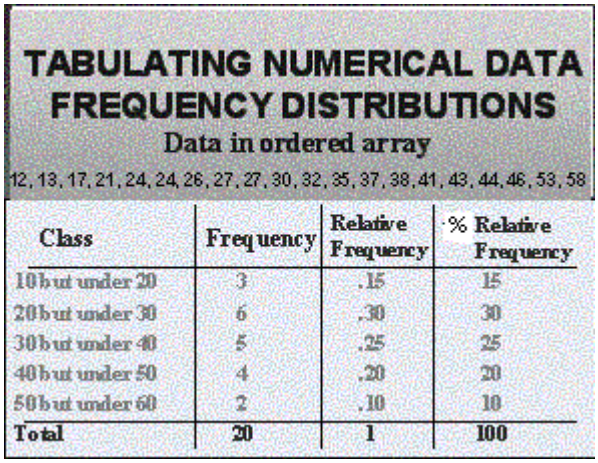
1. There should be between 5 and 15 classes.
2. Choose an odd number as a class width if you want to have classes' midpoints as an integer instead of decimals.
3. The classes must be mutually exclusive. This means that no data value can fall into two different classes
4. The classes must be all inclusive or exhaustive. This means that all data values must be included.
5. The classes must be continuous. There should be no gaps in a frequency distribution. Classes that have no values in them must be included (unless it's the first or last class (es) that could be dropped).
6. The classes must be equal in width. The exception here is the first or last class. It is possible to have a "below ..." as a first class or "... and above" as a last class.

Frequency Distribution: Count Observations & Assign to Class Intervals:
Looking through the data shows that there are three values between 10 and 19. Hence frequency is 3. Similarly, frequency of other class intervals can be found as follows:
10 – 19 : 3
20 - 29 : 6
30 - 39 : 5
40 - 49 : 4
50 - 59 : 2
Total frequency = 3 + 6 + 5 + 4 + 2 = 20

Relative frequency :
Relative Frequency of a class = $\frac{\text{Frequency of the class interval}}{\text{Total Frequency}}$

There are 3 observations in first class interval 10 – 19.
The relative frequency is 3/20 = 0.15.
Similarly relative frequency for other class intervals are calculated.

Percent Relative Frequency:
If we multiply 0.15 by 100, then the % Relative Frequency 15% is obtained.



TABULATING NUMERICAL DATA
FREQUENCY DISTRIBUTIONS
Data in ordered array
12, 13, 17, 21, 24, 24, 26, 27, 27, 30, 32, 35, 37, 38, 41, 43, 44, 46, 53, 58

Class	Frequency	Relative Frequency	% Relative Frequency
10 but under 20	3	.15	15
20 but under 30	6	.30	30
30 but under 40	5	.25	25
40 but under 50	4	.20	20
50 but under 60	2	.10	10
Total	20	1	100

Cumulative Frequency:
If we add frequency of the second interval to the frequency of the first interval , then the cumulative frequency for the second interval is obtained. Cumulative frequency of each class interval is calculated below.
10 – 19 : 3
20 – 29 : 3 + 6 = 9
30 – 39 : 3 + 6 + 5 = 14
40 – 49 : 3 + 6 + 5 + 4 = 18
50 – 59 : 3 + 6 + 5 + 4 + 2 = 20

Percent cumulative relative frequency
This can be calculated same as cumulative frequency except now percent relative frequency for each class interval is considered. The percent cumulative relative frequency of the last class interval is 100% as all observations have been added.
Percent cumulative relative frequency of each class interval is calculated below
10 – 19 : 15
20 – 29 : 15 + 30 = 45
30 – 40 : 15 + 30 + 25 =70
40 – 50 : 15 + 30 + 25 + 20 = 90
50 – 60 : 15 + 30 + 25 + 20 + 10 = 100

LECTURE 25
STATISTICAL REPRESENTATION
MEASURES OF CENTRAL TENDENCY
PART 2

OBJECTIVES

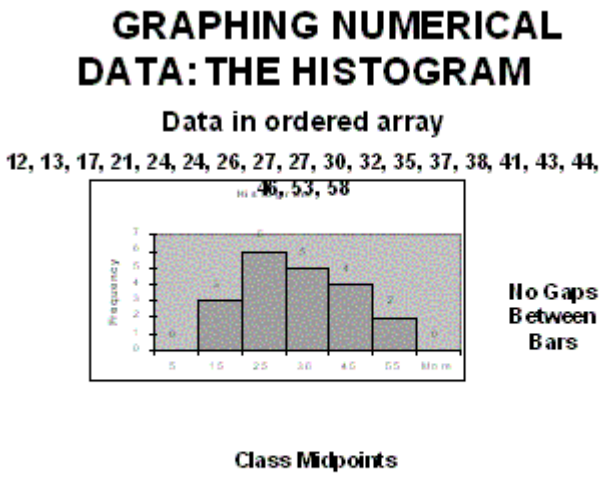
The objectives of the lecture are to learn about:

- Review Lecture 24
- Statistical Representation
- Measures of Central Tendency

Part 2

GRAPHING NUMERICAL DATA: THE HISTOGRAM

Histogram is a bar graph of a frequency distribution in which the widths of the bars are proportional to the classes into which the variable has been divided and the heights of the bars are proportional to the class frequencies. Histogram of the example of frequency distribution discussed at the end of lecture 24 is given below.



MEASURES OF CENTRAL TENDENCY

Measures of central tendency can be summarized as under:

1. Arithmetic Mean
    a. Arithmetic mean for discrete data
        i. Sample Mean
        ii. Population Mean
    b. Arithmetic Mean for grouped data
2. Geometric Mean
3. Harmonic Mean
4. Weighted Mean
5. Truncated Mean or Trimmed Mean
6. Winsorized Mean
7. Median
    a. Median for grouped data
    b. Median for discrete data
8. Mode
    a. Mode for grouped data

b. Mode for discrete data

9. Midrange
10. Midhinge

As you see it is a long list. However, if you look closely you will find that the main measures are Arithmetic Mean, Median, Mode.
All the above measures are used in different situations to understand the behavior of data for decision making. It may be interesting to know the average, median or mode salary in an organization before the company decides to increase the salary level. Comparisons with other companies are also important. The above measures provide a useful summary measure to consolidate large volumes of data. Without such summaries it is not possible to compare large selections of data.
EXCEL has a number of useful functions for calculating different measures of central tendency. Some of these are explained below. You are encouraged to go through EXCEL Help file for detailed descriptions of different functions. For selected functions, the help file has been included in the handouts. The examples are also from the help files.

AVERAGE

Returns the average (arithmetic mean) of the arguments.

Syntax

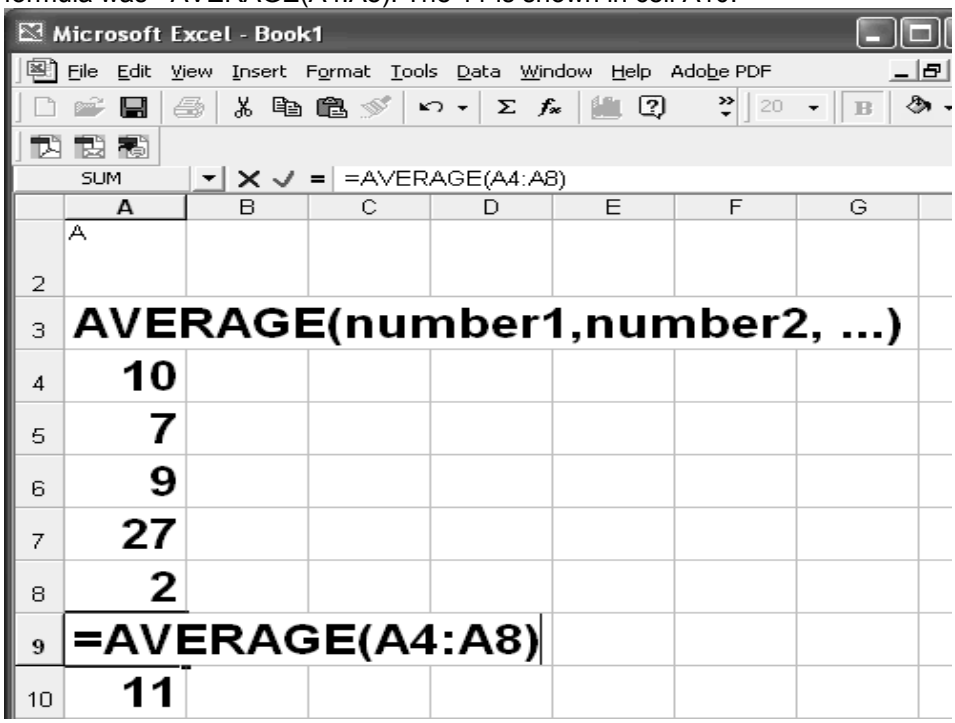
AVERAGE(**number1**,number2,...)
Number1, number2, ... are 1 to 30 numeric arguments for which you want the average.

Remarks

- The arguments must either be numbers or be names, arrays, or references that contain numbers.
- If an array or reference argument contains text, logical values, or empty cells, those values are ignored; however, cells with the value zero are included

Example

An example of AVERAGE is shown below. Data was entered in cells A4 to A8. The formula was =AVERAGE(A4:A8). The 11 is shown in cell A10.



AVERAGEA

Calculates the average (arithmetic mean) of the values in the list of arguments. In addition to numbers, text and logical values such as TRUE and FALSE are included in the calculation.

Syntax

AVERAGEA(**value1**,value2,...)

Value1, value2, ... are 1 to 30 cells, ranges of cells, or values for which you want the average.

Remarks

- The arguments must be numbers, names, arrays, or references.
- Array or reference arguments that contain text evaluate as 0 (zero). Empty text ("") evaluates as 0 (zero). If the calculation must not include text values in the average, use the AVERAGE function.
- Arguments that contain TRUE evaluate as 1; arguments that contain FALSE evaluate as 0 (zero).

Example

	A
1	**Data**
2	10
3	7
4	9
5	2
6	Not available
7	

Formula	Description (Result)
=AVERAGEA(A2:A6)	Average of the numbers above, and the text "Not Available". The cell with the text "Not available" is used in the calculation. (5.6)
=AVERAGEA(A2:A5,A7)	Average of the numbers above, and the empty cell. (7)

MEDIAN

Returns the median of the given numbers. The median is the number in the middle of a set of numbers; that is, half the numbers have values that are greater than the median, and half have values that are less.

Syntax

MEDIAN(**number1**,number2,...)

Number1, number2, ... are 1 to 30 numbers for which you want the median.

Remarks

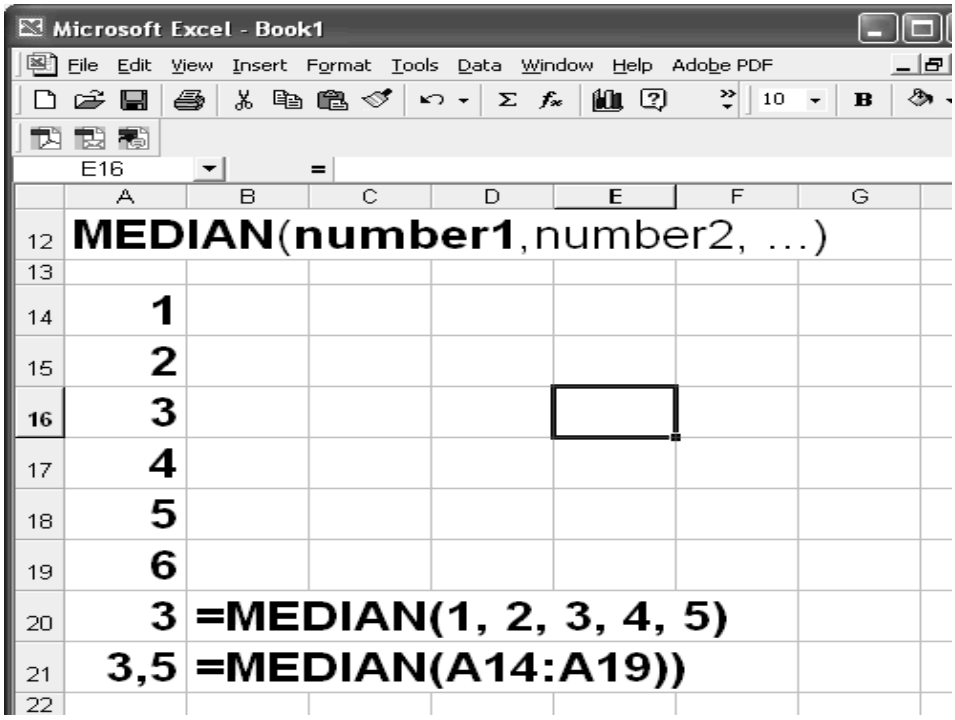
- The arguments should be either numbers or names, arrays, or references that contain numbers. Microsoft Excel examines all the numbers in each reference or array argument.
- If an array or reference argument contains text, logical values, or empty cells, those values are ignored; however, cells with the value zero are included.
- If there is an even number of numbers in the set, then MEDIAN calculates the average of the two numbers in the middle. See the second formula in the example.

Example

The numbers are entered in cells A14 to A19.

In the first formula =MEDIAN(1,2,3,4,5) the actual values are specified. The median as you see is 3, in the middle.

In the next formula =MEDIAN(A14:A19), the entire series was specified. There is no middle value in the middle. Therefore the average of the two values 3 and 4 in the middle was used as the median 3.5.



MODE

Returns the most frequently occurring, or repetitive, value in an array or range of data. Like MEDIAN, MODE is a location measure.

Syntax

MODE(**number1**,number2,...)

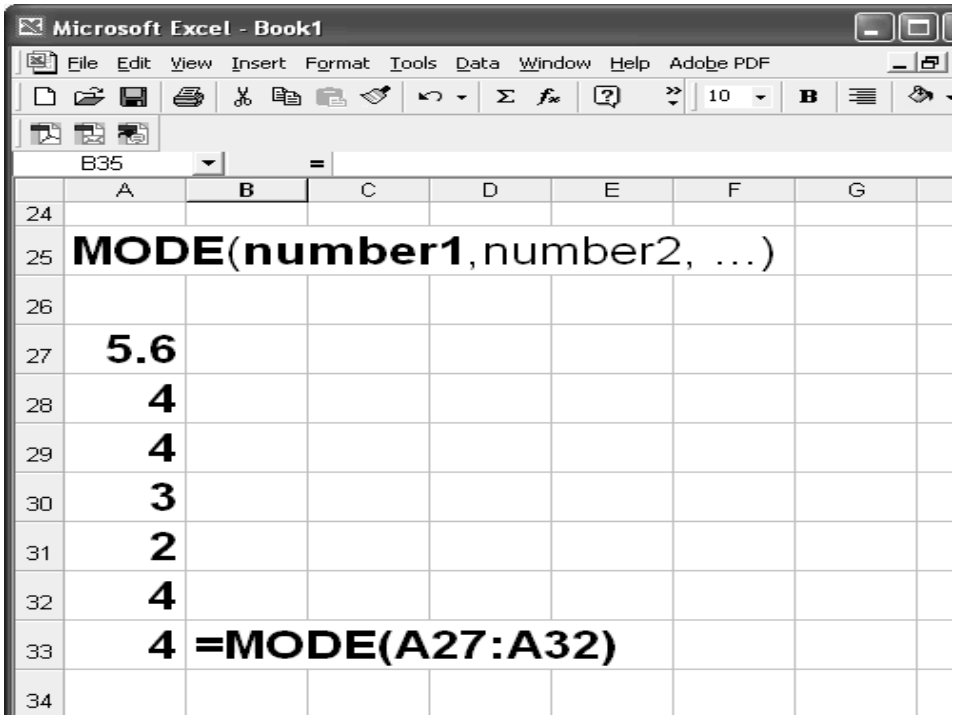
Number1, number2, ... are 1 to 30 arguments for which you want to calculate the mode. You can also use a single array or a reference to an array instead of arguments separated by commas.

Remarks

- The arguments should be numbers, names, arrays, or references that contain numbers.
- If an array or reference argument contains text, logical values, or empty cells, those values are ignored; however, cells with the value zero are included.
- If the data set contains no duplicate data points, MODE returns the #N/A error value. In a set of values, the mode is the most frequently occurring value; the median
- is the middle value; and the mean is the average value. No single measure of central tendency provides a complete picture of the data. Suppose data is clustered in three areas, half around a single low value, and half around two large values. Both AVERAGE and MEDIAN may return a value in the relatively empty middle, and MODE may return the dominant low value.

Example

The data was entered in cells A27 to A32. The formula was =MODE(A27:A32).
The answer 4 is the most frequently occurring value.



COUNT FUNCTION

Counts the number of cells that contain numbers and also numbers within the list of arguments. Use COUNT to get the number of entries in a number field that's in a range or array of numbers.

Syntax

COUNT(**value1**,value2,...)

Value1, value2, ... are 1 to 30 arguments that can contain or refer to a variety of different types of data, but only numbers are counted.

Remarks

- Arguments that are numbers, dates, or text representations of numbers are counted; arguments that are error values or text that cannot be translated into numbers are ignored.
- If an argument is an array or reference, only numbers in that array or reference are counted. Empty cells, logical values, text, or error values in the array or reference are ignored. If you need to count logical values, text, or error values, use the COUNTA function.

Example

	A
1	**Data**
2	Sales
3	12/8/2008
4	
5	19
6	22.24
7	TRUE
8	#DIV/0!

Formula	Description (Result)

=COUNT(A2:A8)	Counts the number of cells that contain numbers in the list above (3)
=COUNT(A5:A8)	Counts the number of cells that contain numbers in the last 4 rows of the list (2)
=COUNT(A2:A8,2)	Counts the number of cells that contain numbers in the list, and the value 2 (4)

FREQUENCY

Calculates how often values occur within a range of values, and then returns a vertical array of numbers. For example, use FREQUENCY to count the number of test scores that fall within ranges of scores.

Remember FREQUENCY returns an array, it must be entered as an array formula.

Syntax

FREQUENCY(**data_array**,**bins_array**)

Data_array is an array of or reference to a set of values for which you want to count frequencies. If data_array contains no values, FREQUENCY returns an array of zeros.

Bins_array is an array of or reference to intervals into which you want to group the values in data_array. If bins_array contains no values, FREQUENCY returns the number of elements in data_array.

Remarks

- FREQUENCY is entered as an array formula after you select a range of adjacent cells into which you want the returned distribution to appear.
- The number of elements in the returned array is one more than the number of elements in bins_array. The extra element in the returned array returns the count of any values above the highest interval. For example, when counting three ranges of values (intervals) that are entered into three cells, be sure to enter FREQUENCY into four cells for the results. The extra cell returns the number of values in data_array that are greater than the third interval value.
- FREQUENCY ignores blank cells and text.
- Formulas that return arrays must be entered as array formulas.

	A	B
1	**Scores**	**Bins**
2	79	70
3	85	79
4	78	89
5	85	
6	50	
7	81	
8	95	
9	88	
10	97	

Formula	**Description (Result)**
=FREQUENCY(A2:A10,B2:B5)	Number of scores less than or equal to 70 (1) Number of scores in the bin 71-79 (2)

Number of scores in the bin 80-89 (4)
Number of scores greater than or equal to 90 (2)

Example

Note The formula in the example must be entered as an array formula. After copying the example to a blank worksheet, select the range A13:A16 starting with the formula cell. Press F2, and then press CTRL+SHIFT+ENTER. If the formula is not entered as an array formula, the single result is 1.

ARITHMETIC MEAN GROUPED DATA

Below is an example of calculating arithmetic mean of grouped data. Here the marks (Classes) and frequency are given. The class marks are the class mid points calculated as average of lower and higher limits. For example, the average of 20 and 24 is 22. The frequency f is multiplied by the class mark to obtain the total number. In first row the value of fx is 1 x 22 = 22. The sum of all fx is 1950. The total number of observations is 50. Hence the arithmetic mean is 1950/50 = 39.

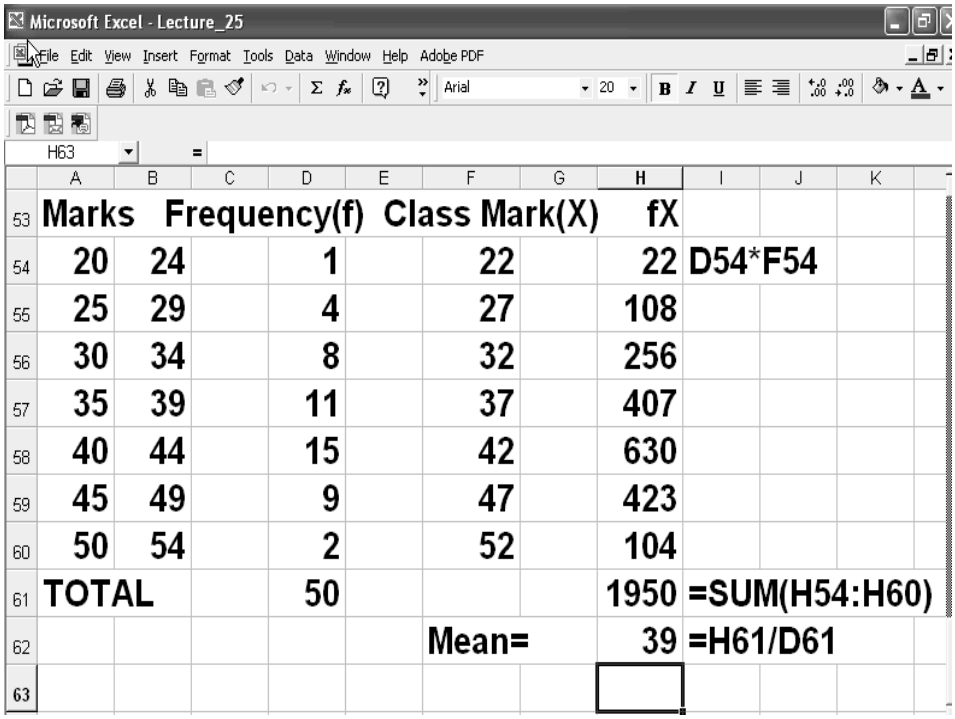
Marks	Frequency	Class Marks	fX
20-24	1	22	22
25-29	4	27	108
30-34	8	32	256
35-39	11	37	407
40-44	15	42	630
45-49	9	47	423
50-54	2	52	104
TOTAL	50		1950

n= 50; Sum(fX) =1950; Mean =1950/50= 39 Marks

EXCEL Calculation

The above calculation would be common in business life. Let us see how we can do it using EXCEL.

The basic data of lower limits is entered in cell range A54:A60. The data of higher limit is entered in cells B54:B60. Frequency is given in cell range D54:D60. Class midpoints or class marks were calculated in cells F54:F60. In cell F54 the formula =(A54+B54)/2 was used to calculate the class mark. This formula was copied in other cells (F55 to F60). The value of fx was calculated in cell H54 using the formula =D54*F54. This formula was copied to other cells H55 to H60. Total frequency was calculated in cell D61 using the formula =SUM(D54:D60). Sum of fx was calculated in cell H61 using the formula =SUM(H54:H60). Mean was calculated in cell H62 using the formula =ROUND(H61/D61,0).



	A	B	C	D	E	F	G	H	I	J	K
53	Marks			Frequency(f)		Class Mark(X)		fX			
54	20	24		1		22		22	D54*F54		
55	25	29		4		27		108			
56	30	34		8		32		256			
57	35	39		11		37		407			
58	40	44		15		42		630			
59	45	49		9		47		423			
60	50	54		2		52		104			
61	TOTAL			50				1950	=SUM(H54:H60)		
62						Mean=		39	=H61/D61		
63											

LECTURE 26
STATISTICAL REPRESENTATION
MEASURES OF DISPERSION AND SKEWNESS
PART 1

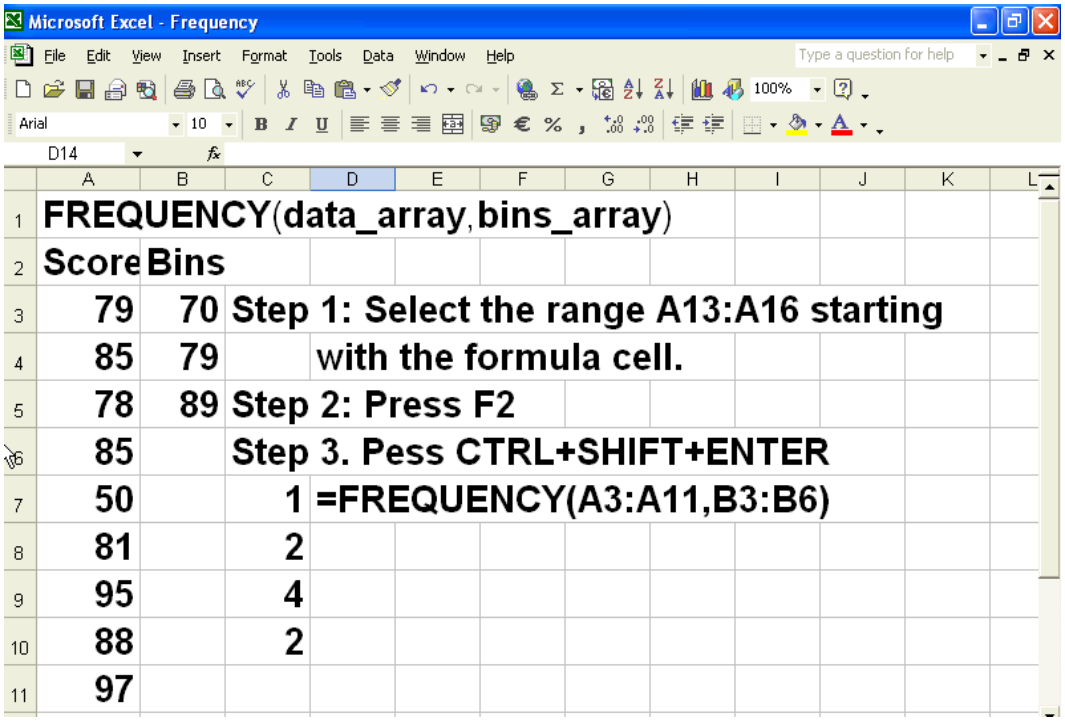
OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 25
- Statistical Representation
- Measures of Dispersion and Skewness

FREQUENCY-EXAMPLE

FREQUENCY Function calculates how often values occur within a range of values, and then returns a vertical array of numbers. For details see handout for lecture 25. The syntax is **FREQUENCY**(**data_array**,**bins_array**).



The data was entered in cells A3 to A11. The Bins array which gives the limits 70, 79 and 89 were entered in cells B3 to B5. Select cells ~~B~~C7 to ~~B~~C10 (one more than the limits) Type the formula =FREQUENCY(A3:A1,B3:B5) Then, CTRL+Shift+Enter were pressed to indicate that we are entering an array formula.
The result is given in cells B7 to B10. We can interpret the result as the frequency of ~~It~~
Less than or equal to 70 is 1
71 to 79 is 2
80 to 88 is 4
89 and above is 2

Application of FREQUENCY function in Frequency Distribution

Consider the example at the end of lecture 24.

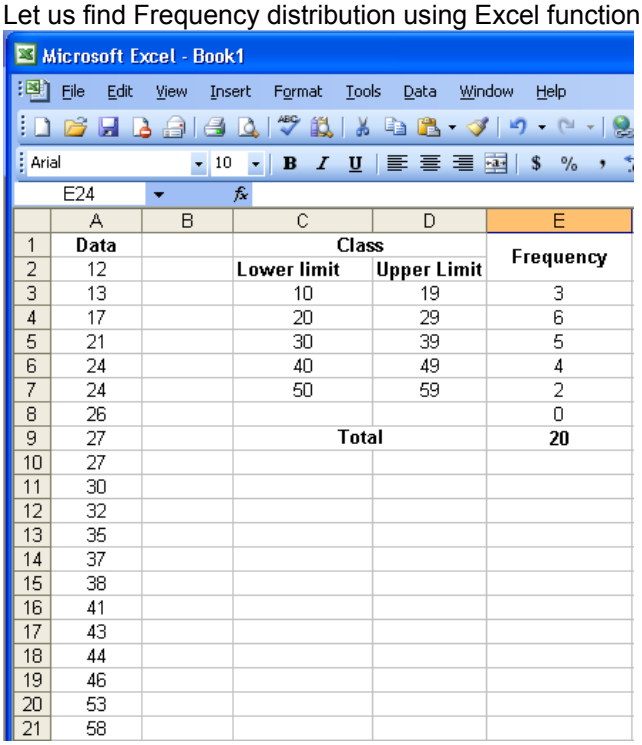
TABULATING NUMERICAL DATA
FREQUENCY DISTRIBUTIONS

Data in ordered array

12, 13, 17, 21, 24, 24, 26, 27, 27, 30, 32, 35, 37, 38, 41, 43, 44, 46, 53, 58

Class	Frequency	Relative Frequency	% Relative Frequency
10 but under 20	3	.15	15
20 but under 30	6	.30	30
30 but under 40	5	.25	25
40 but under 50	4	.20	20
50 but under 60	2	.10	10
Total	20	1	100

Let us find Frequency distribution using Excel function FREQUENCY.

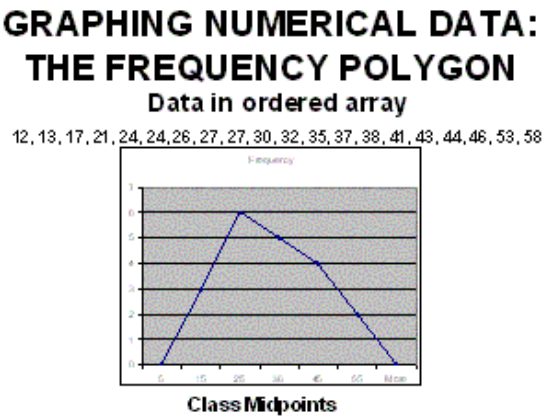


	A	B	C	D	E
1	Data		Class		Frequency
2	12		Lower limit	Upper Limit	
3	13		10	19	3
4	17		20	29	6
5	21		30	39	5
6	24		40	49	4
7	24		50	59	2
8	26				0
9	27		Total		20
10	27				
11	30				
12	32				
13	35				
14	37				
15	38				
16	41				
17	43				
18	44				
19	46				
20	53				
21	58				

Write data in column A. In column C and D write the class's lower and upper limits. Here we will take class's upper limit as bins. Select the cells E3 to E8, under Frequency heading (one cell more than the number of classes). Type =FREQUENCY(A2:A21,D3:D7). Press Ctrl+Shift+Enter. Your Frequency column will get filled with above mentioned values.

FREQUENCY POLYGONS

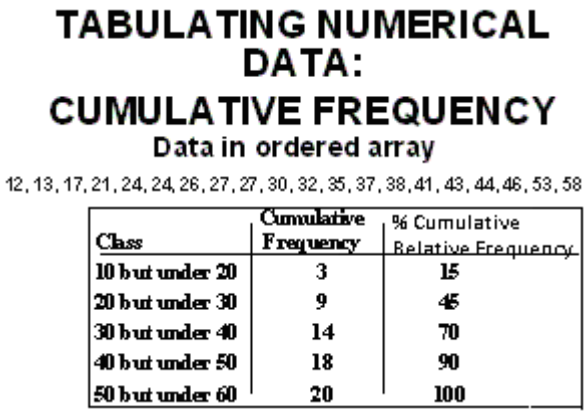
Frequency polygon is a line graph obtained from a frequency distribution by joining with straight lines points, whose abscissa are the midpoints of successive class intervals and whose ordinates are the corresponding class frequencies.



CUMULATIVE FREQUENCY

Relative frequency can be converted into cumulative frequency by adding the current frequency to the previous total. In the slide below, the first interval has the relative as well as cumulative frequency as 3. In the next interval the relative frequency was 6. it was added to the previous value to arrive at 9 as cumulative frequency for interval 20 to 30. What it really means is that 9 values are equal to or less than 30. Similarly, the other cumulative frequencies were calculated. The total cumulative frequency 20 is the total number of observations.

Percent Cumulative frequency is calculated by dividing the cumulative frequency by the total number of observations and multiplying by 100. For the first interval the % cumulative frequency is 3/20*100 = 15%. Similarly other values were calculated.



CUMULATIVE % POLYGON-OGIVE

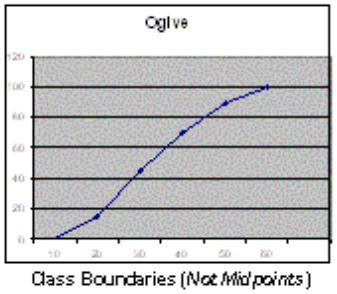
From the % cumulative relative frequency polygon that starts from the first limit (not mid point as in the case of relative frequency polygons) can be drawn. Such a polygon is called Ogive. The maximum value in an Ogive is always 100%. Ogives are determining cumulative frequencies at different values (not limits).

GRAPHING NUMERICAL DATA:

THE OGIVE (CUMULATIVE % POLYGON)

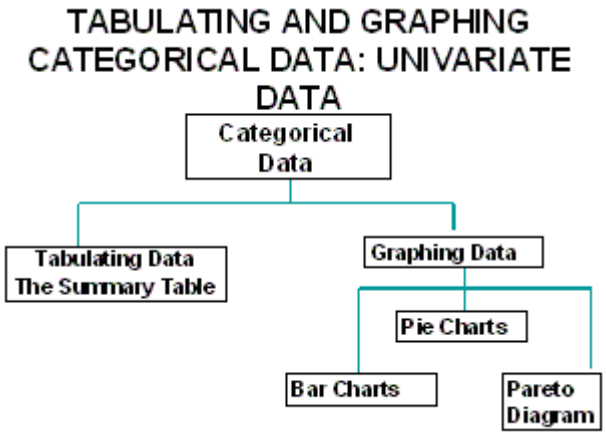
Data in ordered array

12, 13, 17, 21, 24, 24, 26, 27, 27, 30, 32, 35, 37, 38, 41, 43, 44, 46, 53, 58



TABULATING AND GRAPHING UNIVARIATE DATA

Univariate data (one variable) can be tabulated in Summary form or in graphical form. Three types of charts, namely, Bar Charts, Pie Charts or Pareto Diagrams can be prepared.



SUMMARY TABLE

A summary table is built specifically from detailed data. It contains summaries of the data and is used to speed up analysis.

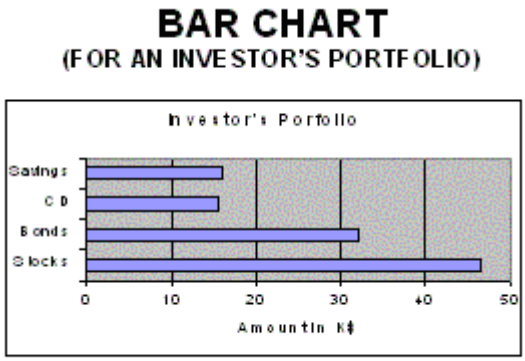
A typical Summary Table for an investor's portfolio is given in the slide. The variables such as stocks etc. are the categories. The table shows the amount and percentage.

Investment Category	Amount(in thousand Rs)	Percentage
Stocks	46.5	42.27
Bonds	32	29.09
Cash Deposit	15.5	14.09
Savings	16	14.55
TOTAL	**110**	**100**

BAR CHART

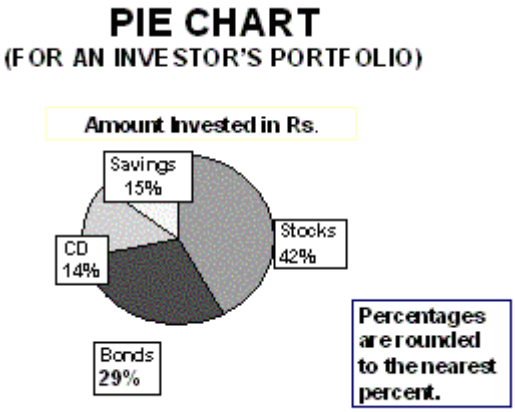
The data of Investor's portfolio can be shown in the form of Bar Chart as shown below. This chart was prepared using EXCEL Chart Wizard. The Wizard makes it

very simple to prepare such graphs. You must practice with the Chart Wizard to prepare different types of graphs.



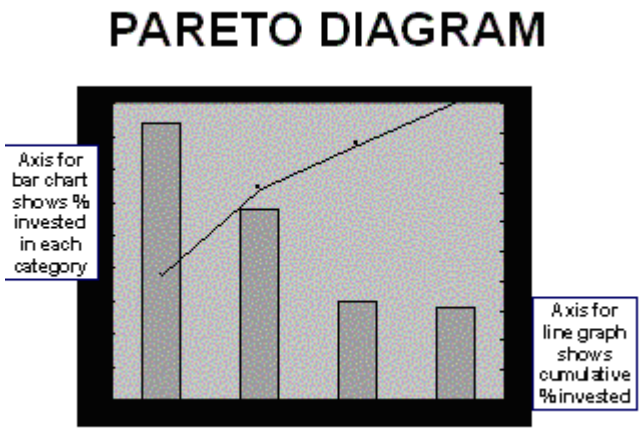
PIE CHARTS

Pie Charts are very useful charts to show percentage distribution. These charts are made with the help of Chart Wizard. You may notice how Stocks and bonds stand out.



PARETO DIAGRAMS

A Pareto diagram is a cumulative distribution with the first value as first relative frequency, in this case 42%. The point is drawn in the middle of bar for the first category stocks. Next the category Bonds was added. The total is 71%. Next the savings 15% were added to 71% to obtain cumulative frequency 86%. Adding the 14% for CD gives 100%. Thus, the Pareto diagram gives both relative and cumulative frequency.



CONTINGENCY TABLES

Another form of presentation of data is the contingency table. An example is shown in the slide below. The table shows a comparison of three investors along with their combined total investment.

TABULATING CATEGORICAL DATA: BIVARIATE DATA

Contingency Table

Investment in Thousands of Rupees

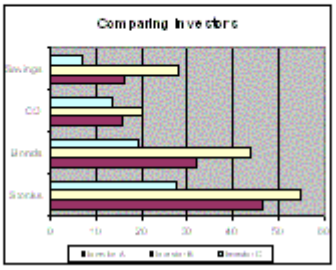
Investment Category	Investor A	Investor B	Investor C	Total
Stocks	46.5	55	27.5	129
Bonds	32	44	19	95
CD	15.5	20	13.5	49
Savings	16	28	7	51
Total	110	147	67	324

SIDE BY SIDE CHARTS

The same investor data can be shown in the form of side by side charts where different colours were used to differentiate the investors. This graph is a complete representation of the contingency table.

GRAPHING CATEGORICAL DATA: BIVARIATE DATA

Side by Side Chart



GEOMETRIC MEAN

Geometric mean is defined as the root of product of individual values. Typical syntax is as under:

G=(x1.x2.x3.....xn)^(1/n)

Example

Find GM of 130, 140, 160

GM = (130*140*160) ^ (1/3)

= 142.8

HARMONIC MEAN

Harmonic mean is defines as under:

$$HM = \frac{n}{(1/x1+1/x2+.....1/xn)} = \frac{n}{Sum(1/xi)}.$$

Example

Find HM of 10, 8, 6

$$HM = \frac{3}{(1/10+1/8+ 1/6)}. = 7.66$$

QUARTILES

Quartiles divide data into 4 equal parts
Syntax
1st Quartile Q1=(n+1)/4
2nd Quartile Q2= 2(n+1)/4
3rd Quartile Q3= 3(n+1)/***4***
Grouped data
Qi= ith Quartile = l + h/f[Sum f/4*i – cf)
l = lower boundary
h = width of CI
cf = cumulative frequency

DECILES
Deciles divide data into 10 equal parts
Syntax
1st Decile D1=(n+1)/10
2nd Decile D2= 2(n+1)/10
9th Deciled D9= 9(n+1)/10
Grouped data
Qi = ith Decile (i=1,2,.9) = l + h/f[Sum f/10*i – cf)
l = lower boundary
h = width of CI
cf = cumulative frequency

PERCENTILES
Percentiles divide data into 100 equal parts
Syntax
1st Percentile P1=(n+1)/100
2nd Decile D2= 2(n+1)/100
99th Deciled D9= 99(n+1)/100
Grouped data
Qi = ith Decile(i=1,2,.9) = l + h/f[Sum f/100*i – cf)
l = lower boundary
h = width of CI
cf = cumulative frequency

Symmetrical Distribution

mean = median = mode
Positively Skewed Distribution
(Tilted to left)
mean > median > mode
Negatively Skewed Distribution
mode < median < mean
(Tilted to right)

EMPIRICAL RELATIONSHIPS
Moderately Skewed and Unimodal Distribution
Mean – Mode = 3(Mean – Median)
Example
mode = 15, mean = 18, median = ?
Median = 1/3[mode + 2 mean]
= 1/3[15 + 2(18)]
= [15+36]/3 = 51/3 = 17

A trimmed or truncated mean is a measure of central tendency and is one type of modified mean. In this, first we sort the data. Then according to the problem, discard an equal number of data at both ends. Most often 25 percent of the ends are discarded. That is, values below first

quartile and above third quartile are removed. Mean of the remaining data is called trimmed mean or truncated mean.

WINSORIZED MEAN

It involves the calculation of the mean after replacing given parts of a data at the high and low end with the most extreme remaining values. Most often 25 percent of the ends are replaced. That is, values below first quartile and above third quartile are replaced.

Example

Find trimmed and winsorized mean.
9.1, 9.2, 9.3, 9.2, 9.2, 9.9
Arrange the data in ascending order
9.1, 9.2, 9.2, 9.2, 9.3, 9.9
Position of Q1 = (6+1)/4 =1.75
Q1 = 2nd value approximately = 9.2
Position of Q3 = 3(6+1)/4= 5.25
Q3 = 5^{th} value approximately = 9.3
Trimmed Mean = (9.2 + 9.2 + 9.2 + 9.3) / 4 = 9.225
Winsorized Mean = (9.2 + 9.2 + 9.2 + 9.2 + 9.3 + 9.3) / 6 = 9.233

DISPERSION OF DATA

Definition

The degree to which numerical data tend to spread about an average is called the dispersion of data

TYPES OF MEASURES OF DISPERSION

Absolute measures
Relative measures (coefficients)

DISPERSION OF DATA

Types Of Absolute Measures:

- ***Range***
- ***Quartile Deviation***
- ***Mean Deviation***
- ***Standard Deviation or Variance***

Types Of Relative Measures

- ***Coefficient of Range***
- ***Coefficient of Quartile Deviation***
- ***Coefficient of Mean Deviation***
- ***Coefficient of Variation***

LECTURE 27
STATISTICAL REPRESENTATION
MEASURES OF DISPERSION AND SKEWNESS
PART 2

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 26
- Measures of Dispersion and Skewness

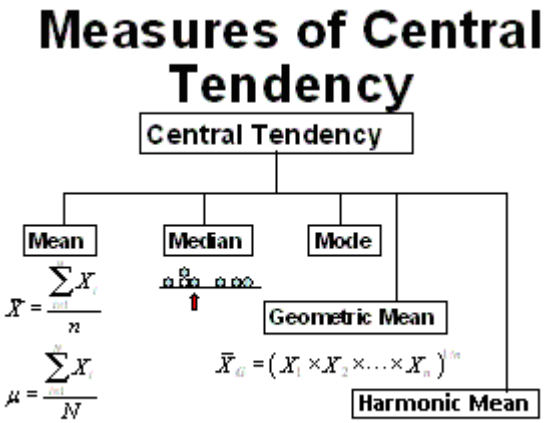
MEASURES OF CENTRAL TENDENCY, VARIATION AND SHAPE FOR A SAMPLE

There are many different measures of central tendency as discussed in the last lecture handout. These include:

- Mean, Median, Mode, Midrange, Quartiles, Midhinge
- Range, Interquartile Range
- Variance, Standard Deviation, Coefficient of Variation
- Right-skewed, Left-skewed, Symmetrical Distributions
- Measures of Central Tendency, Variation and ShapeExploratory Data Analysis
- Five-Number Summary
- Box-and-Whisker Plot
- Proper Descriptive Summarisation
- Exploring Ethical Issues
- Coefficient of Correlation

MEANS

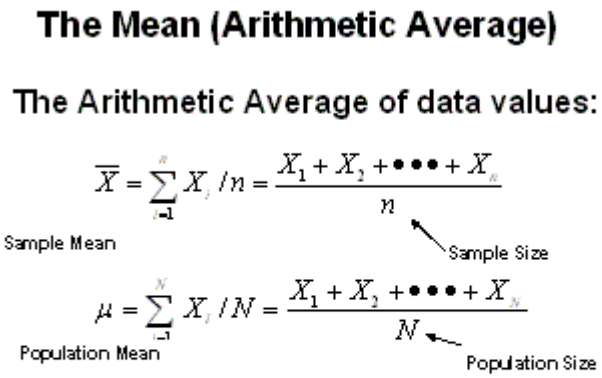
The most common measure of central tendency is the mean. The slide below shows the Mean (Arithmetic), Median, Mode and Geometric mean. Another mean not shown is the Harmonic mean. Each of these has its own significance and application. The mean is the arithmetic mean and represents the overall average. The median divides data in two equal parts. Mode is the most common value. Geometric mean is used in compounding such as investments that are accumulated over a period of time. Harmonic mean is the mean of inverse values. Each has its own utility. The slide shows the formulas for mean and geometric mean.



THE MEAN

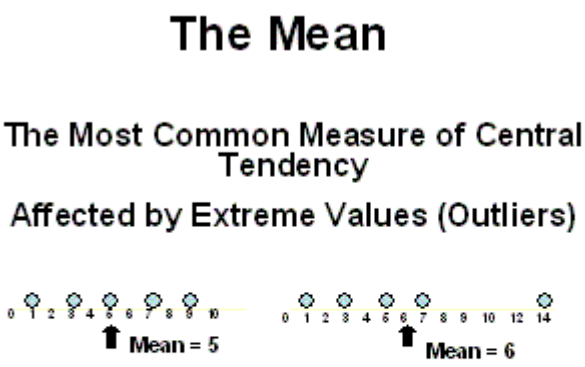
The formula for Arithmetic Mean is given in the slide. It is the sum of all values divided by the number. In the case of mean of a sample, the number n is the total sample size.

When the sample data is to be used for estimating the value of mean, then the number is reduced by 1 to improve the estimate. In reality this will be a slight overestimation of the population mean. This is done to avoid errors in estimation based on sample data that may not be truly represented of the population.



EXTREME VALUES

An important point to remember is that arithmetic mean is affected by extreme values. In the following slide mean of 5 values 1, 3, 5, 7 and 9 is 5. In the second case where the data values are 1, 3, 6, 7 and 14, the value 14 is an outlier as it is considerably different from the other values. In this case the mean is 6. in other words the mean increased by 1 or about 20% due to the outlier. While preparing data for mean, it is important to spot and eliminate outlier values.



THE MEDIAN

The Median is derived after ordering the array in ascending order. If the number of

The Median

Important Measure of Central Tendency

In an ordered array, the median is the "middle" number.

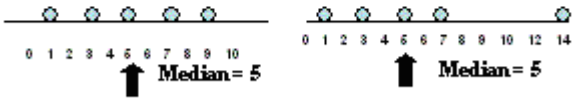
If n is *odd*, the median is the *middle number*

If n is *even*, the median is the *average of the 2 middle numbers*

observations is odd, it is the middle value otherwise it is the the average of the two middle values. It is not affected by extreme values.

The Median

Not Affected by Extreme Values



THE MODE

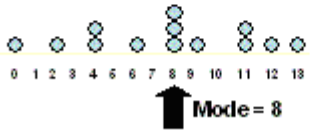
The mode is the value that occurs most frequently. In the example shown on the slide, 8 is the most frequently occurring value. Hence the mode is 8. Mode is also not affected by extreme values.

The Mode

A Measure of Central Tendency

Value that Occurs Most Often

Not Affected by Extreme Values



An important point about Mode is that there may not be a Mode at all (no value is occurring frequently). There may be more than one mode. The mode can be used for numerical or categorical data. The slide shows two examples where there is no mode or there are two modes.

The Mode

There May Not be a Mode
There May be Several Modes

Used for Either Numerical or Categorical Data



RANGE

Another measure of dispersion of data is the Range. It is the difference between the largest and smallest value. The slides show examples where the value of range was calculated

DISPERSION OF DATA

Range

R = Largest – Smallest Value

Example

Find range:

31, 26, 15, 43, 19, 27, 22, 12, 36, 33, 30, 24, 20

Largest value = 43

Smallest = 12

Range = 43 – 12 = 31

The Range

Measure of Variation Difference Between Largest & Smallest Observations.

Range = Largest – Smallest Value

Ignores How Data Are Distributed:



MIDRANGE

Midrange is the average of smallest and largest value. In other words it is half of a range. Midrange is affected by extreme values as it is based on smallest and largest values

Midrange

A Measure of Central Tendency

Average of Smallest and Largest Observation:

$$\text{Midrange} = \frac{X_{largest} + X_{smallest}}{2}$$

Midrange

Affected by Extreme Value

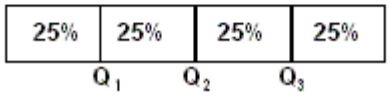


QUARTILES

Quartiles are not exclusively measures of central tendency. However, they are useful for dividing the data in 4 equal parts, each containing 25% of the data. So there are three quartiles. 25% data falls below first quartile, 50% below second and 75% below third quartile.

Quartiles

Not a measure of central tendency
Split ordered data into 4 quarters



Position of ith quartile = $\frac{i\,(n + 1)}{4}$

Where i = 1, 2, 3 and n is the number of data points.

To understand the procedure of finding the value of each quartile let us consider an example.

Example:
Find first, second and third quartile of the following data
11, 22, 17, 16, 12, 21, 16, 13, 18

Arrange the data in ascending order.
11, 12, 13, 16, 16, 17, 18, 21, 22

Here n = number of data points = 9

Position of first quartile Q1 = $\frac{9 + 1}{4}$ = 2.5

Since we get position of first quartile as decimal fraction, so we proceed as follows

Q1 = 2nd value + 0.5 × (3rd value – 2nd value)

= 12 + 0.5 × (13 – 12)

= 12 + 0.5 × 1

= 12.5

Position of second quartile Q2 = $\frac{2 \times (9 + 1)}{4}$ = 5

So, Q2 = 5th value = 16

Position of third quartile Q3 = $\frac{3 \times (9 + 1)}{4}$ = 7.5

So, Q3 = 7th value + 0.5 × (8th value – 7th value)

= 18 + 0.5 × (21 – 18)

= 18 + 0.5 × 3

= 19.5

MIDHINGE:

Midhinge is the average of the first and third quartiles.

Midhinge = $\frac{Q1 + Q3}{2}$

QUARTILE DEVIATION

Quartile Deviation is the average of 1st and 3rd Quartile.

Q.D = $\frac{(Q3 – Q1)}{2}$

Example

Find Q.D of the following data

14, 10, 17, 5, 9, 20, 8, 24, 22, 13

Here number of data points = n = 10

Position of Q1 = (n+1)/4 = (10+1)/4 = 2.75

So Q1 = 2nd value + 0.75 × (3rd value – 2nd value)

= 8 + 0.75 × (9 - 8)

= 8 + 0.75 x 1

= 8.75

Position of Q3 = 3 × (10 + 1) /4 = 3 × (2.75) = 8.25

So Q3 = 8th value + 0.25(9th value – 8th value)

= 20 + 0.25 × (22 –20)

= 20.50

Q.D = $\frac{(20.50 – 8.75)}{2}$ = 5.875

SHAPE OF DISTRIBUTION

1. Symmetrical Distribution
2. Asymmetrical Distribution
    a. Right-skewed or positively-skewed distribution
    b. Left-skewed or negatively-skewed distribution

We can find the shape of distribution using 5-number summary.

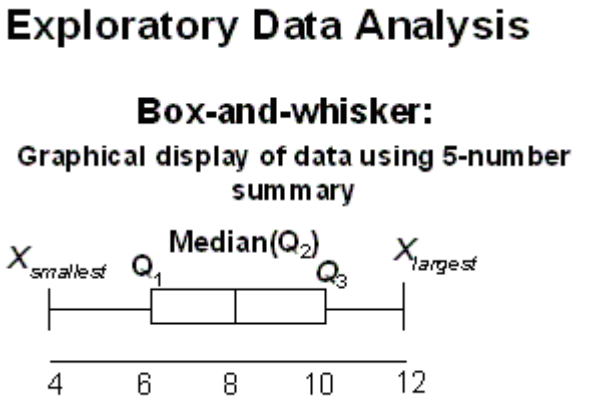
5-NUMBER SUMMARY:

5 number summary is:

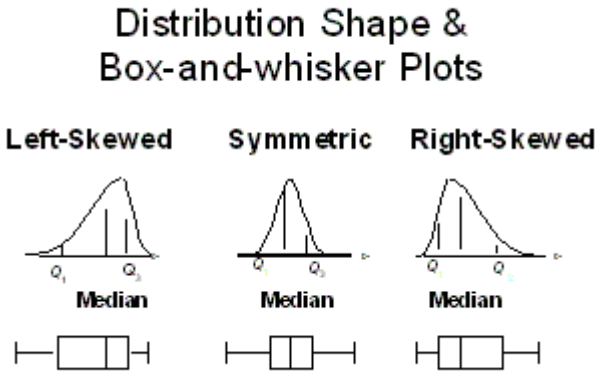
- Smallest value
- 1st Quartile (Q1)
- Median(Q2)
- 3rd Quartile (Q3)
- Largest value

BOX AND WHISKER PLOT:

Box and whisker plot shows 5-number summary.



The plot gives a good idea about the shape of the distribution as detailed below. Box and whisker plot for symmetrical, left-skewed and right-skewed distribution are shown below.



1. Symmetrical Distribution:

Data is perfectly symmetrical if:

- Distance from Q1 to Median = Distance from Median to Q3
- Distance from $X_{smallest}$ to Q1 = Distance from Q3 to $X_{largest}$

That is,

Median = Midhinge = Midrange

2. **Asymmetrical Distribution**:

a) Right-skewed distribution

Distance from $X_{largest}$ to Q3 greatly exceeds distance from Q1 to $X_{smallest}$

That is,

Median < Midhinge < Midrange

b) Left-skewed distribution

Distance from Q1 to $X_{smallest}$ greatly exceeds distance from $X_{largest}$ to Q3

That is,
Median > Midhinge > Midrange

Example

Suppose there are nine homes valued at Rs. 150,0000, Rs. 140,0000, Rs.160,0000, Rs.150,0000, Rs.160,0000, Rs.170,0000, Rs.160,0000, Rs.150,0000, and Rs160,0000 in a new area. There is one small empty lot in the area and someone builds small home with a valuation of Rs. 20,0000. Find either the frequency polygon of these values is negatively skewed or positively skewed.

Following sample represents annual costs (in '000 Rs) for attending 10 conferences.
13.0, 14.5, 14.9, 15.2, 15.2, 15.4, 15.6, 16.2, 17, 23.1
Find 5-number summary and shape of the distribution.

Position of Q1 = (10 + 1) / 4 = 2.75
Q1 = 14.5 + 0.75 × (14.9 – 14.5) = 14.8

Position of Q3 = 3 * (10 + 1) / 4 = 8.25
Q3 = 16.2 + 0.25 × (17 - 6.2) = 16.4

Median = (15.2 + 15.4) / 2 = 15.3

Thus 5-number summary is

- Smallest value = 13.0
- 1st Quartile (Q1) = 14.8
- Median(Q2) = 15.3
- 3rd Quartile (Q3) = 16.4
- Largest value = 23.1

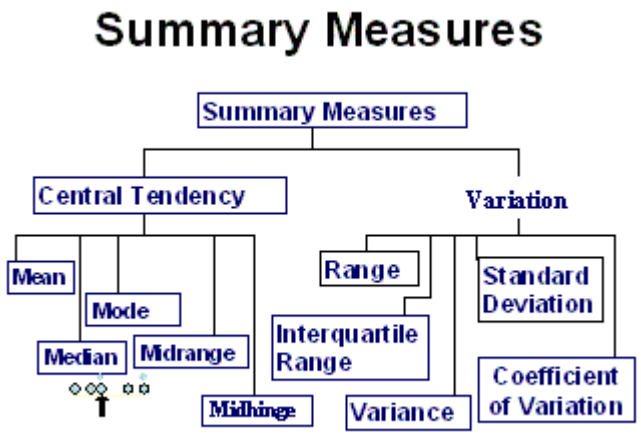
$$\text{Midrange} = \frac{\text{Largest value} + \text{Smallest value}}{2} = \frac{23.1 + 13.0}{2} = 18.05$$

$$\text{Midhinge} = \frac{Q1 + Q3}{2} = \frac{14.8 + 16.4}{2} = 15.6$$

Find the relationship between median, midhinge and midrange
Median < Midhinge < Midrange
Thus the shape of distribution is right skewed.

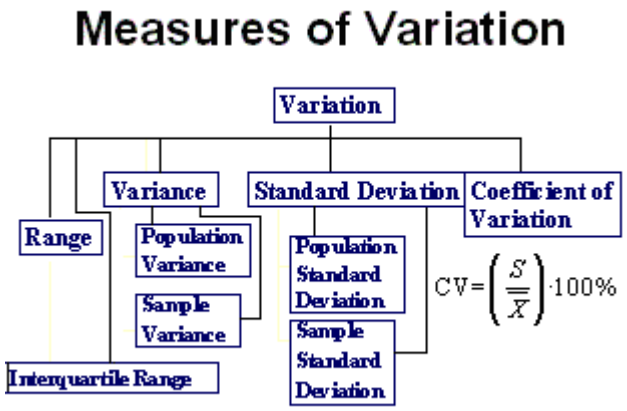
SUMMARY MEASURES

The slide shows summary of measures of central tendency and variation. In variation there are range, Interquartile range, standard deviation, variance, and coefficient of variation. The measures of central tendency have been discussed already



MEASURES OF VARIATION

In measures of variation, there are the sample and population standards deviation and variance the most important measures. The coefficient of variation is the ratio of standard deviation to the mean in %.



INTERQUARTILE RANGE

Interquartile range is the difference between the 1st and 3rd quartile.

Interquartile Range

Measure of Variation

Also Known as Midspread:

Spread in the Middle 50%

Difference Between Third & First Quartiles: Interquartile Range =

Data in Ordered Array: 11 12 13 16 16 17 17 18 21

$Q_3 - Q_1 = 17.5 - 12.5 = 5$

Not Affected by Extreme Values

LECTURE 28
MEASURES OF DISPERSION
CORRELATION
PART 1

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 27
- Measures of Dispersion
- Correlation

MODULE 6

Module 6 covers the following:
Correlation
(Lecture 28-29)
Line Fitting
(Lectures 30-31)
Time Series and Exponential Smoothing
(Lectures 32-33)

VARIANCE

Variance is the one of the most important measures of dispersion. Variance gives the average square of deviations from the mean. In the case of the population, the

Variance

Important Measure of Variation

Shows Variation About the Mean:

For the Population: $\sigma^2 = \frac{\sum (X_i - \mu)^2}{N}$

For the Sample: $s^2 = \frac{\sum (X_i - \bar{X})^2}{n-1}$

For the Population: use N in the denominator.	For the Sample : use n - 1 in the denominator.

Sum of square of deviations is divided by N the number of values in the population. In the case of variance for the sample the number of observations less 1 is used.

STANDARD DEVIATION

Standard deviation is the most important and widely used measure of dispersion. The square root of square of deviations divided by the number of values for the population and number of observations less 1 gives the standard deviation.

Standard Deviation

Most Important Measure of Variation

Shows Variation About the Mean

Same unit of measurement as the observations

For the Population: $\sigma = \sqrt{\frac{\sum (X - \mu)^2}{N}}$

For the Sample: $s = \sqrt{\frac{\sum (X_i - \bar{X})^2}{n-1}}$

For the Population: use N in the denominator.	For the Sample : use n - 1 in the denominator

Example

Let us do this for a simple data-set shown below:
The Number of Fatalities in Motorway Accidents in one Week:

Day	Number of fatalities X
Sunday	4
Monday	6
Tuesday	2
Wednesday	0
Thursday	3
Friday	5
Saturday	8
Total	28

The arithmetic mean number of fatalities per day is

$$\bar{X} = \frac{\sum X}{n} = \frac{28}{7} = 4$$

Taking the deviations of the X-values from their mean, and then squaring these deviations, we obtain:

X	$(x - \bar{x})$	$(x - \bar{x})^2$
4	0	0
6	+ 2	4
2	– 2	4
0	– 4	16
3	– 1	1
5	+ 1	1
8	+ 4	16
		42

Hence $\sum(x - \bar{x})^2 = 42$ is now positive, and this positive value has been achieved without 'bending' the rules of mathematics. Averaging these squared deviations, the variance is given by:

Variance:

$$= \frac{\sum (x-\bar{x})^2}{n-1}$$
$$= \frac{42}{6} = 7$$

The variance is frequently employed in statistical work, but it should be noted that the figure achieved is in 'squared' units of measurement.
In the example that we have just considered, the variance has come out to be "6 squared fatalities", which does not seem to make much sense!
In order to obtain an answer which is in the original unit of measurement, we take the positive square root of the variance. The result is known as the standard deviation.

Standard Deviation

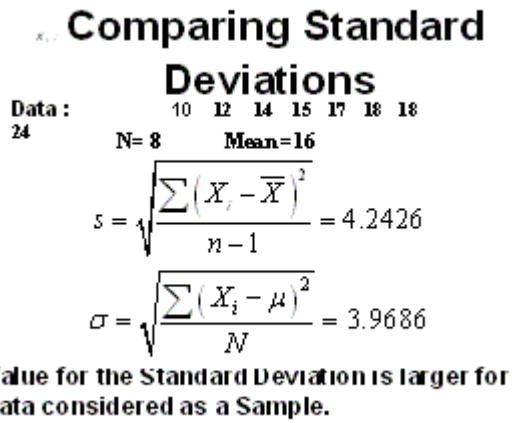
$$\text{S.D} = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{42}{6}} = 2.65 \text{ fatalities}$$

The formulae that we have just discussed are valid in case of raw data. In case of grouped data i.e. a frequency distribution, each squared deviation round the mean must be multiplied by the appropriate frequency figure.

COMPARING STANDARD DEVIATIONS

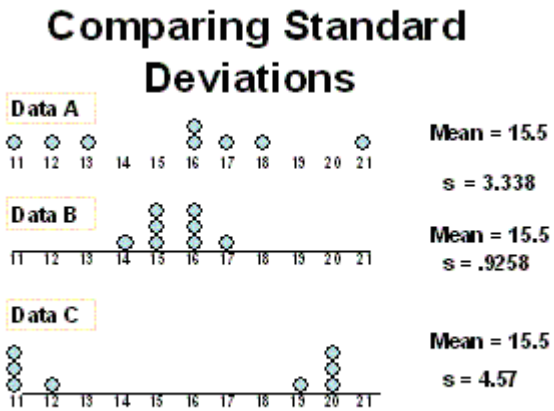
In many situations it becomes necessary to calculate population standard deviation (SD) on the basis of SD of the sample where n-1 is used for



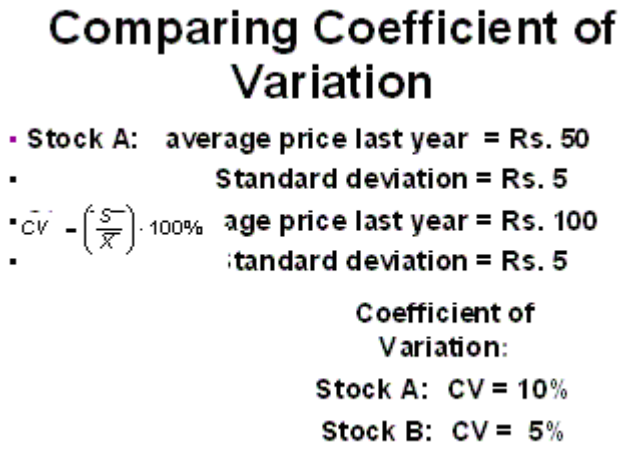
division. In the slide the same data is first treated as the sample and the value of SD is 4.2426. When we treat it as the population the SD is 3.9686, which is slightly less than the SD for the sample. You can see how the sample SD will be overestimated if used for the population.

COMPARING STANDARD DEVIATIONS

The slide shows three sets of data A, B and C. All the three datasets have the same mean 15.5 but different standard deviations (A: s=3.338; B: s=0.9258 and C: s=4.57). It is clear that SD is an important measure to understand how different sets of data differ from each other. Mean and SD together form a complete description of the central tendency of data.



COEFFICIENT OF VARIATION



Coefficient of variation (CV) shows the dispersion of the standard deviation about the mean. In the slide you see two stocks A and B with CV=10% and 5% respectively. This comparison shows that in the case of stock A there was a much greater variation in price with reference to the mean.

EXAMPLE-

Suppose that, in a particular year, the mean weekly earnings of skilled factory workers in one particular country were $ 19.50 with a standard deviation of $ 4, while for its neighboring country the figures were Rs. 75 and Rs. 28 respectively.

From these figures, it is not immediately apparent which country has the GREATER VARIABILITY in earnings.

The coefficient of variation quickly provides the answer:

COEFFICIENT OF VARIATION

For country No. 1:

$$\frac{4}{19.5} \times 100 = 20.5 \text{ per cent,}$$

And for country No. 2:

$$\frac{28}{75} \times 100 = 37.3 \text{ per cent.}$$

From these calculations, it is immediately obvious that the spread of earnings in country No. 2 is greater than that in country No. 1, and the reasons for this could then be sought.

MEAN DEVIATION

Other useful measures are Mean Deviation about the Mean and median.

The mean deviation of a set of data is defined as the arithmetic mean of the deviations measured either from the mean or from the median. The symbolic definition of the **mean deviation about the mean** is

$$M.D = \frac{1}{n}\sum_{i=1}^{n} |x - \bar{x}| \text{ , for sample}$$

.

$$M.D = \frac{1}{N}\sum_{i=1}^{N} | x - \mu |, \text{ For a population}$$

Note that first take the absolute value of difference of data point and the mean and then add those **absolute values**. . The absolute value of a number is the number without its sign.

Example:

Calculate the mean deviation from mean and median, of the following set of examination marks

45, 32, 37, 46, 39, 36, 41, 48, and 36.

Solution:

Median = 39

$$Mean = \frac{\sum x}{n}$$

Mean = 360/9 = 40 marks

	x_i	$x_i - \bar{x}$	$\left\|x_i - \bar{x}\right\|$	$\left\|x_i - Median\right\|$
	32	-8	8	7
	36	-4	4	3
	36	-4	4	3
	37	-3	3	2
	39	-1	1	0
	41	1	1	2
	45	5	5	6
	46	6	6	7
	48	8	8	9
Sum	360	0	40	39

Mean Deviation from Mean = $\frac{\sum \left|x_i - \bar{x}\right|}{n} = 40/9 = 4.4$ marks

Mean Deviation from Median = $\frac{\sum \left|x_i - Median\right|}{n} = 39/9 = 4.3$ marks

In excel ABS function returns the absolute value of a number.

Syntax

ABS(**number**)

Number is the real number of which you want the absolute value.

For the data organized into a grouped frequency distribution having **k** classes with midpoints x_1, x_2, x_3, ...,x_k and the corresponding frequencies f_1, f_2, f_3, ..., f_i ($\Sigma f_i = N$), the mean deviation about mean for grouped data is given by

$$M.D = \frac{\sum_{i=1}^{n} f_i \mid x_i - \bar{x} \mid}{\sum f_i}$$

Similarly we can define mean deviation about median, just by replacing the mean value in formula with the median.

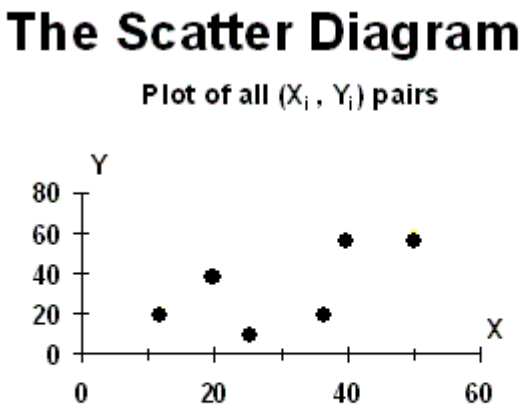
REGRESSION ANALYSIS

The primary objective of regression analysis is the development of a regression model to explain the association between two or more variables in the given population.
A regression model is the mathematical equation that provides prediction of value of dependent variable based on the known values of one or more independent variables.

In regression Analysis, we shall encounter different types of regression models. One of the main functions of regression analysis is determining the simple linear regression equation. What are the different Measures of variation in regression and correlation? What are the Assumptions of regression and correlation? What is Residual analysis? How do we make
Inferences about the slope? How can you estimate predicted values? What are the Pitfalls in regression? What are the ethical issues?
An important point in regression analysis is the purpose of the analysis.

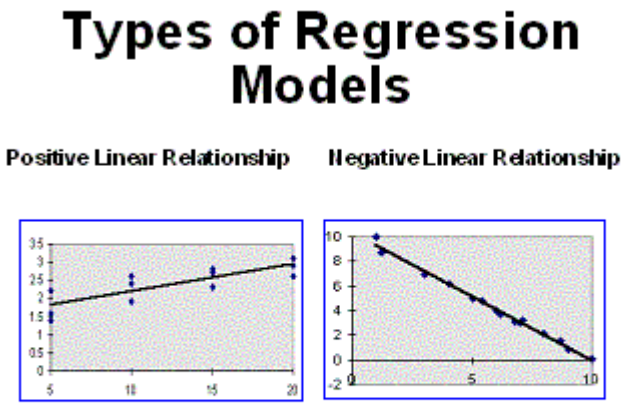
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SCATTER DIAGRAM
The first step in regression analysis is to plot the values of the dependent and independent variable in the form of a scatter diagram as shown below. The form of the scatter of the points indicates whether there is any degree of association between them. In the scatter diagram below you can see that there seems to be a fairly distinct correlation between the two variables. It appears as if the points were located around a straight line.
.



Types of Regression Models

There are two types of linear models as shown in the slide below. These are positive and negative linear relationships. In the positive relationship, the value of the dependent variable increases as the value of the independent variable increases. In the case of negative linear relationship, the value of the dependent variable decreases with increase in the value of independent variable.



LECTURE 29
MEASURES OF DISPERSION
CORRELATION
PART 2

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 28
- Correlation

CORRELATION

Correlation is a measure of the strength or the degree of relationship between two RANDOM variables.

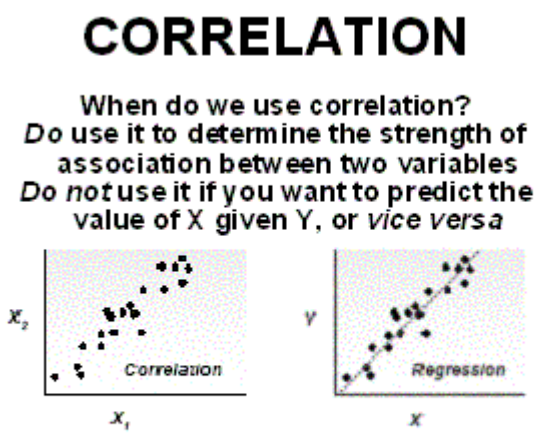
When do we use correlation?

It will be used when we wish to establish whether there is a degree of association between two variables. If this association is established, then it makes sense to proceed further with regression analysis. Regression analysis determines the constants of the regression. You can not make any predictions with results of correlation analysis. Predictions are based on regression equations.

CORRELATION ANALYSIS

To analyze the strength of the relationship or co-variation between two variables, we use correlation analysis.

Correlation analysis contributes to the understanding of economics behavior, aids in locating the critically important variables on which others depends, may reveal to the economist the connections by which disturbances spread and suggest to him the path through which stabilizing forces may become effective



SIMPLE LINEAR CORRELATION VERSUS SIMPLE LINEAR REGRESSION

The calculations for linear correlation analysis and regression analysis are the same.

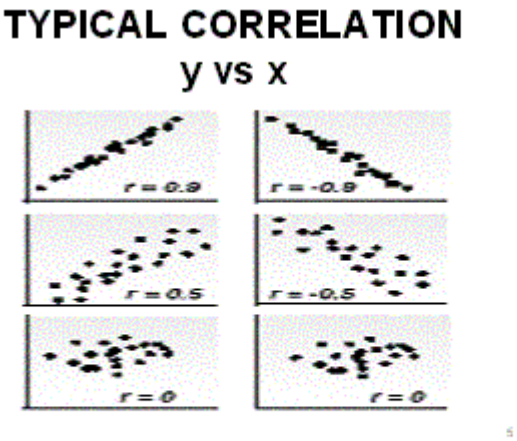
In correlation analysis, one must sample randomly both X and Y.

Correlation deals with the association (importance) between variables whereas Regression deals with prediction (intensity).

The slide shows three types of correlation for both positive and negative linear relationships. In the first figure (r = .0.9), the data points are practically in a straight line. This kind of association or correlation is near perfect. This applies to negative correlation also.

The graphs where r = 0.5, the points are more scattered, there is a clear association but this association is not very pronounced.

In graphs where r = 0, there is no association between variables.



CORRELATION COEFFICIENT

For calculation of correlation coefficient:

1. A standardised transform of the covariance (sxy) is calculated by dividing it by the product of the standard deviations of X (sx) & Y (sy).
2. It is called the population correlation coefficient is defined as:

$$r = s_{XY}/s_X s_Y$$

or

$$r = \frac{Cov(X,Y)}{\sqrt{Var(X)\,Var(Y)}}$$

Where, covariance of X and Y is defined as

$$Cov(X,Y) = \frac{\sum (X-\bar{X})(Y-\bar{Y})}{n}$$

This formula is a bit cumbersome to apply. Therefore, we may use the following short cut formula:

Short Cut Formula

$$r = \frac{\sum XY - \left(\sum X\right)\left(\sum Y\right)/n}{\sqrt{\left[\sum X^2 - \left(\sum X\right)^2/n\right]\left[\sum Y^2 - \left(\sum Y\right)^2/n\right]}}$$

It should be noted that

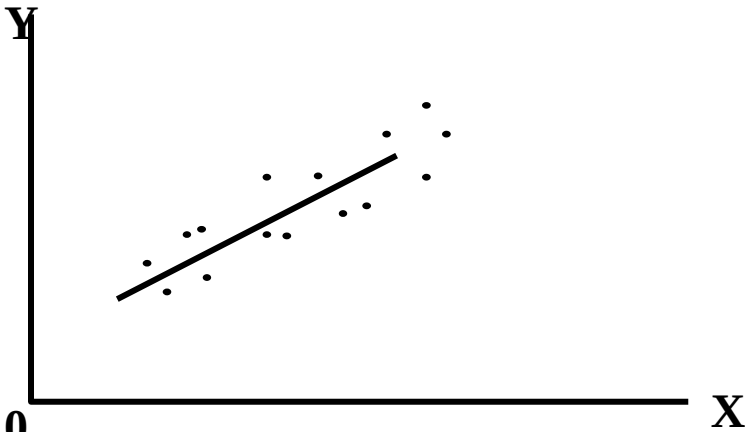
r is a pure number that lies between –1 and 1 i.e.

$-1 < r < 1$

Actually, the mathematical expression that you have just seen is a combination of three different mathematical expressions:

Case 1:
Positive correlation: $0 < r < 1$

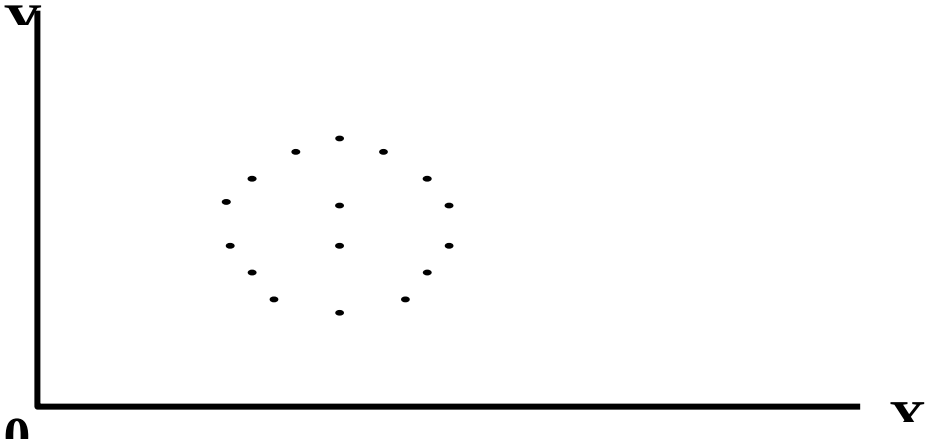
In case of a positive linear relationship, r lies between 0 and 1.



In this case, the closer the points are to the UPWARD-going line, the STRONGER is the positive linear relationship, and the closer r is to 1.

Case 2:
No correlation: $r = 0$

The extreme of dissociation (zero correlation (r = 0)):



In such a situation, X and Y are said to be *uncorrelated*.

Case 3:
Negative correlation: $-1 < r < 0$

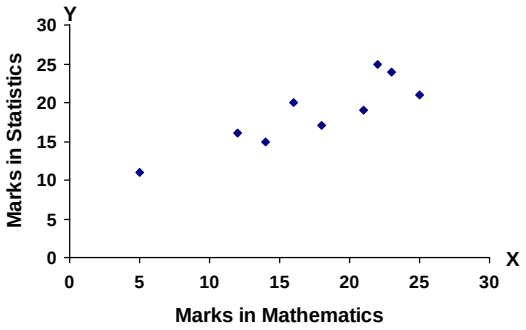
Warning

- Existence of a high correlation does not mean there is causation, which means that there may be a correlation but it does not make things happen because of that.
- There can exist spurious correlations. And correlations can arise because of the action of a third unmeasured or unknown variable. In many situations correlation can be high without any solid foundation.

EXAMPLE
Suppose that the principal of a college wants to know if there exists any correlation between grades in Mathematics and grades in Statistics. Suppose that he selects a random sample of 9 students out of all those who take this combination of subjects. The following information is obtained:

Student	Marks in Mathematics (Total Marks: 25)	Marks in Statistics (Total Marks: 25)
	X	Y
A	5	11
B	12	16
C	14	15
D	16	20
E	18	17
F	21	19
G	22	25
H	23	24
I	25	21

SCATTER DIAGRAM



In order to compute the correlation coefficient, we carry out the following computations:

X	Y	X^2	Y^2	XY
5	11	25	121	55
12	16	144	256	192
14	15	196	225	210
16	20	256	400	320
18	17	324	289	306
21	19	441	361	399
22	25	484	625	550
23	24	529	576	552
25	21	625	441	525
156	168	3024	3294	3109

$$r = \frac{\sum XY - (\sum X)(\sum Y)/n}{\sqrt{\left[\sum X^2 - (\sum X)^2/n\right]\left[\sum Y^2 - (\sum Y)^2/n\right]}}$$

$$= \frac{3109 - (156)(168)/9}{\sqrt{\left[3024 - (156)^2/9\right]\left[3294 - (168)^2/9\right]}}$$

$$= \frac{3109 - 2912}{\sqrt{[3024 - 2704][3294 - 3136]}}$$

$$= \frac{197}{\sqrt{\quad}} = \frac{197}{\quad} = 0.88$$

There exists a strong *positive* linear correlation between marks in Mathematics and marks in Statistics for these 9 students who have been taken into consideration.

EXCEL Tools

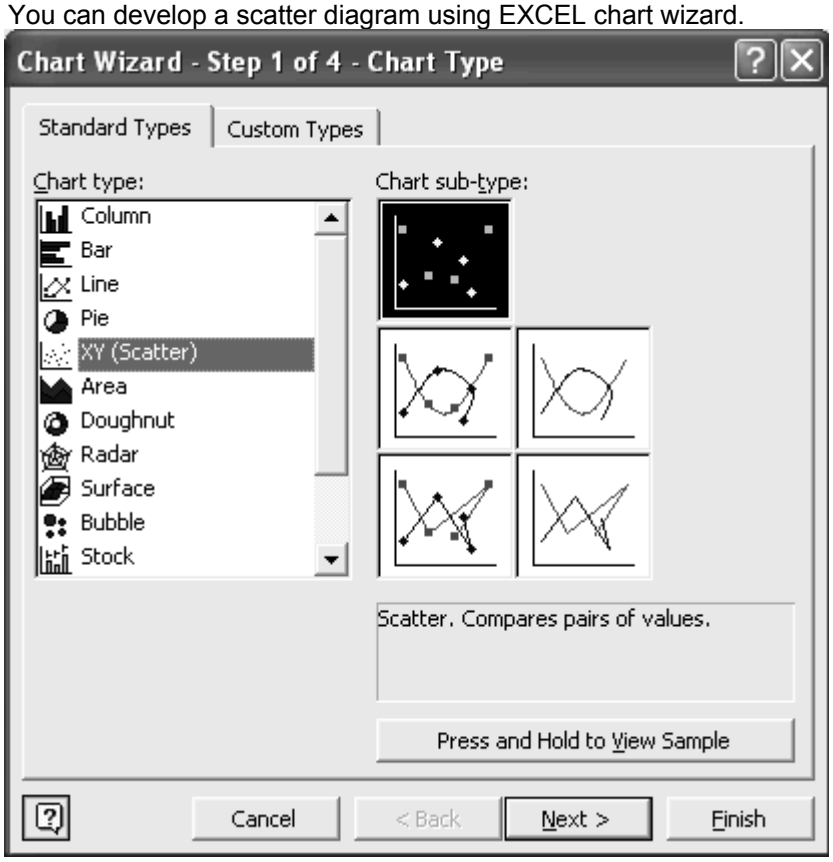
- For summary of sample statistics, use:
  Tools / Data Analysis / Descriptive Statistics
- For individual sample statistics, use:
  Insert / Function / Statistical
  and **select the function** you need

EXCEL Functions

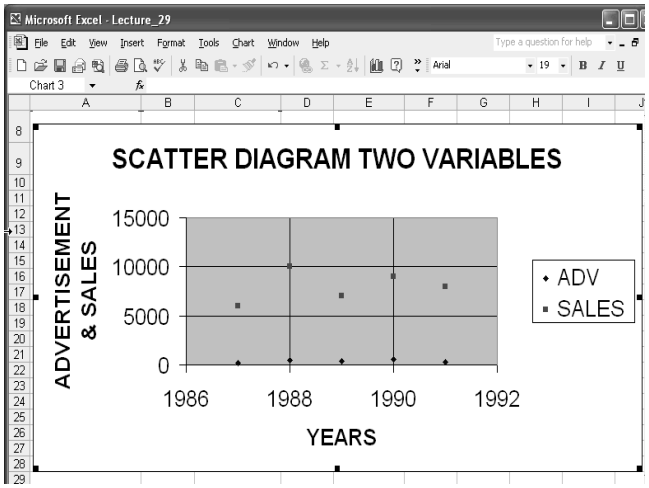
- In **EXCEL**, use the **CORREL** function to calculate correlations
- The correlation coefficient is also given on the output from **TOOLS, DATA ANALYSIS, CORRELATION or REGRESSION**

Scatter Diagram Two Variables

You can develop a scatter diagram using EXCEL chart wizard.

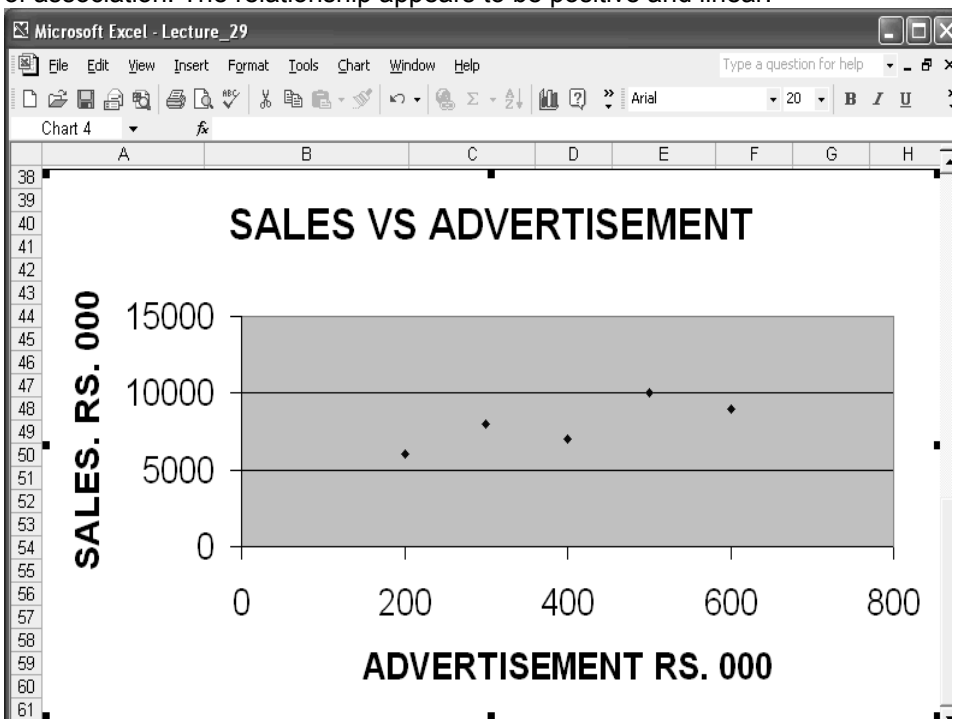


The slide shows a scatter diagram of Advertisement and Sales over the years. The graph was made using EXCEL chart Wizard. As you can see one cannot draw any conclusions about the degree of association between advertising from this graph.



SALES VERSUS ADVERTISEMENT

The scatter diagram for sale versus advertisement shows a fairly high degree of association. The relationship appears to be positive and linear.



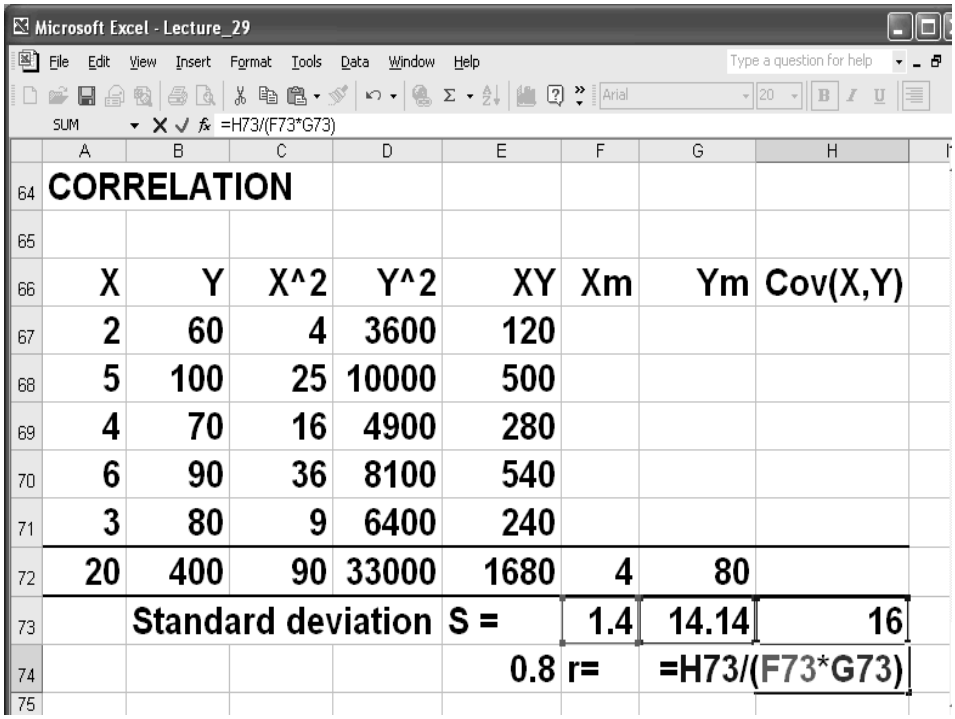
CORRELATION COEFFICIENT USING EXCEL

Correlation Coefficient for correlation between two steams of data was calculated using the formula Cov(x,y)/Sx.Sy as given above.

The data for variable x was entered in cells A67 to A71. Data for variable y was entered in cells B67 to B71.Calculations for square of x, square of y, product of x and y, Xm, Ym and cov(x,y) were made in columns C, D, E, F and G respectively. Other calculations were made as follows:

Cell A72: Sum of x (=SUM(A67:A71)
Cell B72: Sum of y (=SUM(B67:B71)
Cell C72: Sum of square of x (=SUM(C67:C71)
Cell D72: Sum of square of y (=SUM(D67:D71)
Cell E72: Sum of product of x and y (=SUM(E67:E71)
Cell F72: Mean of x (=A72/5), where 5 is the number of observations
Cell G72: Mean of y (=B72/5), where 5 is the number of observations
Cell F73: Sx (=SQRT(C72/5-F72*F72))
Cell G73: Sy =(SQRT(D72/5-G72*G72))

Cell H73: Cov(x,y) (=E72/5-F72*G72)
Cell H74: Correlation coefficient (=H73/(F73*G73))
The above formulas are in line with formulas described earlier.



	A	B	C	D	E	F	G	H
64	CORRELATION							
65								
66	X	Y	X^2	Y^2	XY	Xm	Ym	Cov(X,Y)
67	2	60	4	3600	120			
68	5	100	25	10000	500			
69	4	70	16	4900	280			
70	6	90	36	8100	540			
71	3	80	9	6400	240			
72	20	400	90	33000	1680	4	80	
73		Standard deviation			S =	1.4	14.14	16
74					0.8	r=		=H73/(F73*G73)
75								

- **CORREL**

Returns the correlation coefficient of the array1 and array2 cell ranges. Use the correlation coefficient to determine the relationship between two properties. For example, you can examine the relationship between a location's average temperature and the use of air conditioners.

Syntax

CORREL(**array1**,**array2**)

Array1 is a cell range of values.

Array2 is a second cell range of values.

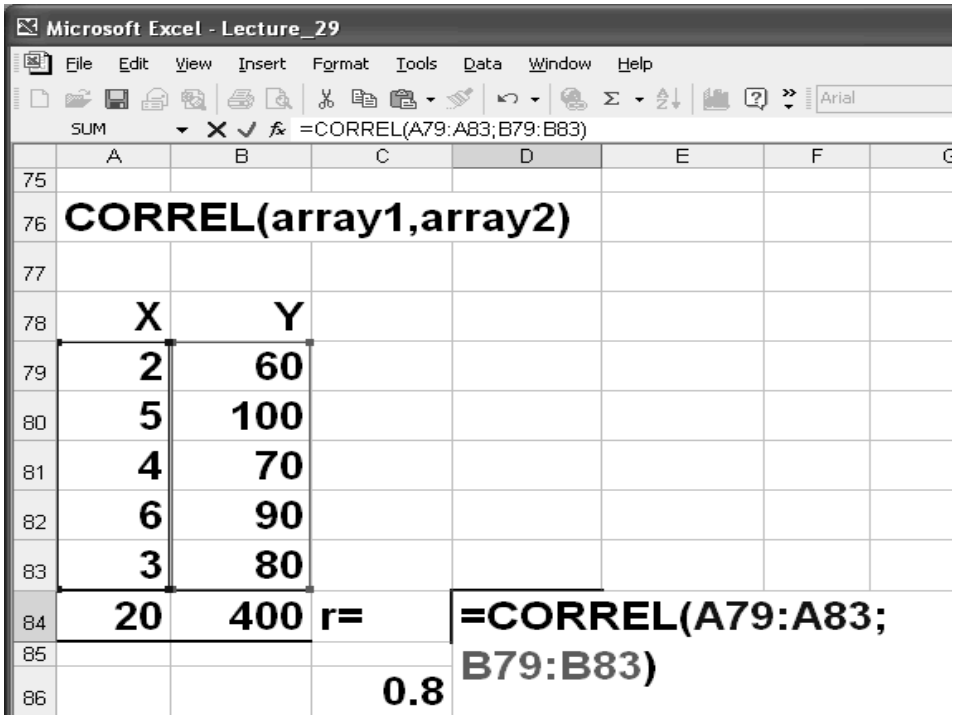
Remarks

- The arguments must be numbers, or they must be names, arrays, or references that contain numbers.
- If an array or reference argument contains text, logical values, or empty cells, those values are ignored; however, cells with the value zero are included.
- If array1 and array2 have a different number of data points, CORREL returns the #N/A error value.

If either array1 or array2 is empty, or if s (the standard deviation) of their values equals zero, CORREL returns the #DIV/0! error value.

EXCEL Calculation

The X and Y arrays are in cells A79 to A83 and B79 to B83 respectively. The formula for correlation coefficient was entered in cell D84 as =CORRE(A79:A83;B79:B83). The value or r (0.8) is shown in cell C86.


Microsoft Excel - Lecture_29
File Edit View Insert Format Tools Data Window Help
SUM
=CORREL(A79:A83;B79:B83)
CORREL(array1,array2)
X
Y
2
60
5
100
4
70
6
90
3
80
20
400
r=
0.8
=CORREL(A79:A83;
B79:B83)

LECTURE 30
Measures of Dispersion
LINE FITTING
PART 1

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 29
- Line Fitting

EXCEL SUMMARY OF SAMPLE STATISTICS

For summary of sample statistics, use:
Tools > Data Analysis > Descriptive Statistics
For individual sample statistics, use:
Insert > Function > Statistical
and select the function you need

EXCEL STATISTICAL ANALYSIS TOOL

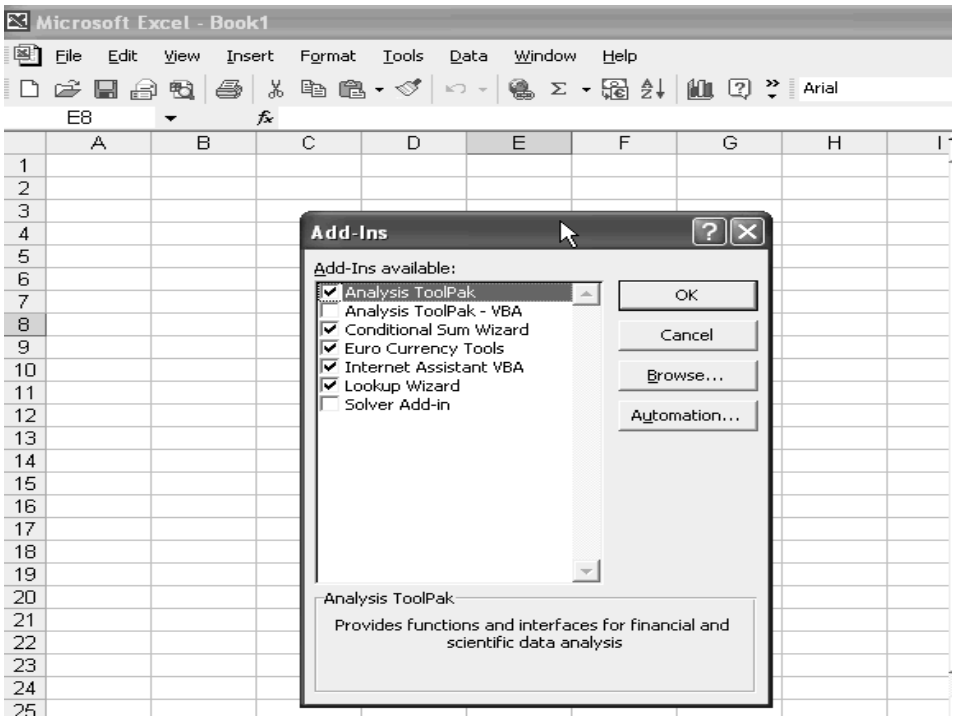
You can use **EXCEL** to perform a statistical analysis:

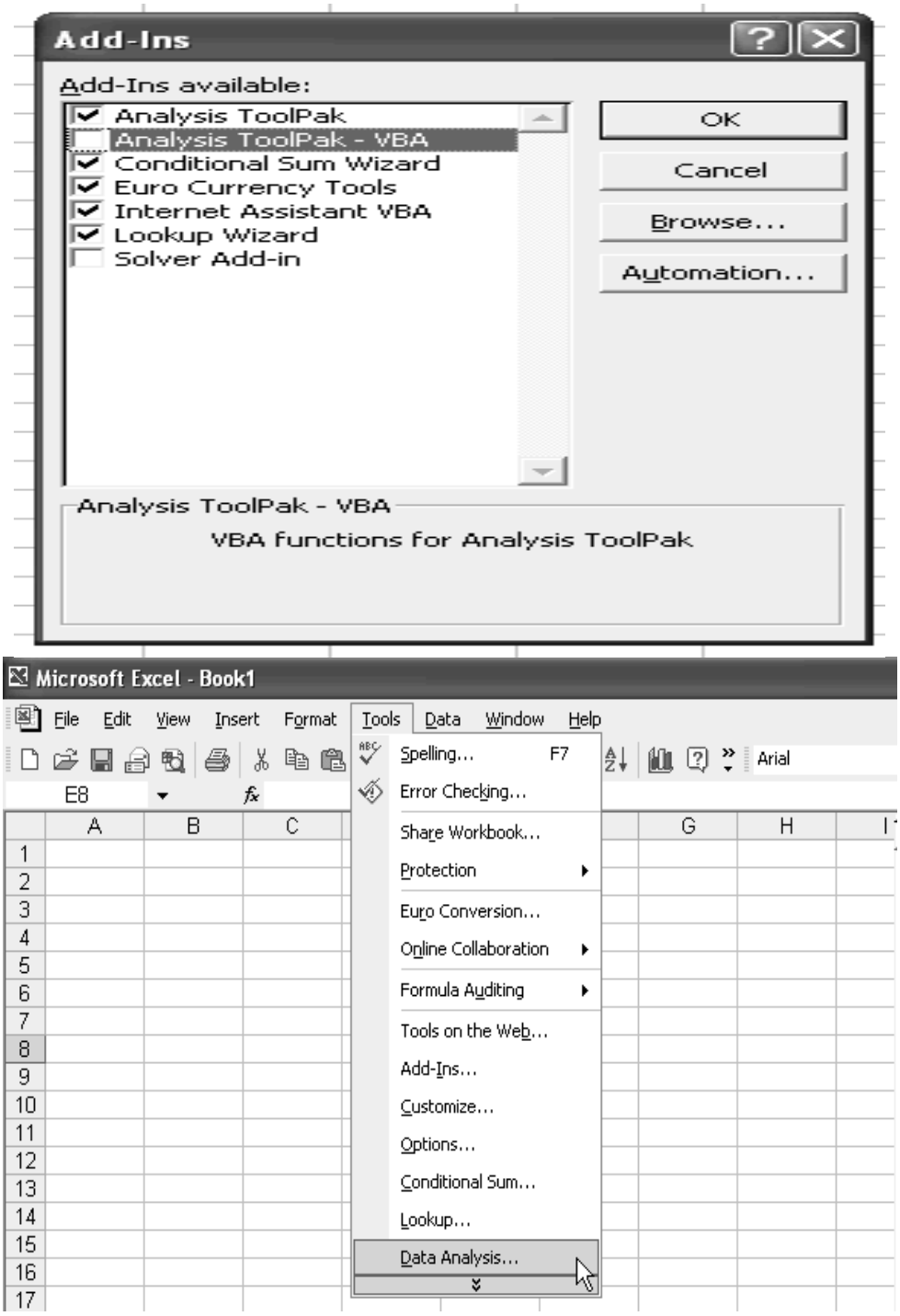
- On the **Tools** menu, click Data **Analysis**. If **Data Analysis** is not available, load the **Analysis** ToolPak.
- In the **Data Analysis** dialog box, click the **name of the analysis tool** you want to use, and then click OK.
- In the **dialog box** for the **tool** you selected, set the **analysis options you want.**
- You can use the Help button on the dialog box to get more information about the options.

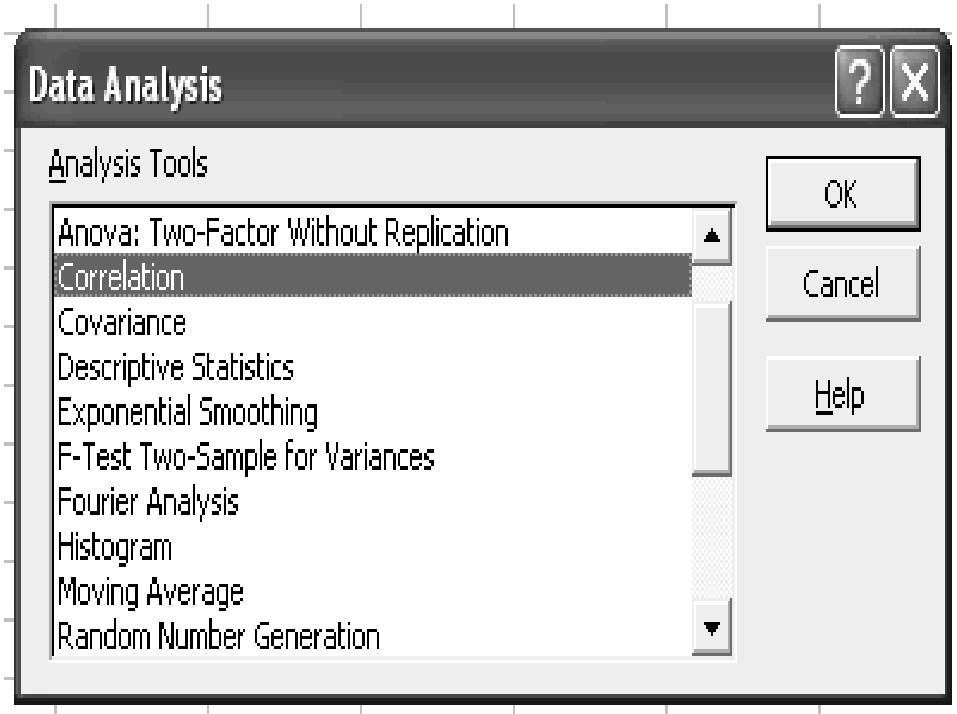
LOAD THE ANALYSIS TOOLPAK

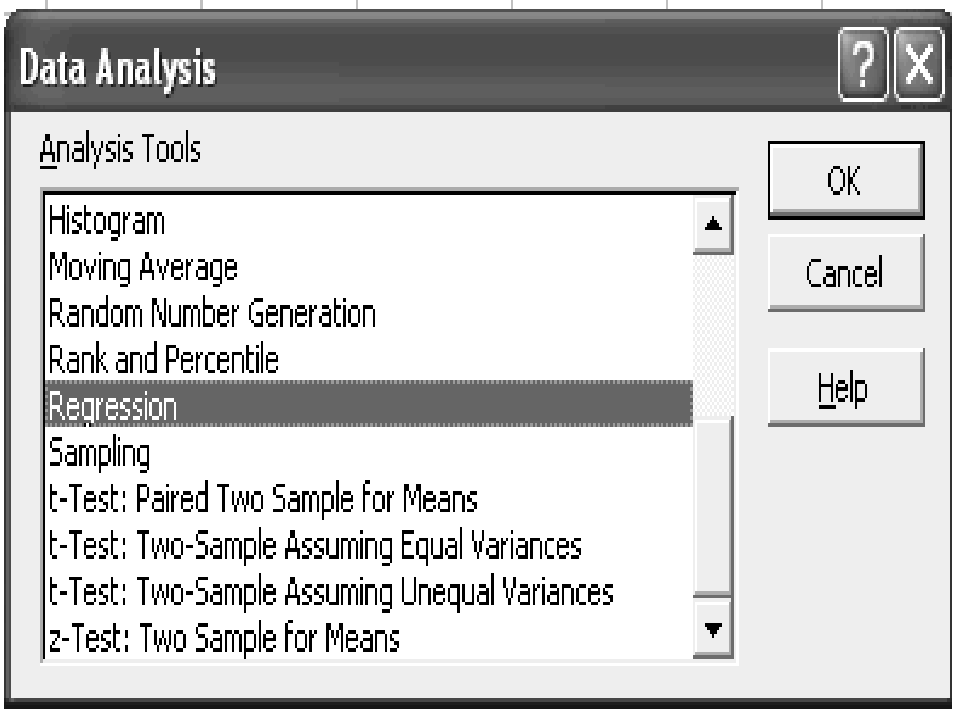
You can load the EXCEL Analysis ToolPak **as follows:**

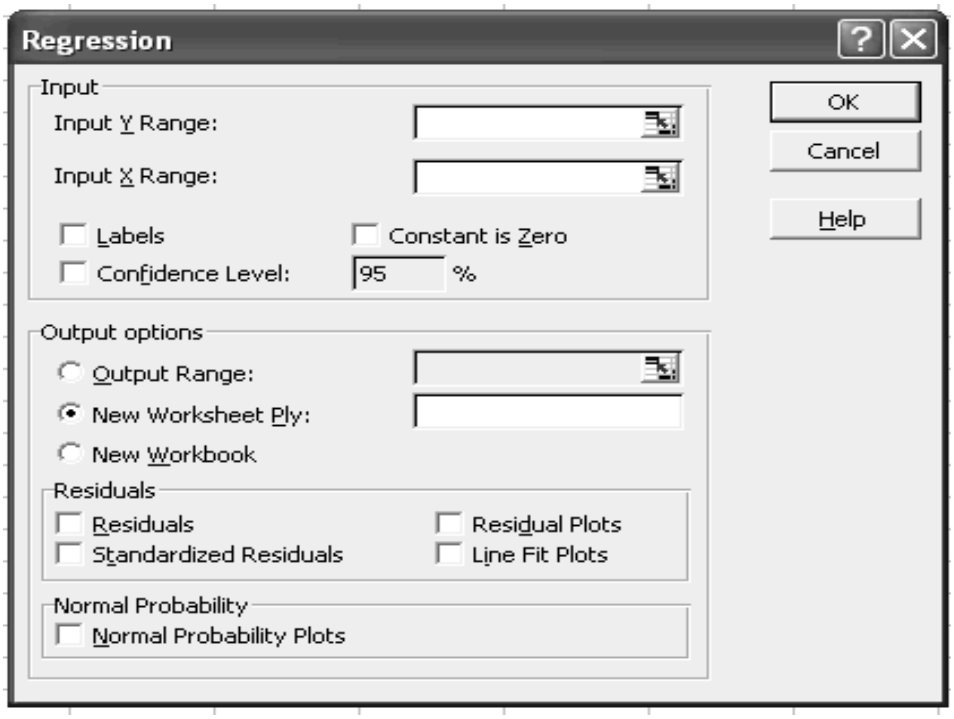
On the Tools **menu, click** Add-Ins**.**
In the **Add-Ins** available **list**, select the **Analysis ToolPak** box, and then click **OK**.
If necessary, follow the instructions in the setup program




Add-Ins
Add-Ins available:
Analysis ToolPak
Analysis ToolPak - VBA
Conditional Sum Wizard
Euro Currency Tools
Internet Assistant VBA
Lookup Wizard
Solver Add-in
OK
Cancel
Browse...
Automation...
Analysis ToolPak - VBA
VBA functions for Analysis ToolPak
Microsoft Excel - Book1
File Edit View Insert Format Tools Data Window Help
Arial
E8
Spelling... F7
Error Checking...
Share Workbook...
Protection
Euro Conversion...
Online Collaboration
Formula Auditing
Tools on the Web...
Add-Ins...
Customize...
Options...
Conditional Sum...
Lookup...
Data Analysis...


Data Analysis
Analysis Tools
Anova: Two-Factor Without Replication
Correlation
Covariance
Descriptive Statistics
Exponential Smoothing
F-Test Two-Sample for Variances
Fourier Analysis
Histogram
Moving Average
Random Number Generation
OK
Cancel
Help


Data Analysis
Analysis Tools
Histogram
Moving Average
Random Number Generation
Rank and Percentile
Regression
Sampling
t-Test: Paired Two Sample for Means
t-Test: Two-Sample Assuming Equal Variances
t-Test: Two-Sample Assuming Unequal Variances
z-Test: Two Sample for Means
OK
Cancel
Help



SLOPE

Returns the slope of the linear regression line through data points in known_y's and known_x's. The slope is the vertical distance divided by the horisontal distance between any two points on the line, which is the rate of change along the regression line.

Syntax

SLOPE(**known_y's**,**known_x's**)

Known_y's is an array or cell range of numeric dependent data points.

Known_x's is the set of independent data points.

Remarks

The arguments must be numbers or names, arrays, or references that contain numbers.

If an array or reference argument contains text, logical values, or empty cells, those values are ignored; however, cells with the value zero are included.

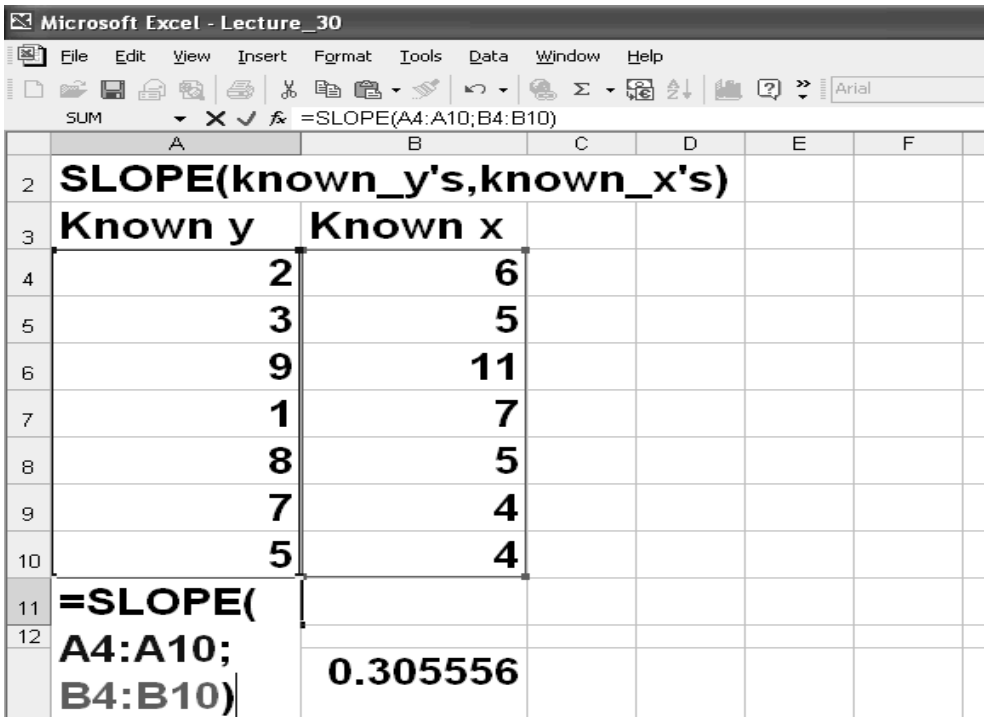
If known_y's and known_x's are empty or have a different number of data points, SLOPE returns the #N/A error value.

The equation for the slope of the regression line is:

$$b = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{n\sum x^2 - \left(\sum x\right)^2}$$

Example

The known y-values and x-values were entered in cells A4 to A10 and B4 to B10 respectively. The formula =SLOPE(A4:A10;B4:B10) was entered in cell A11. The result 0.305556 is the value of slope in cell B12.



INTERCEPT

Calculates the point at which a line will intersect the y-axis by using existing x-values and y-values. The intercept point is based on a best-fit regression line plotted through the known x-values and known y-values. Use the INTERCEPT function when you want to determine the value of the dependent variable when the independent variable is 0 (zero). For example, you can use the INTERCEPT function to predict a metal's electrical resistance at 0°C when your data points were taken at room temperature and higher.

Syntax

INTERCEPT(**known_y's**,**known_x's**)

Known_y's is the dependent set of observations or data.

Known_x's is the independent set of observations or data.

Remarks

The arguments should be either numbers or names, arrays, or references that contain numbers.

If an array or reference argument contains text, logical values, or empty cells, those values are ignored; however, cells with the value zero are included.

If known_y's and known_x's contain a different number of data points or contain no data points, INTERCEPT returns the #N/A error value.

The equation for the intercept of the regression line is:

$$a = \overline{Y} - b\overline{X}$$

where the slope is calculated as:

$$b = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n\Sigma x^2 - (\Sigma x)^2}$$

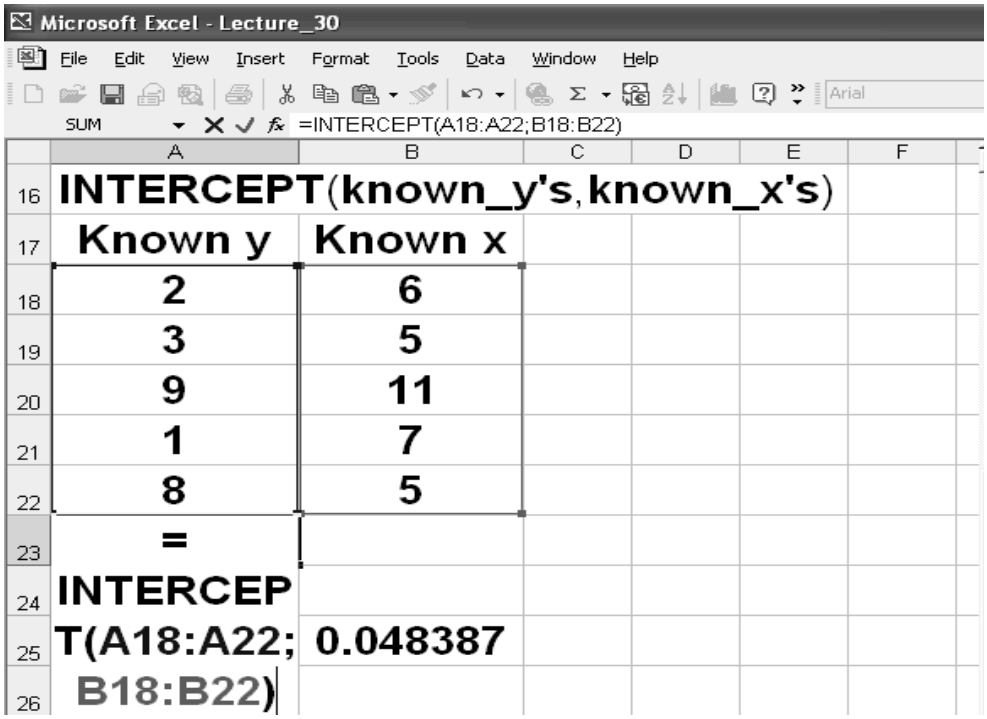
Example

The data for y-values was entered in cells A18 to A22.

The data for x-values was entered in cells B18 to B22.

The formula =INTERCEPT(A18:A22;B18:B22) was entered in cell A24.

The answer 0.048387 is shown in cell B25.


Microsoft Excel - Lecture_30
File Edit View Insert Format Tools Data Window Help
SUM
=INTERCEPT(A18:A22;B18:B22)
INTERCEPT(known_y's,known_x's)
Known y
Known x
2
6
3
5
9
11
1
7
8
5
=
INTERCEPT(A18:A22;
B18:B22)
0.048387

LECTURE 31
LINE FITTING
PART 2

OBJECTIVES

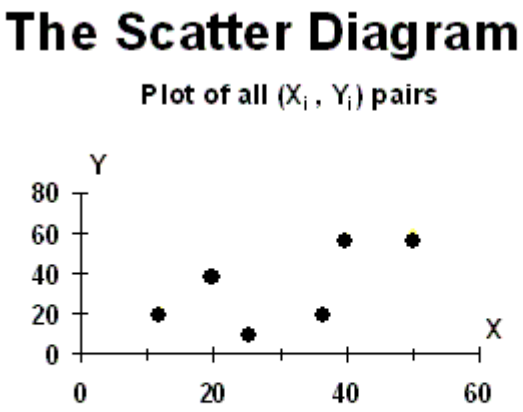
The objectives of the lecture are to learn about:

- Review Lecture 30
- Line Fitting

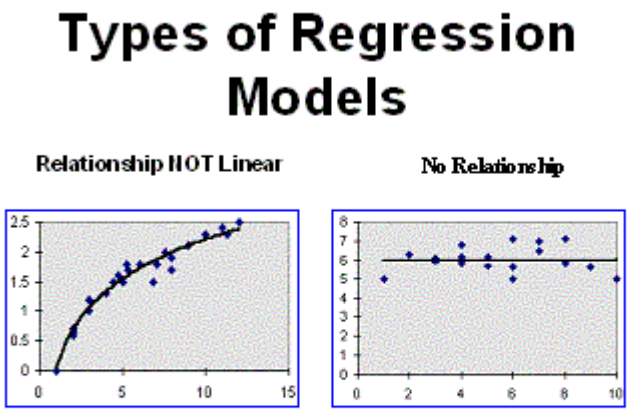
Types of Regression Models

There are different types of regression models. The simplest is the Simple Linear Regression Model or a relationship between variables that can be represented by a straight line equation.

To determine whether a linear relationship exists, a Scatter Diagram is developed first.



In linear regression two types of models are considered. The first one is the Population Linear Regression that represents the linear relationship between the variables of the entire population (i.e. all the data). It is quite customary to carry out sample surveys and determine linear relationship between two variables on the basis of sample data. Such regression analysis is called Sample Linear Regression.



Relationship between Variables is described by a Linear Function. The change of one variable causes the other variable to change. The relationship describes the dependency of one variable on the other.

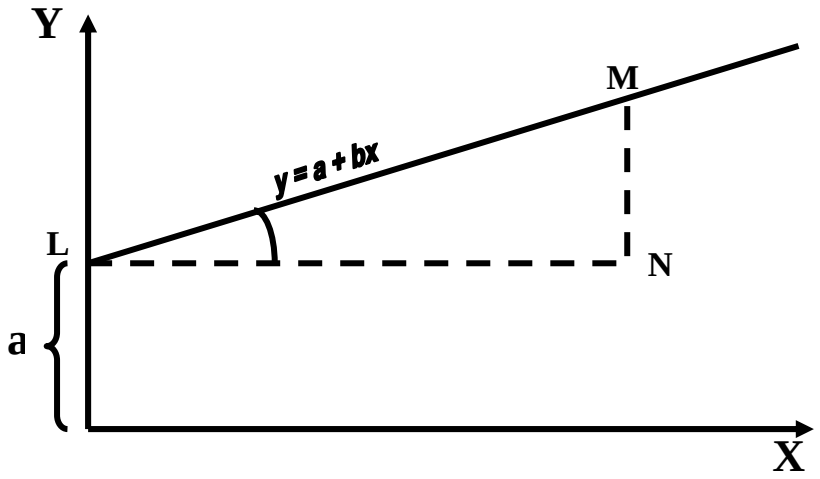
If the relationship between the variables is exactly linear, then the mathematical equation describing the liner relation is written as

$Y = a + bX.$

Where
Y represents the dependent variable
X represents the independent variable
a represents the Y-intercept
(i.e. the value of Y when X is equal to zero)
b represents the slope of the line
(i.e. the value of the tan θ, where θ represents the angle between the line and the horizontal axis)

Interpretation of 'a' and 'b':

$$b = \tan\theta = \frac{MN}{NL}$$



A very important point to note is that MANY lines can be drawn through the same scatter diagram.

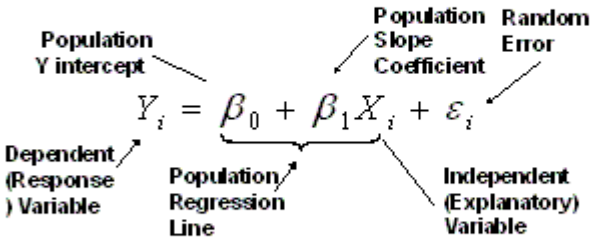
In contrast to the above, the liner relationship in some situations is not exact. For this we add an unknown random error variable as

$Y_i = a + bX_i + e_i$

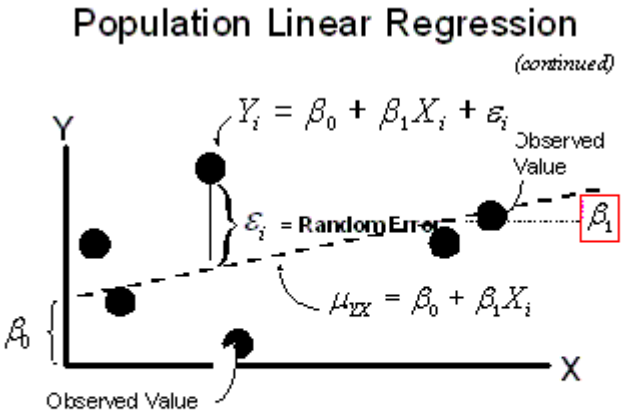
We assume that the liner relationship between the dependent variable Y_i and the value X_i of the regressor X is.

Population Linear Regression

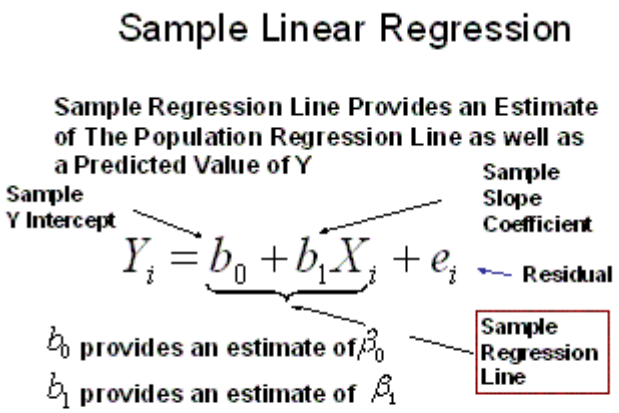
- Population Regression Line Is A Straight Line that Describes The Dependence of The Average Value of One Variable on The Other



The slide below shows the graphical representation of the population regression equation. It may be seen that the distance of the points from the regression line (obtained by inserting values of X in the equation) is the random error. The intercept is shown on the Y-axis.



The slide below shows the regression equation for the sample. Note that the intercept in this case has a notation B_0. The slope is B_1. The random error is e1. Different notations are used to distinguish between population regression and sample regression.



REGRESSION EQUATION EXAMPLE

Computer the least square regression equation of Y on X for the following data.

X	5	6	8	10	12	13	15	16	17
Y	16	19	23	28	36	41	44	45	50

Solution:

The estimated regression line of Y on X is

$$\hat{Y} = a + bX$$

In the previous lecture we learn that the equation for the slope of the regression line is:

$$b = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{n\sum x^2 - \left(\sum x\right)^2}$$

And the equation for the intercept of the regression line is:

$$a = \overline{Y} - b\overline{X}$$

Now from the given data $\overline{X} = \frac{\sum X}{n} = 102/9 = 11.33$

$$\overline{Y} = \frac{\sum Y}{n} = 302/9 = 33.56$$

$$b = \frac{n\sum XY - (\sum X)(\sum Y)}{n\sum X^2 - (\sum X)^2} = 2.831$$

$$a = \overline{Y} - b\overline{X} = 33.56\text{-}(2.831)(11.83) = 1.47$$

Hence the desired estimated regression line is

$$\hat{Y} = 1.47 + 2.831X$$

REGRESSION EXAMPLE

Regression Analysis can be carried out easily using EXCEL Regression Tool. Let us see how it can be done. We chose to carry out regression on data given in the slide below. Y-values are 60, 100, 70, 90 and 80. X-values are 2, 5, 4, 6

and 3.

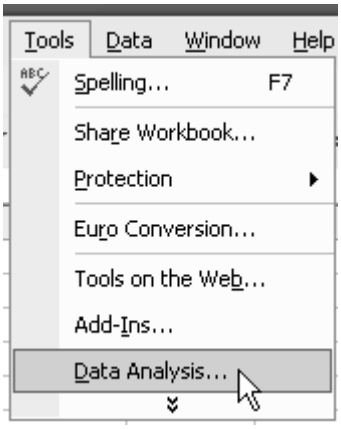
Microsoft Excel - Lecture_31

File Edit View Insert Format Tools Data Window Help

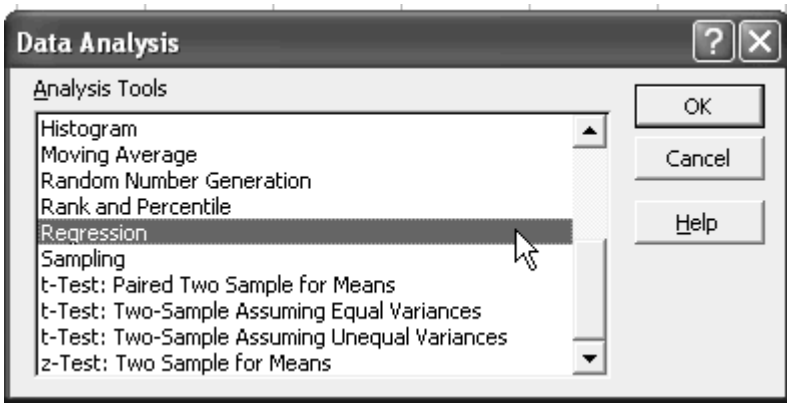
D51

	A	B	C
37	REGRESSION ANALYSIS		
38	EXAMPLE 1		
39			
40	Y	X	
41	60	2	
42	100	5	
43	70	4	
44	90	6	
45	80	3	
46			

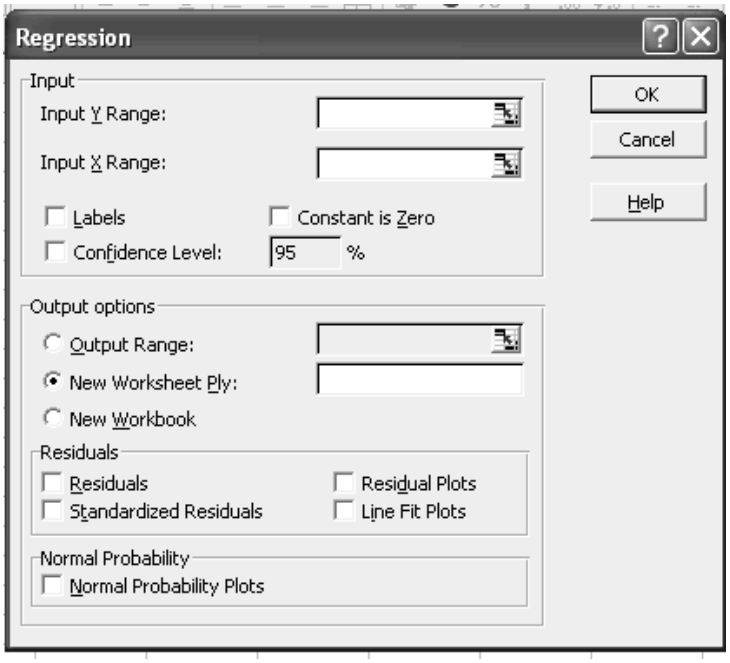
We start the regression analysis by going to the Tools menu and selecting the Data Analysis menu as shown below.



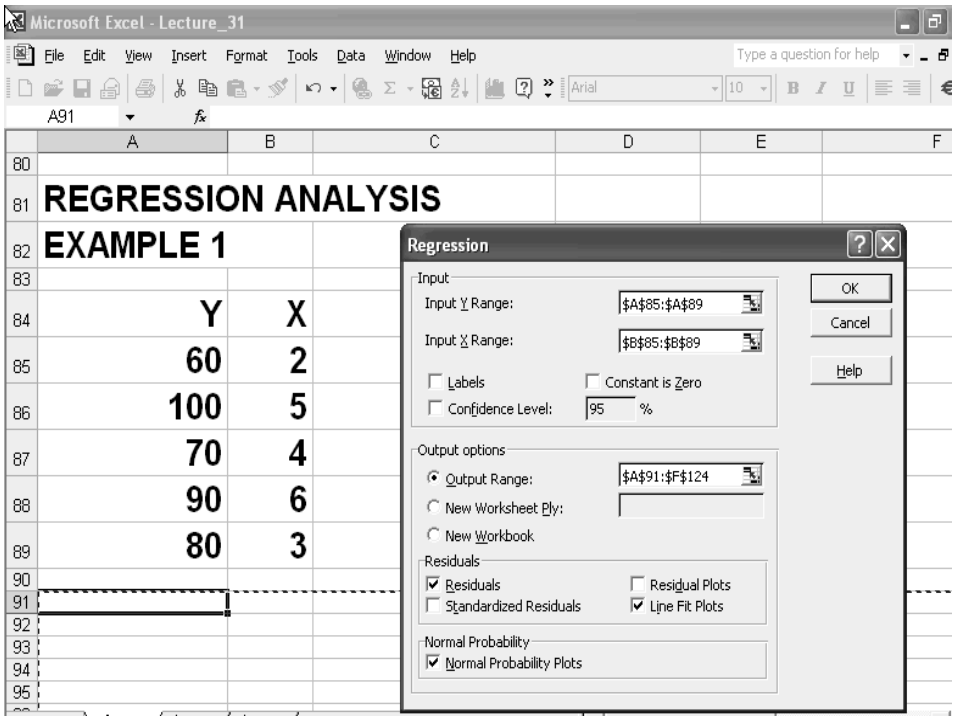
The Regression dialog box opens as shown in the following slide. You click the Regression analysis tool and then OK.



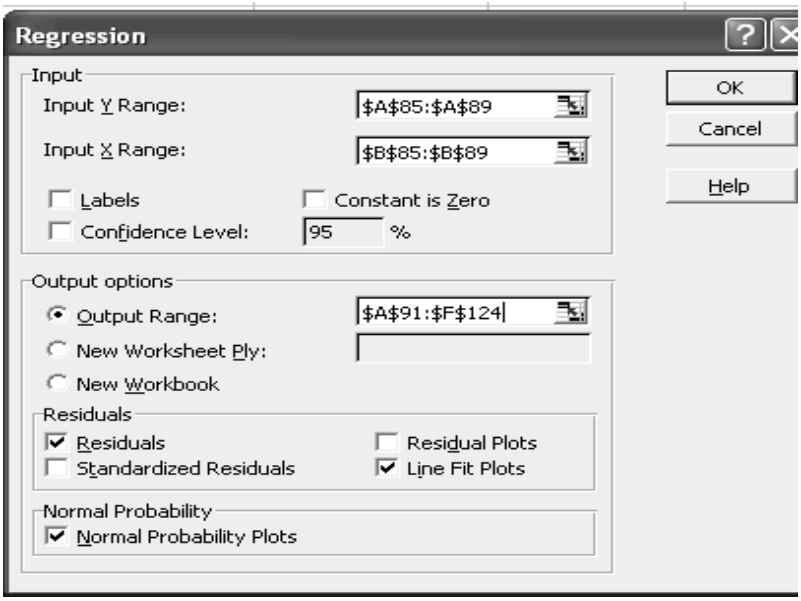
The regression dialog box opens as shown below. In this dialog box, Input range for X and Y is required. One can specify labels, confidence level and output etc.



For the sample data the input Y range was selected by clicking in the text box for input y range data first and then selecting the Y range (A85:A89). The regression tool adds the $ sign in front of the column and row number to fix its location. The input range for X was specified in a similar fashion. No labels were chosen. The default value of 95% confidence interval was accepted. The output range was also selected in an arbitrary fashion. All you need to do is to select a range of cells for the output tables and the graphs. The range A91:F124 was selected as output range by selecting cell A91 and then dragging the mouse in such a manner that the last cell selected on the right was F124.



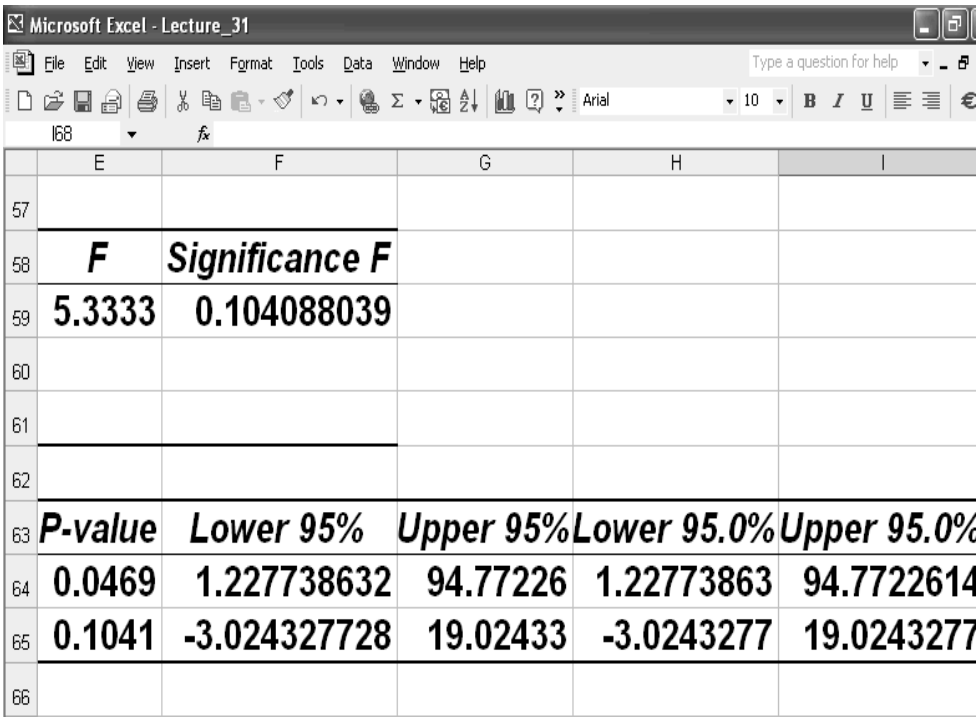
The Regression dialog box with data is shown below for clarity.



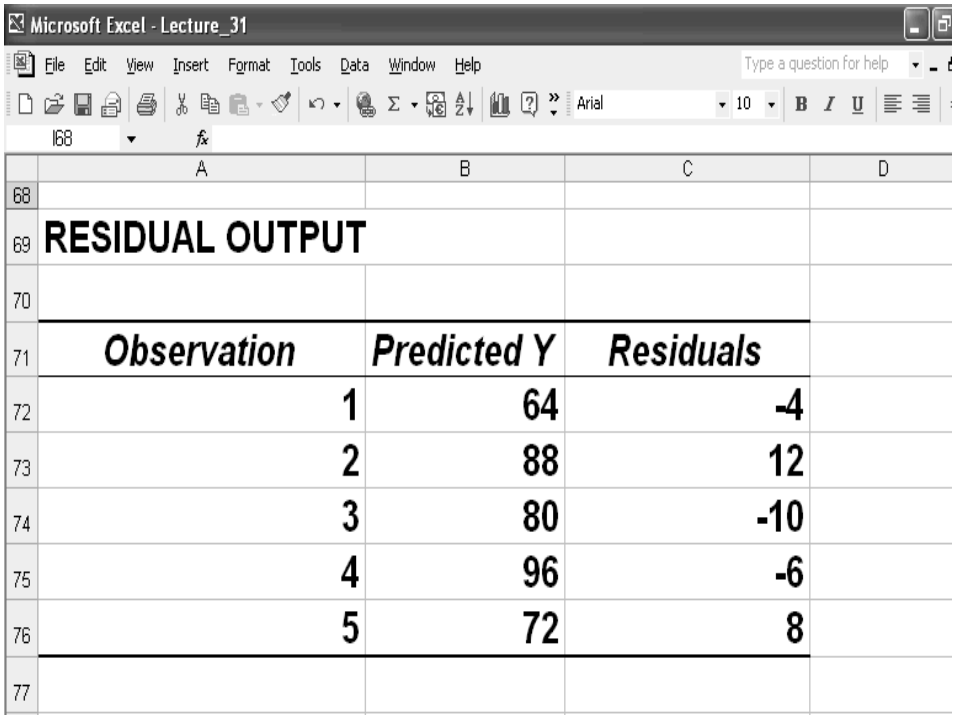
When you click OK on the Regression tool box a detailed SUMMARY OUTPUT is generated by the Regression Tool. This output is shown in parts below.

SUMMARY OUTPUT		
Regression Statistics		
Multiple R	0.8	
R Square	0.64	
Adjusted R Square	0.52	
Standard Error	10.9544512	
Observations	5	

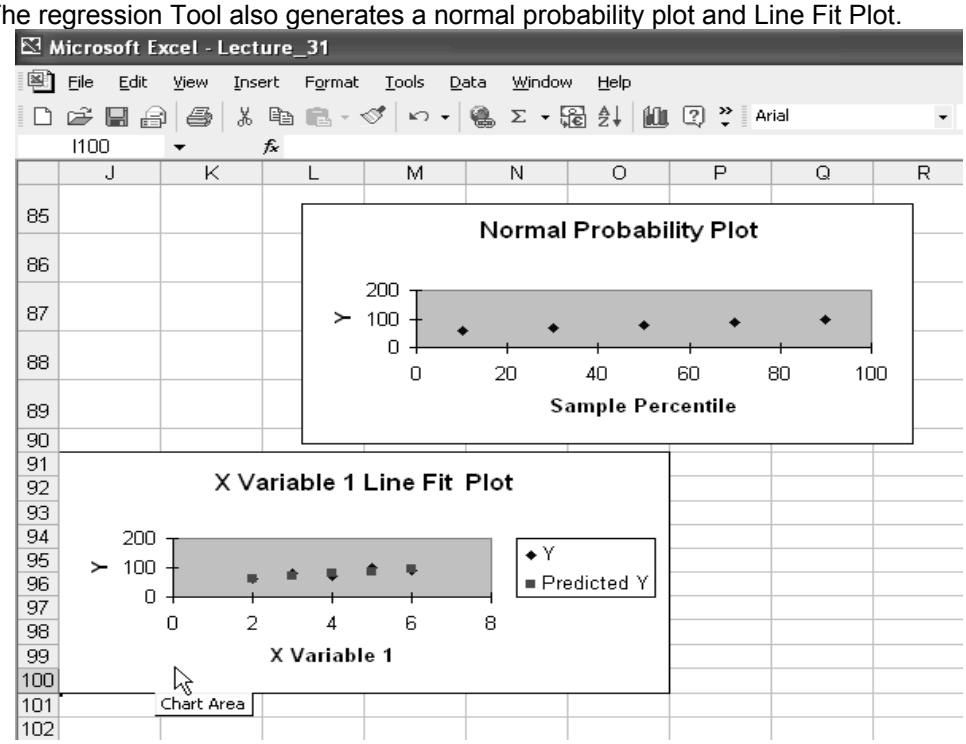
ANOVA			
	df	*SS*	*MS*
Regression	1	640	64
Residual	3	360	12
Total	4	1000	
	Coefficients	*Standard Error*	*t Stat*
Intercept	48	14.69693846	3.2659
X Variable 1	8	3.464101615	2.309



	E	F	G	H	I
57					
58	*F*	*Significance F*			
59	5.3333	0.104088039			
60					
61					
62					
63	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
64	0.0469	1.227738632	94.77226	1.22773863	94.7722614
65	0.1041	-3.024327728	19.02433	-3.0243277	19.0243277
66					



The regression Tool also generates a normal probability plot and Line Fit Plot.



EXCEL REGRESSION TOOL OUTPUT

In the regression Tool output there are a number of outputs for detailed analysis including Analysis of Variance (ANOVA) that is not part of this course. The main points of our interest for simple linear regression are:

Multiple R
Correlation Coefficient

R Square
Coefficient of determination

STEM-Standard Error of mean:
Standard deviation of population/sample size

T-Statistic
= (sample slope – population slope) / Standard error

RSQ

There is a separate function RSQ in EXCEL to calculate the coefficient of determination square of r. Description of this function is as follows:
Returns the square of the Pearson product moment correlation coefficient through data points in known_y's and known_x's. For more information, see PEARSON. The r-squared value can be interpreted as the proportion of the variance in y attributable to the variance in x.

Syntax
RSQ(**known_y's**,**known_x's**)
Known_y's is an array or range of data points.
Known_x's is an array or range of data points.

Remarks
The arguments must be either numbers or names, arrays, or references that contain numbers.
If an array or reference argument contains text, logical values, or empty cells, those values are ignored; however, cells with the value zero are included.
If known_y's and known_x's are empty or have a different number of data points, RSQ returns the #N/A error value.
The equation for the r value of the regression line is:

$$r = \frac{n(\Sigma XY) - (\Sigma X)(\Sigma Y)}{\sqrt{\left[n\Sigma X^2 - (\Sigma X)^2\right]\left[n\Sigma Y^2 - (\Sigma Y)^2\right]}}$$

Example

	A	B
1	**Known y**	**Known x**
2	2	6
3	3	5
4	9	11
5	1	7
6	8	5
7	7	4
8	5	4

	Formula	**Description (Result)**
	=RSQ(A2:A8,B2:B8)	Square of the Pearson product moment correlation

coefficient through data points above (0.05795)

P-VALUE

In the EXCEL regression Tool, the P-Value is defined as under:
P-value is the Probability of not getting a sample slope as high as the calculated value. Smaller the value more significant the result. In our example P-value=0.000133.
It means that slope is very significantly different from zero.

Conclusion

X and y are strongly associated

SAMPLING DISTRIBUTION IN r

It is possible to construct a sampling distribution for r similar to those for sampling distributions for means and percentages.
Tables at the end of books give minimum values of r (ignoring sign) for a given sample size to demonstrate a significant non-zero correlation at various significance levels (0.1, 0.05, 0.02, 0.01 and 0.001) and degrees of freedom (1 to 100).
It is to be noted that v = degrees of freedom = n -2 in all these calculations.

SAMPLING DISTRIBUTION IN r-EXAMPLE1

Look at a sample size n =5.
Null hypothesis: r = 0.
Calculated coefficient = 0.8.
Test the significance at 5% confidence level.

Solution:

Look in the table at row with v=n-2= 3 and column headed by 0.05.

Pearson Product-Moment Correlation Coefficient

Table of Critical Values

df= N-2 N = number of pairs of data	Level of significance for two-tailed test			
	.10	.05	.02	.01
1	.988	.997	.9995	.9999
2	.900	.950	.980	.990
3	.805	.878	.934	.959
4	.729	.811	.882	.917
5	.669	.754	.833	.874
6	.622	.707	.789	.834
7	.582	.666	.750	.798
8	.549	.632	.716	.765
9	.521	.602	.685	.735
10	.497	.576	.658	.708

11	.476	.553	.634	.684
12	.458	.532	.612	.661
13	.441	.514	.592	.641
14	.426	.497	.574	.628
15	.412	.482	.558	.606
16	.400	.468	.542	.590
17	.389	.456	.528	.575
18	.378	.444	.516	.561
19	.369	.433	.503	.549
20	.360	.423	.492	.537
21	.352	.413	.482	.526
22	.344	.404	.472	.515
23	.337	.396	.462	.505
24	.330	.388	.453	.495
25	.323	.381	.445	.487
26	.317	.374	.437	.479
27	.311	.367	.430	.471
28	.306	.361	.423	.463
29	.301	.355	.416	.456
30	.296	.349	.409	.449
35	.275	.325	.381	.418
40	.257	.304	.358	.393
45	.243	.288	.338	.372
50	.231	.273	.322	.354
60	.211	.250	.295	.325
70	.195	.232	.274	.302
80	.183	.217	.256	.284
90	.173	.205	.242	.267
100	.164	.195	.230	.254

You will find the Tabulated value = 0.878.
Sample value of 0.8 is less than 0.878.

Conclusion
Correlation is not significantly different from zero at 5% level.
Variables are not strongly associated.

SAMPLING DISTRIBUTION IN r-EXAMPLE 2
Look at a smple size n = 5.
Null hypothesis: r = 0
Calculated coefficient = -0.95
Test the significance at 5% confidence level.

Solution:
Look at row with v = 3 and column headed by 0.05.
Tabulated value = 0.878.
Sample value of 0.95 (ignoring sign) is greater than 0.8783

Conclusion
Correlation is significantly different from zero at 5% level.
Variables are strongly associated.

LECTURE 32
TIME SERIES AND EXPONENTIAL SMOOTHING
PART 1

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 31
- Time Series and Exponential Smoothing

SIMPLE LINEAR REGRESSION EQUATION. EXAMPLE

The slide below shows the data from 7 stores covering square ft and annual sales. The question is whether there is a relationship between the area and the sale for these stores. It is required to find the regression equation that best fits the data.

Simple Linear Regression Equation: Example

You wish to examine the relationship between the square footage of produce stores and their annual sales. Sample data for 7 stores were obtained. Find the equation of the straight line that fits the data best

Store	Square Feet	Annual Sales Rs. (000)
1	1,726	3,681
2	1,542	3,395
3	2,816	6,653
4	5,555	9,543
5	1,292	3,318
6	2,208	5,563
7	1,313	3,760

First of all a scatter diagram is prepared using EXCEL Chart Wizard as shown below. The points on the scatter diagram clearly show a positive linear relationship between the annual sale and the area of store. It means that it will make sense to proceed further with regression analysis.

The estimated regression line of Y on X is

$$\hat{Y} = a + bX$$

In the previous lecture we learn that the equation for the slope of the regression line is:

$$b = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{n\sum x^2 - \left(\sum x\right)^2}$$

And the equation for the intercept of the regression line is:

$$a = \overline{Y} - b\overline{X}$$

Stores	X	Y	XY	X^2
1	1726	3681	6353406	2979076
2	1542	3395	5235090	2377764
3	2816	6653	18734848	7929856
4	5555	9543	53011365	30858025
5	1292	3318	4286856	1669264
6	2208	5563	12283104	4875264
7	1313	3760	4936880	1723969
$\sum$	$\sum X$ =16452	$\sum Y$ =35913	$\sum XY$ =104841549	$\sum X^2$ =52413218

Now from the given data $\bar{X} = \frac{\sum X}{n}$ = 16452/7= 2350.29

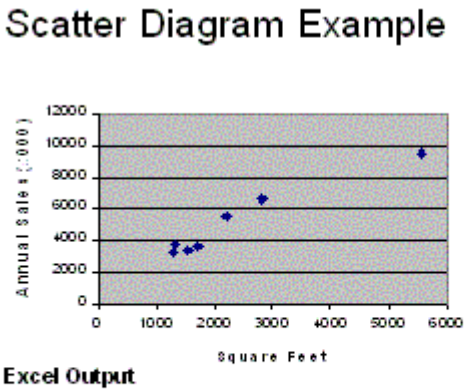
$$\bar{Y} = \frac{\sum Y}{n} = 35913/7 = 5130.429$$

$$b = \frac{n\sum XY - (\sum X)(\sum Y)}{n\sum X^2 - (\sum X)^2} = 1.4866$$

$a = \bar{Y} - b\bar{X} =$ 5130.43 – (1.4866)(2350.29) = 1636.489

Hence the desired estimated regression line is

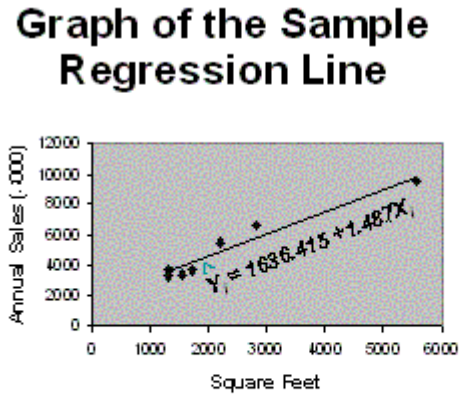
$$\hat{Y} = 1636.489 + 1.4866X$$



Excel Output

Using the EXCEL Regression Tool, the regression equation was derived as given below.

The graph of the regression line was prepared using the regression Tool. The result shows the data points, regression line and text showing the equation. As you see, it is possible to carry out linear regression very easily using Excel's Regression Tool.



Interpreting the Results

The slide below gives the main points, namely, that for every increase of 1 sq. ft. there is a sale of 1.487 units or 1407 Rs. As each unit was equal to 1,000. Now that the equation has been developed, we can estimate sale of stores of other sizes using this equation.

Interpreting the Results

$$\hat{Y}_i = 1636.415 + 1.487X_i$$

The slope of 1.487 means that each increase of one unit in X, we predict the average of Y to increase by an estimated 1.487 units.

The model estimates that for each increase of 1 square foot *in the size of the store, the expected annual sales are predicted to increase by Rs.1487*

Interpreting the Results

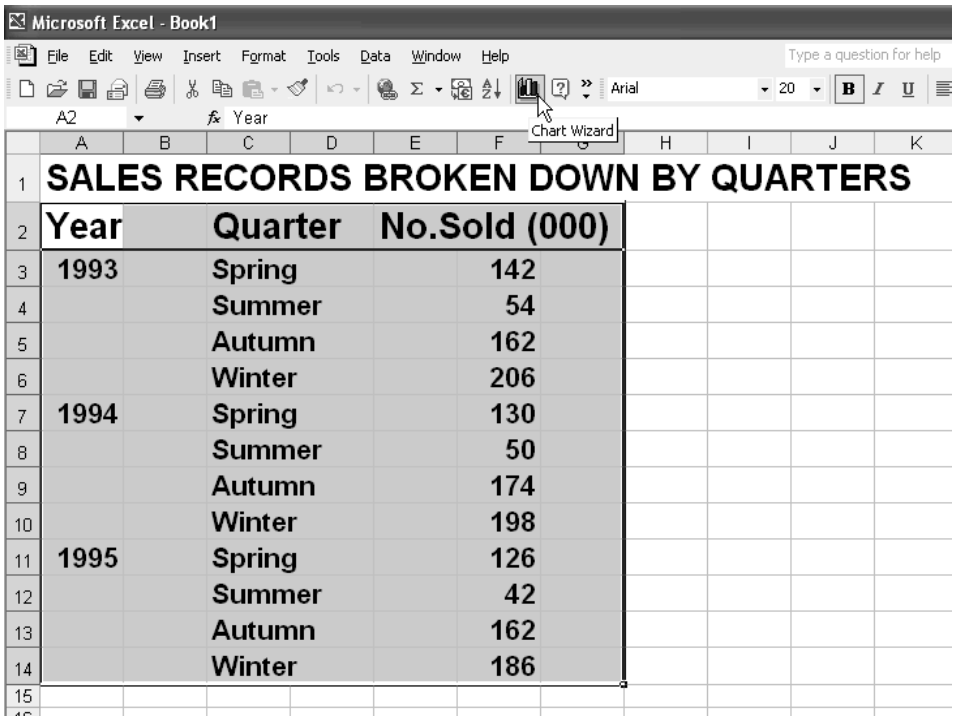
$$\hat{Y}_i = 1636.415 + 1.487X_i$$

The slope of 1.487 means that each increase of one unit in X, we predict the average of Y to increase by an estimated 1.487 units.

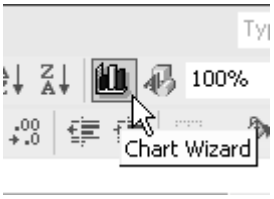
The model estimates that for each increase of 1 square foot *in the size of the store, the expected annual sales are predicted to increase by Rs.1487*

CHART WIZARD

Let us look at how we can use the Chart Wizard. We wish to study the problem shown in the slide below.

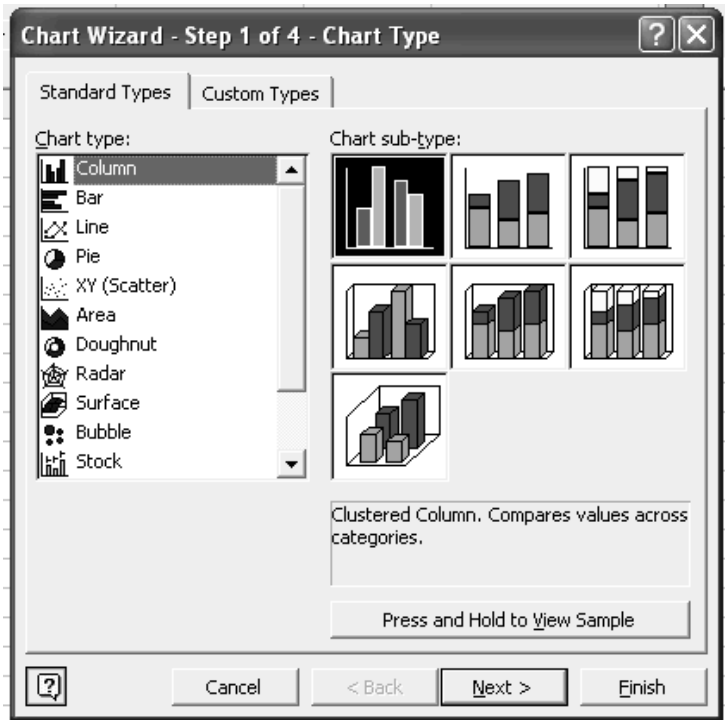


	A	B	C	D	E	F	G	H	I	J	K
1	SALES RECORDS BROKEN DOWN BY QUARTERS										
2	Year		Quarter		No.Sold (000)						
3	1993		Spring		142						
4			Summer		54						
5			Autumn		162						
6			Winter		206						
7	1994		Spring		130						
8			Summer		50						
9			Autumn		174						
10			Winter		198						
11	1995		Spring		126						
12			Summer		42						
13			Autumn		162						
14			Winter		186						
15											

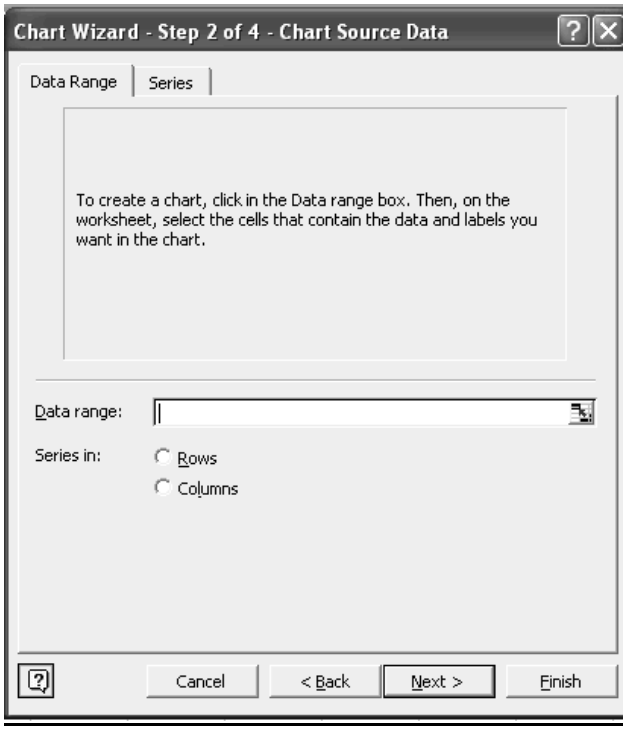


You can start with the Chart Icon as shown on the right.
There are 4 steps in using the Chart wizard as shown below:

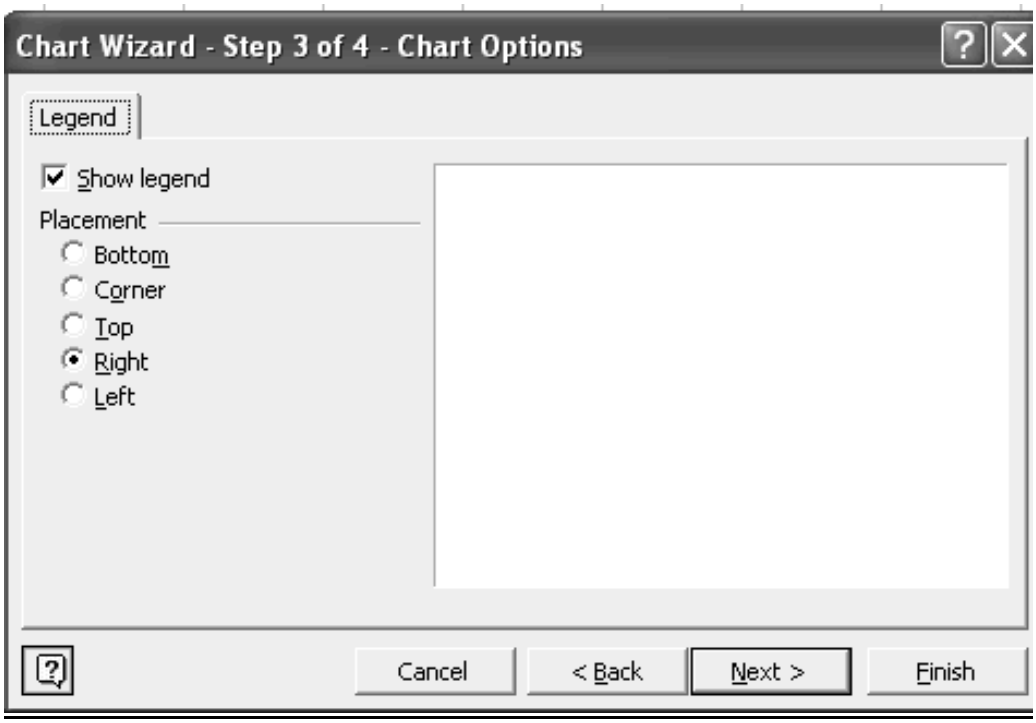
Step 1



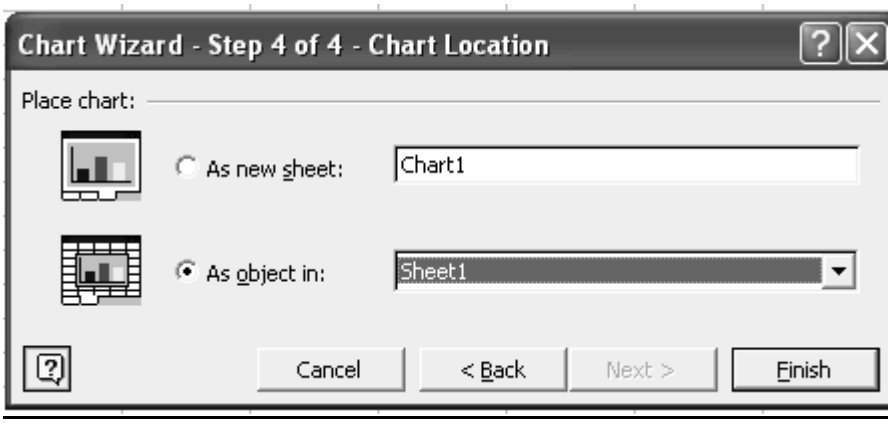
Step 2



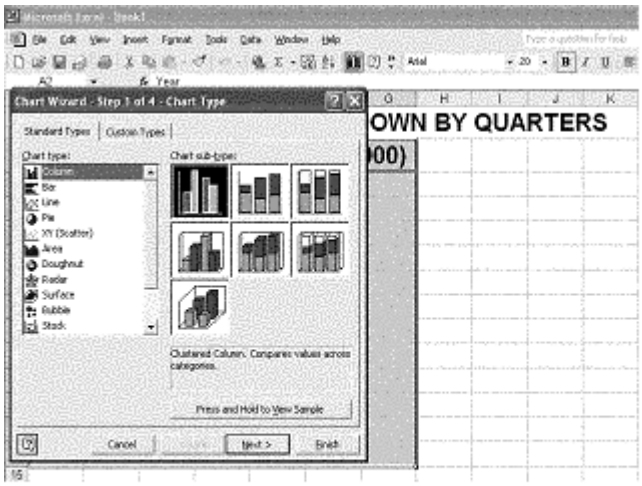
Step 3



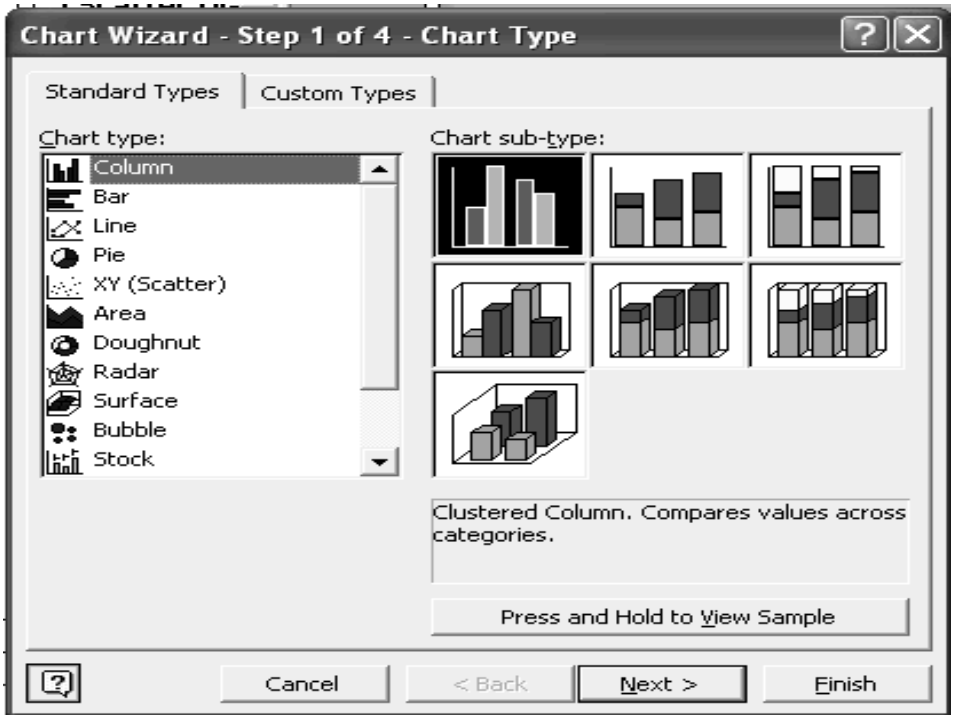
Step 4



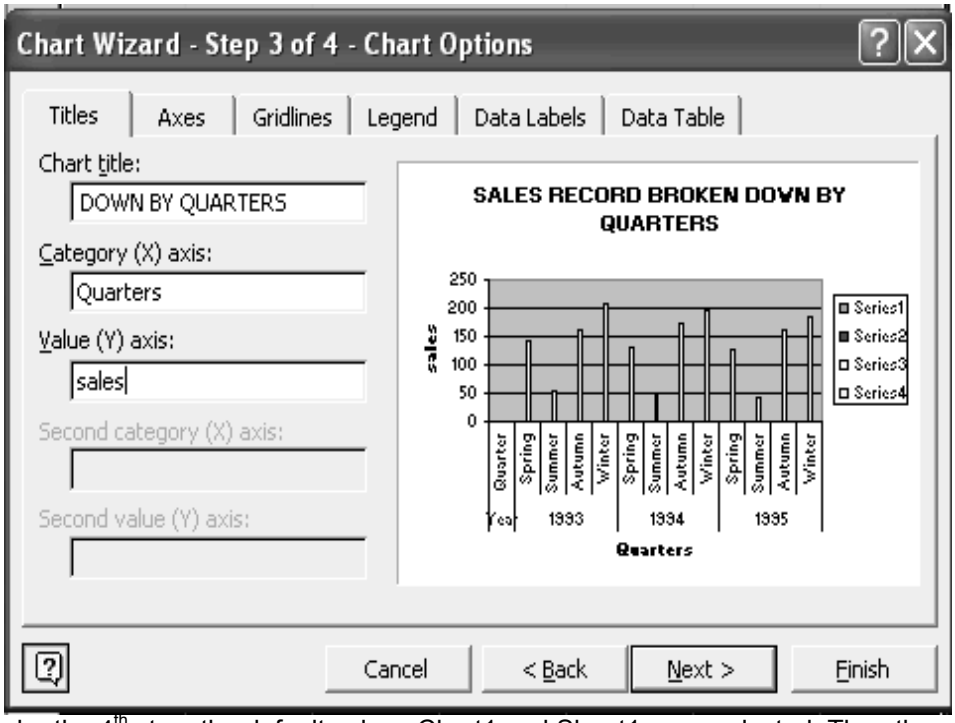
The dialog boxes are self-explanatory. Let us look at the example above and see how Chart wizard was used.
First the data was selected on the worksheet. Next the Chart Wizard was selected.
We chose Column Graph as the option as you can see in the slide below.
We clicked Next.



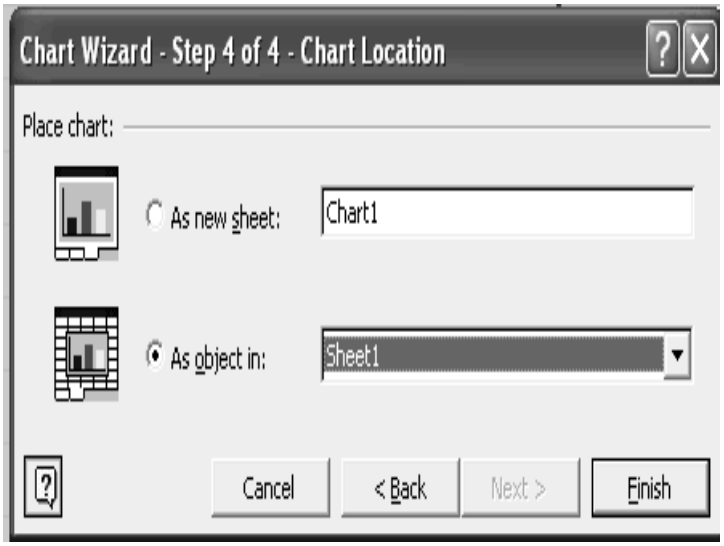
You can see the selection of Column graph in the slide below.



Under Step 2, the Chart Title, Category (X) axis and value (Y) were entered as shown in the slide. Then the button Next was clicked.



Under the 4th step, the default values Chart1 and Sheet1 were selected. Then the button Finish was clicked.



The result is shown below as a column graph.

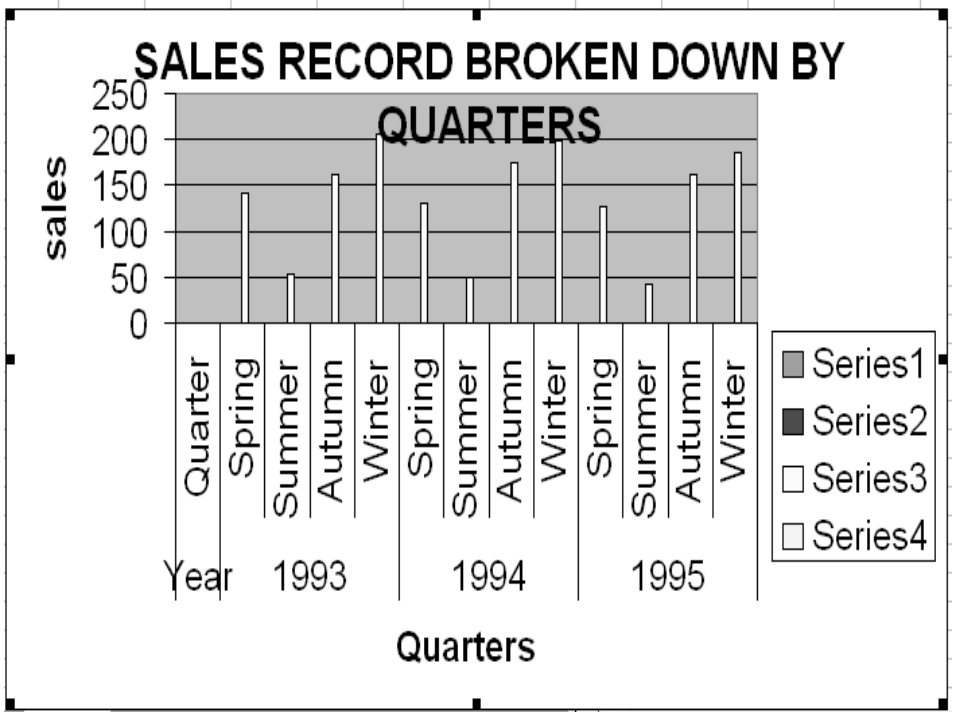
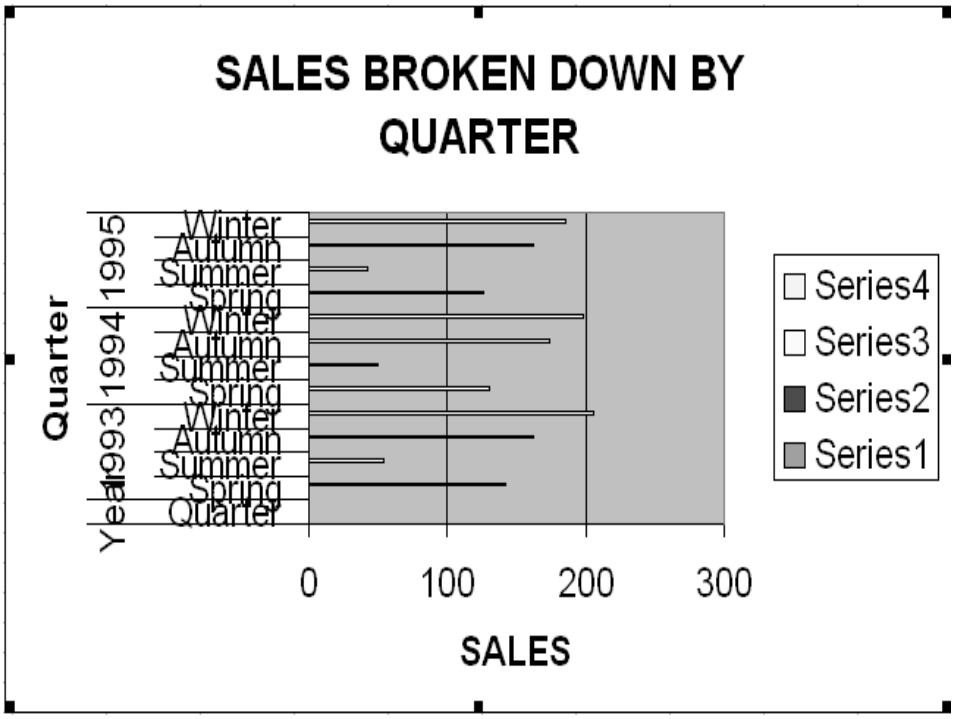
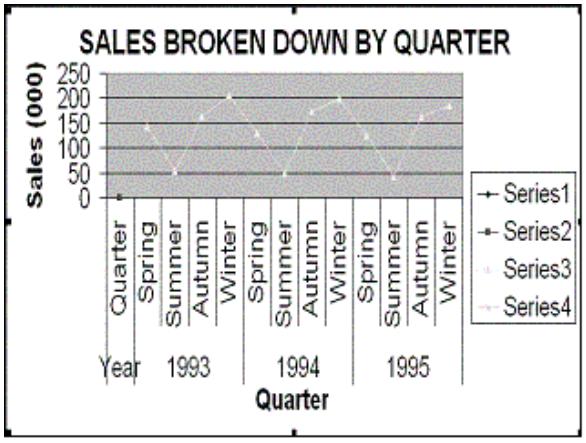


Chart Wizard was used again to draw a Side by Side chart using the same data. The result is shown below.



A line graph of the data was also prepared as shown below. This graph shows the seasonal variations in the values of sales.



EXAMINATION OF GRAPH TREND

The graph shows that there is a general upward or downward steady behaviour of figures. There are Seasonal Variations also. These are variations which repeat themselves regularly over short term, less than a year. There is also a random effect that is variations due to unpredictable situations. There are cyclical variations which appear as alternation of upward and downward movement.

EXTRACTING THE TREND FROM DATA

Look at the following data:

170, 140, 230, 176, 152, 233, 182, 161, 242

There is no explanation regarding time periods. What to do?

First step

Plot figures on graph

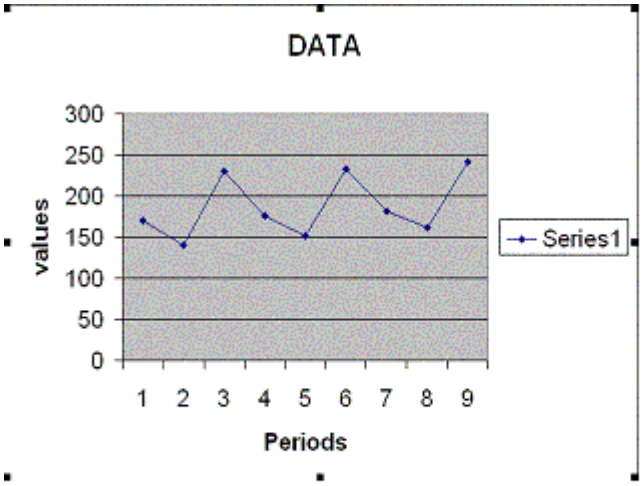
Horizontal as period 1

Vertical as period 2

Conclusion

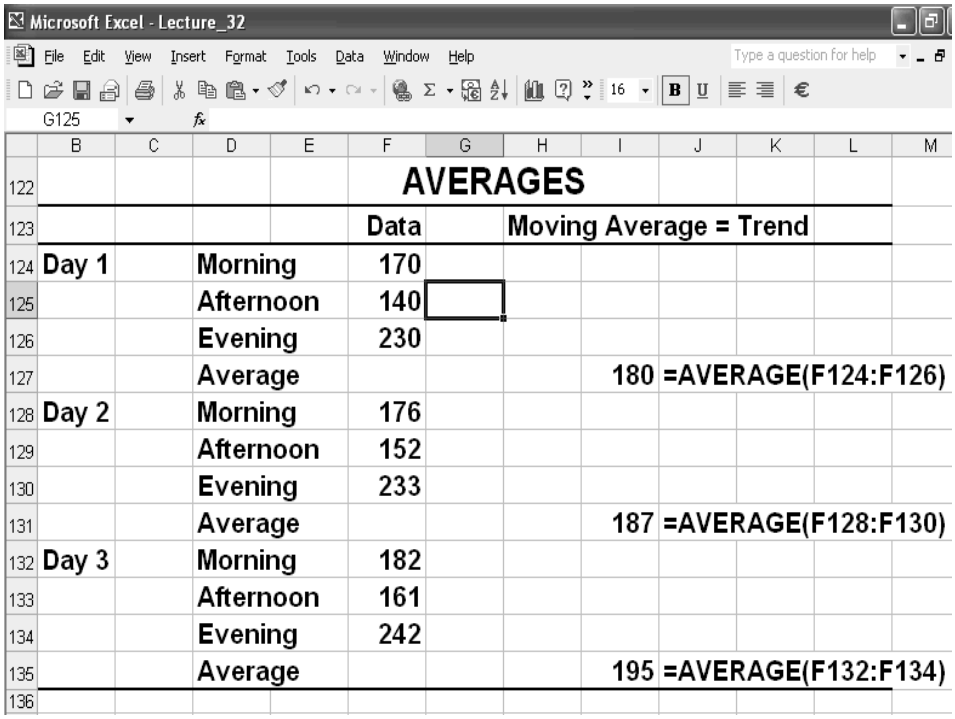
There is a marked pattern that repeats itself.

There is a well established method to extract trend with strong repeating pattern



MOVING AVERAGES

Look at the data in the slide below. There is sales data for morning, afternoon and evening for day 1, 2 and 3. We can calculate averages for each day as shown. These are simple averages for each day.



Now let us look at the idea of moving averages.

First Average- Day 1

= (170 + 140 + 230)/3 = 540/3 = 180

Next Average-Morning

= (140 + 230 + 176)/3 =546/3 = 182

Next Average-Afternoon

= (230 + 176 + 152)/3 = 186

Another method

Drop 170; Add 176; = (176-170)/3 = 6/3 = 2

Last average + 2 = 180 + 2 = 182

Caution

You may make a mistake

You saw how it is possible to start with the first 3 values 170, 140 and 230 for the first day and work out the average (180). Next we dropped 170 and added 152 the morning value from day 2. This gave us an average of 182. Similarly, the next value was calculated. Look at the worksheet below for the complete calculation. These averages are called moving averages. You could have used the alternative method but you may make a mistake in mental arithmetic. So let us only use EXCEL worksheets.

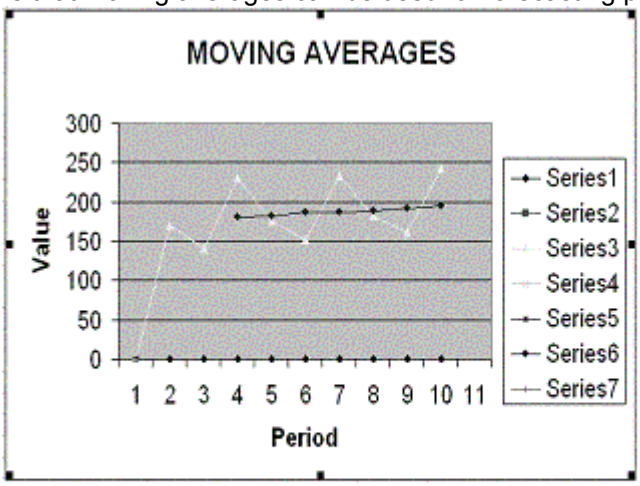
Microsoft Excel - Lecture_32

File Edit View Insert Format Tools Data Window Help

N141

	B	C	D	E	F	G	H	I	J	K	L	M
138					MOVING AVERAGES							
139			Period		Data		Moving Average		= Trend			
140	Day 1		Morning		170							
141			Afternoon		140			180	=AVERAGE(F140:F142)			
142			Evening		230			182	=AVERAGE(F141:F143)			
143	Day 2		Morning		176			186	=AVERAGE(F142:F144)			
144			Afternoon		152			187	=AVERAGE(F143:F145)			
145			Evening		233			189	=AVERAGE(F144:F146)			
146	Day 3		Morning		182			192	=AVERAGE(F145:F147)			
147			Afternoon		161			195	=AVERAGE(F146:F148)			
148			Evening		242							
149												
150												

The moving averages were plotted as shown below. You can see that the seasonal variation has disappeared. Instead you see a clear trend of increase in sales. This plot shows that moving averages can be used for forecasting purposes.



ANALYSING SEASONAL VARIATIONS

Let us find out how much each period differs from trend

Calculate Actual – trend for each period

Day 1, Afternoon

Actual = 140, Trend = 180

Actual – Trend = 140 – 180 = -40

Here, -40 is the seasonal variation.

Similarly, other seasonal variations can be worked out.

LECTURE 33
TIME SERIES AND EXPONENTIAL SMOOTHING
PART 2

OBJECTIVES

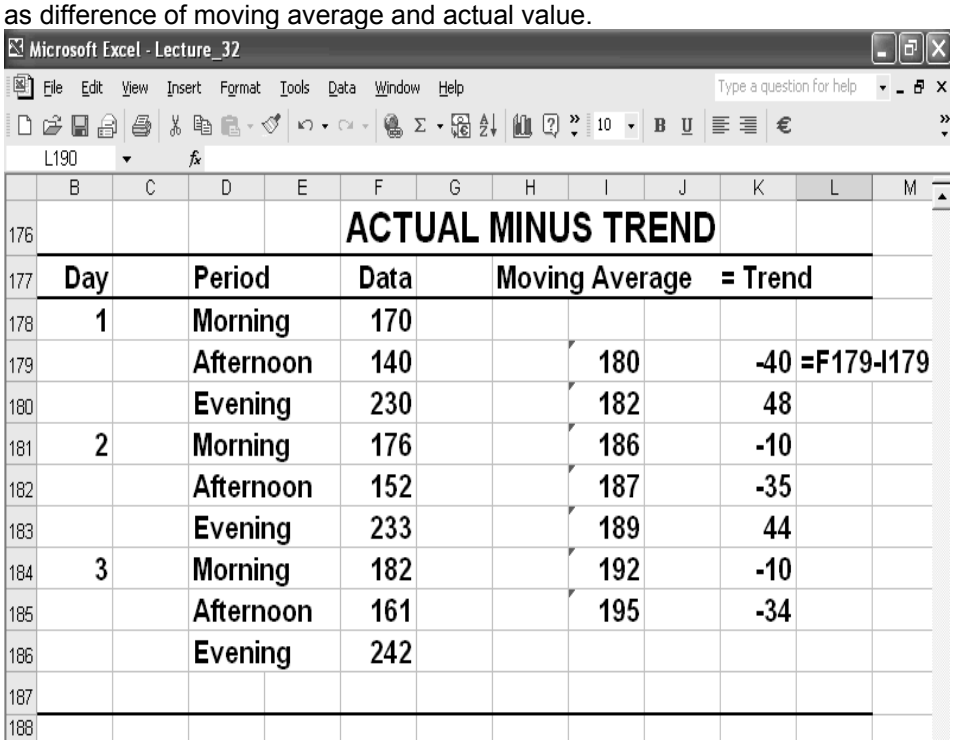
The objectives of the lecture are to learn about:

- Review Lecture 32
- Time Series and Exponential Smoothing.

TREND

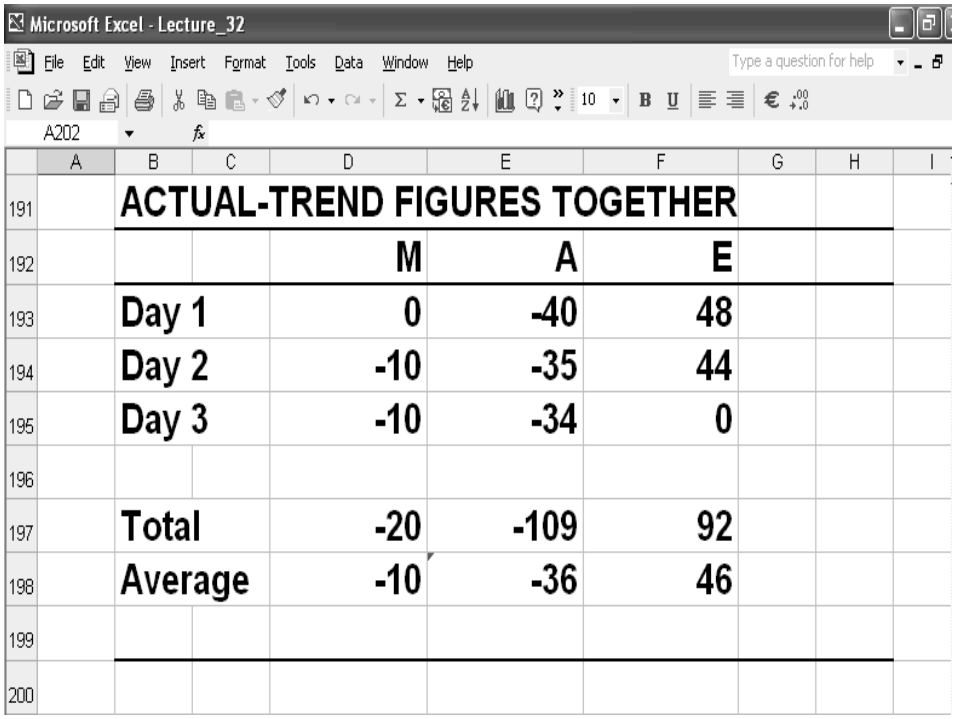
As discussed briefly in the handout for lecture 32, the trend is given by the moving average minus the actual data. Look at the slide shown below. The average of the morning, afternoon and evening of the first day is 180. This value is written in cell I179, which is the middle value for first day. The next moving average is written in cell I180. This means that the last moving average will be written in cell I185 as the moving average of the morning, afternoon and evening of 3rd day will be written against the middle value in cell F185.

Now that all the moving averages have been worked out we can calculate the trend as difference of moving average and actual value.



	B	C	D	E	F	G	H	I	J	K	L	M
176					ACTUAL MINUS TREND							
177	Day		Period		Data		Moving Average			= Trend		
178	1		Morning		170							
179			Afternoon		140			180		-40	=F179-I179	
180			Evening		230			182		48		
181	2		Morning		176			186		-10		
182			Afternoon		152			187		-35		
183			Evening		233			189		44		
184	3		Morning		182			192		-10		
185			Afternoon		161			195		-34		
186			Evening		242							
187												
188												

The actual trend figures are now written as shown in the slide below with M for morning, A for afternoon and E for evening. The titles Day 1, day 2 and Day 3 were written on the left hand side of the table. Further Total for each column was calculated. The total was divided by the non-zero values in the column. For example, in column M, there are 2 non-zero values. Hence, the total 20 was divided by 2 to obtain the average -10. Similarly, the averages in column A and E were calculated. This data is the seasonal variation and can now be used for estimating trend and random variations.



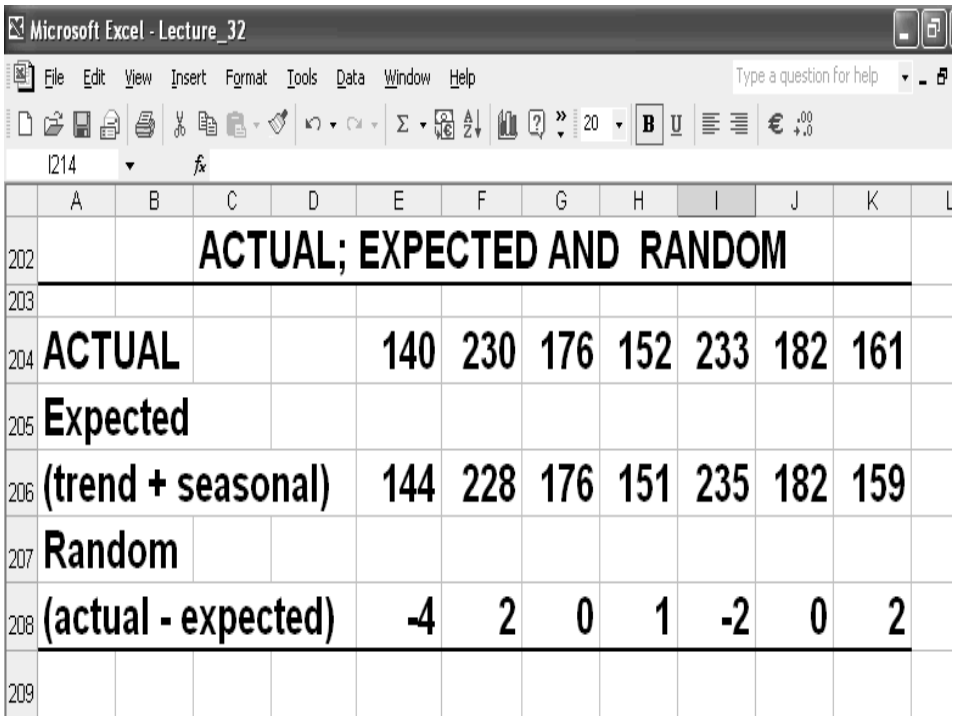
EXTRACTING RANDOM VARIATIONS

Day 1

Afternoon trend = 180
Afternoon seasonal variation = 36
Trend – variation = 180 – 36 = 144
Actual value = 140
Random variation = 140 – 144 = -4

Conclusion

Expected = Trend + Seasonal
Random = Actual – expected



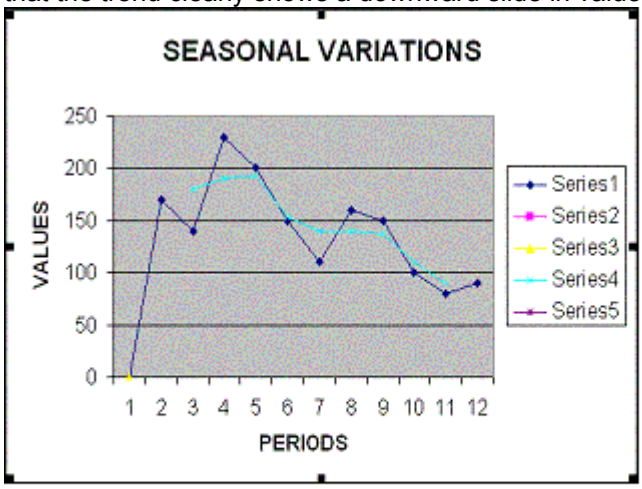
Forecast for day 4

=
Trend for afternoon of day 4
+
Seasonal adjustment for afternoon period
Trend = 180 to 195 (6 intervals)
= 15/6 = 2.5 per period
Figure for evening of day 3 = 195 + 2.5 = 197.5
Morning of day 4 = 197.5+ 2.5 = 200
Afternoon of day 4 = 200 + 2.5 = 202.5
After adjustment of seasonal variation = -36
= 202.5 – 36 = 166.5 or 166

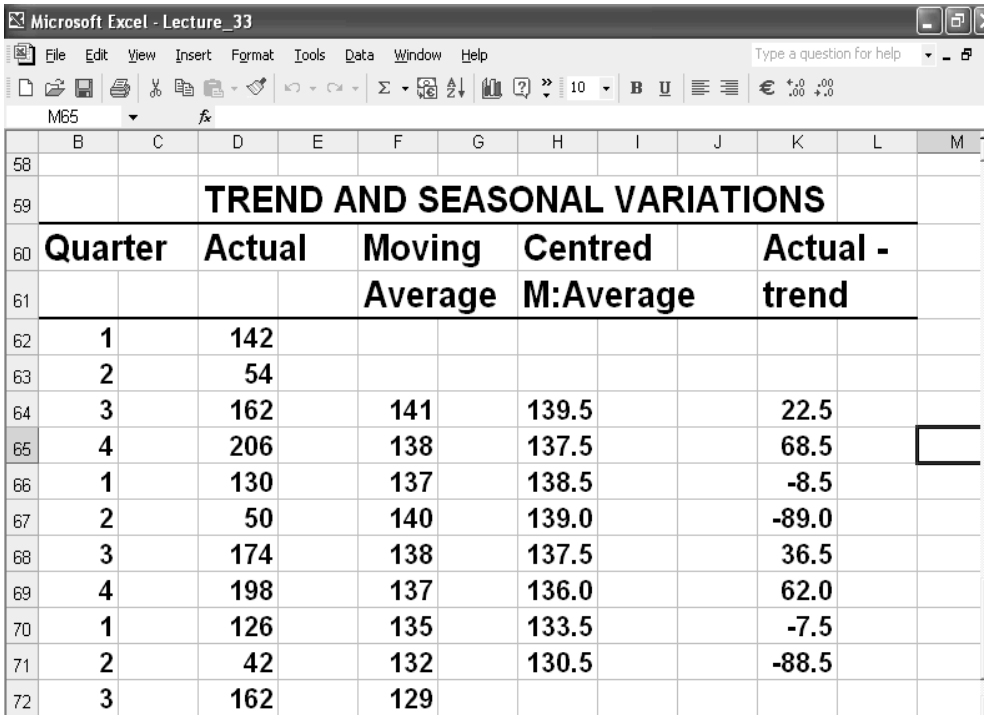
SEASONABLE VARIATIONS

Seasonal Variations are regarded as constant amount added to or subtracted from the trends. This is a reasonable assumption as seasonal peaks and troughs are roughly of constant size. In practice Seasonal variations will not be constant. These will themselves vary as trend increases or decreases. Peaks and troughs can become less pronounced

Seasonal variations as well as the trend are shown in the graph below. You can see that the trend clearly shows a downward slide in values.



In the following slide, the actual values are for 4 quarters per year. Here there is no middle value per year. The moving averages were therefore summarised against the 3rd quarter. As this does not reflect the correct position, the average of the first two moving averages was calculated and written as centred moving average in column H. The first centred moving average is the average of 141 and 138 or 139.5. This is used as the trend and the value Actual-Trend is the difference of Actual – Centred Moving Average. Here also the last row does not have a value as the moving average was shifted one position upwards.

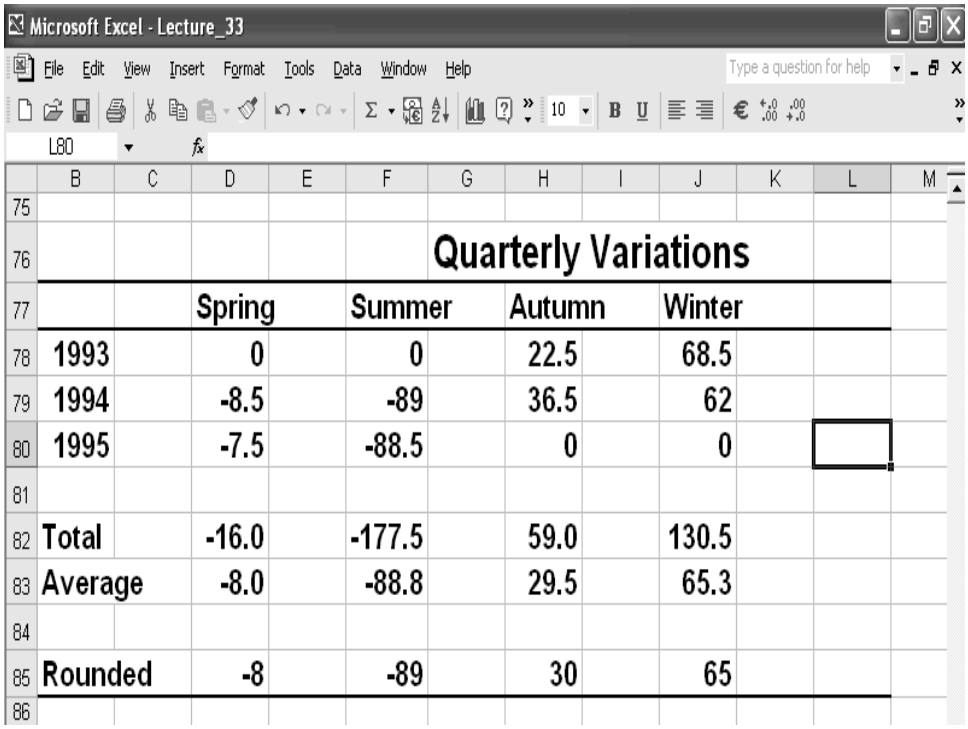

Microsoft Excel - Lecture_33

File Edit View Insert Format Tools Data Window Help

M65

	B	C	D	E	F	G	H	I	J	K	L	M
58												
59			TREND AND SEASONAL VARIATIONS									
60	Quarter		Actual		Moving		Centred			Actual -		
61					Average		M:Average			trend		
62	1		142									
63	2		54									
64	3		162		141		139.5			22.5		
65	4		206		138		137.5			68.5		
66	1		130		137		138.5			-8.5		
67	2		50		140		139.0			-89.0		
68	3		174		138		137.5			36.5		
69	4		198		137		136.0			62.0		
70	1		126		135		133.5			-7.5		
71	2		42		132		130.5			-88.5		
72	3		162		129							

The data from the previous slide was summarised as in the following slide using the approach described earlier. It may be seen that the average seasonal variation for Spring, Summer, Autumn and Winter is -8, -88.8, 29.5 and 65.3 respectively.


Microsoft Excel - Lecture_33

File Edit View Insert Format Tools Data Window Help

L80

	B	C	D	E	F	G	H	I	J	K	L	M
75												
76						Quarterly Variations						
77			Spring		Summer		Autumn		Winter			
78	1993		0		0		22.5		68.5			
79	1994		-8.5		-89		36.5		62			
80	1995		-7.5		-88.5		0		0			
81												
82	Total		-16.0		-177.5		59.0		130.5			
83	Average		-8.0		-88.8		29.5		65.3			
84												
85	Rounded		-8		-89		30		65			
86												

The expected value now is the sum of centred moving average and random

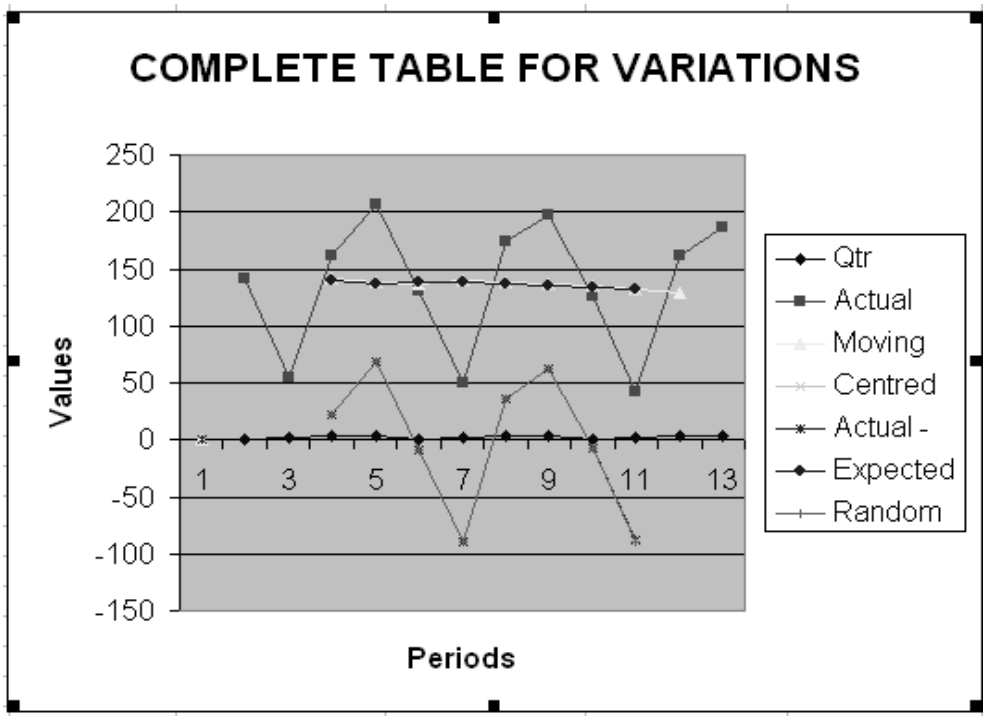
Microsoft Excel - Lecture_33

File Edit View Insert Format Tools Data Window Help

D104

	B	C	D	E	F	G	H	I
87				COMPLETE TABLE				
88	Qtr	Actual	Moving	Centred	Actual -	Expected	Random	
89			Average	M:Average	trend			
90	1	142						
91	2	54				=E92+H85	=C92-G92	
92	3	162	141	139.5	22.5	169.5	-7.5	=C92-G9
93	4	206	138	137.5	68.5	203.5	2.5	
94	1	130	137	138.5	-8.5	130.5	-0.5	
95	2	50	140	139.0	-89.0	50.0	0.0	
96	3	174	138	137.5	36.5	167.5	6.5	
97	4	198	137	136.0	62.0	202.0	-4.0	
98	1	126	135	133.5	-7.5	125.5	0.5	
99	2	42	132	130.5	-88.5	43	-1.0	
100	3	162	129					
101	4	186						

variation. The random variation is the difference between the Actual and Expected value. This gives us a complete table with all the values. The values in this table were plotted using the EXCEL Chart Wizard as shown below. You can see that the different components can now be seen clearly.



FORECASTING APPLE PIE SALES

Forecast

Sale steadily declined from 139.0 to 130.5.
Over 4 quarters, the sales declined by = 139.0 – 130.5 = 8.5
Trend in Spring 1995 was 133.5.
We can assume annual decrease as on the basis of decline over the last 4 quarters = 8.5
Trend in 1996 = trend in 1995 less decline = 133.5 – 8.5 = 125
Seasonal variation as already worked out = -8

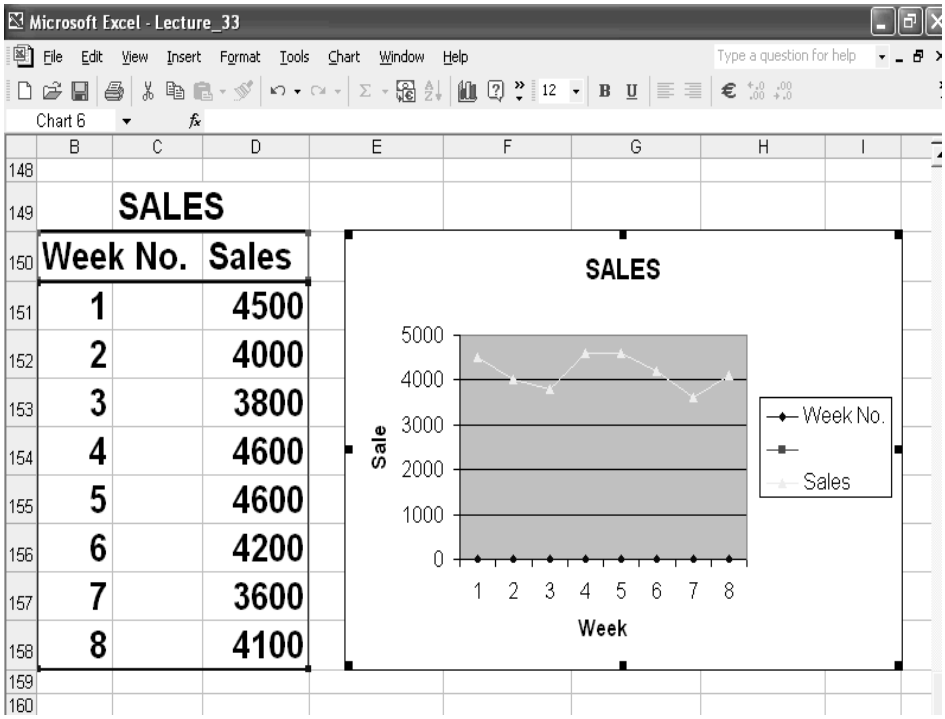
Hence:

Final forecast = 125 – 8 = 117

FORECASTING IN UNPREDICTABLE SITUATIONS

Two methods were studied above. Each one has certain features. If there is steady increase in data and repeated seasonal variations, there are many cases that do not conform to these patterns. There may not be a trend. There may not be a short term pattern. Figures may hover around an average mark. How to forecast under such conditions?

Data for sales over a period of 8 weeks is summarized and plotted in the slide below. You may see that the values hover around an average value without any particular pattern. This problem requires a different solution.



FORECAST

Let us assume that the forecast for week 2 is the same as the actual data for week 1, that is 4500.

Week no.	Actual sales	Forecast
1	4500	-
2	4000	4500

The Actual sale was 4000. Thus, the Forecast is 500 too high.
Another approach would be to incorporate the proportion of error in the estimate as follows:

new forecast = old forecast + proportion of error α

Or

new forecast = old forecast + α x (old actual – old forecast)

This method is called Exponential Smoothing. We shall learn more about this method in lecture 34.

LECTURE 34
FACTORIALS
PERMUTATIONS AND COMBINATIONS

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 33
- Factorials
- Permutations and Combinations

Module 7

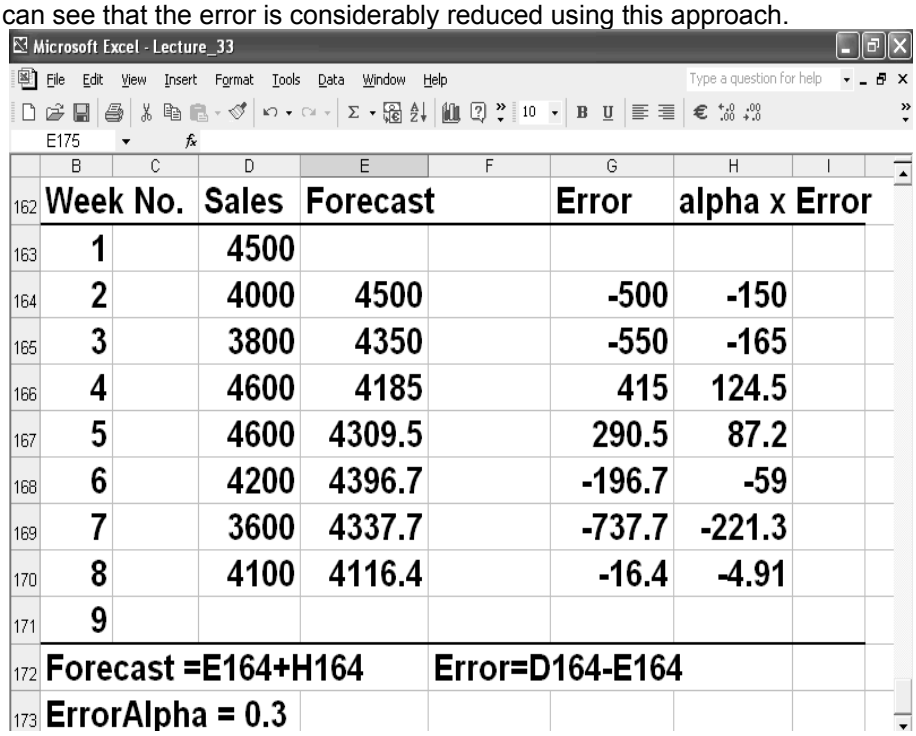
Module 7 covers the following:
Factorials
Permutations and Combinations
(Lecture 34)
Elementary Probability
(Lectures 35-36)
Chi-Square
(Lectures 37)
Binomial Distribution
(Lectures 38)

FORECAST

Please refer to the Example discussed in Handout 33.
Let $\alpha = 0.3$
Then:
Forecast week 3 = week 2 forecast + α x (week 2 actual sale – week 2 forecast)
= 4500 – 0.3 x 500 = 4350

Conclusion

Overestimate is reduced by 30% of the error margin 500.
The slide below shows the calculation for normal error as well as alpha x error. You can see that the error is considerably reduced using this approach.



	B	C	D	E	F	G	H	I
162	Week No.		Sales	Forecast		Error	alpha x Error	
163	1		4500					
164	2		4000	4500		-500	-150	
165	3		3800	4350		-550	-165	
166	4		4600	4185		415	124.5	
167	5		4600	4309.5		290.5	87.2	
168	6		4200	4396.7		-196.7	-59	
169	7		3600	4337.7		-737.7	-221.3	
170	8		4100	4116.4		-16.4	-4.91	
171	9							
172	Forecast =E164+H164				Error=D164-E164			
173	ErrorAlpha = 0.3							

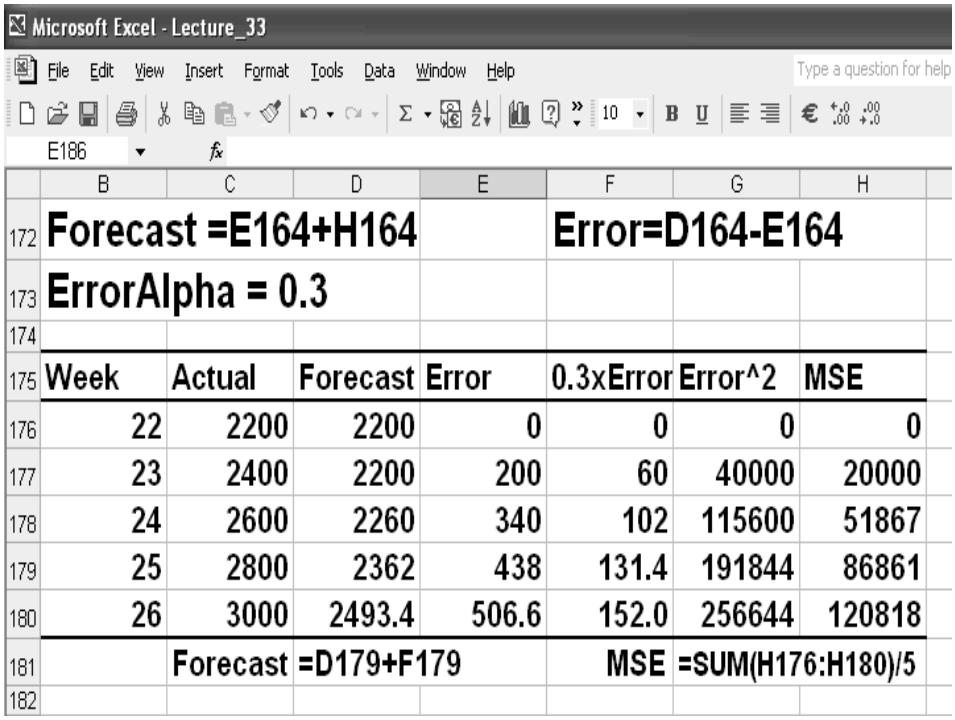
The forecast is now calculated by adding alpha x Error to the actual sales. The error is the difference between the actual sales and the forecast. The first value is the same as the sale last week. Use of alpha =0.3 is considered very

common. This method is called Exponential Smoothing and alpha is Smoothing Constant.

Rule for obtaining a forecast:
Let A= Actual and F= Forecast.
Then:
F (t) = F (t-1) + α(A (t-1) – F (t-1))
= αA (t-1) + (1- α) F (t-1)
F (t-1) = α A (t-2) + (1- α) F (t-2)
Substituting
F3 = α A (t-1) + (1- α) [αA (t-2) + (1- α) F (t-2)]
= α [A (t-1) + (1- α) A (t-2)] + (1- α) ^2F (t-2)
Replacing F (t-2) by a (t-3) + (1- α) F (t-3)
F (t) = α[A (t-1) + (1- α) A (t-2) + (1- α) ^2A (t-3)] +
(1- α)F (t-3)

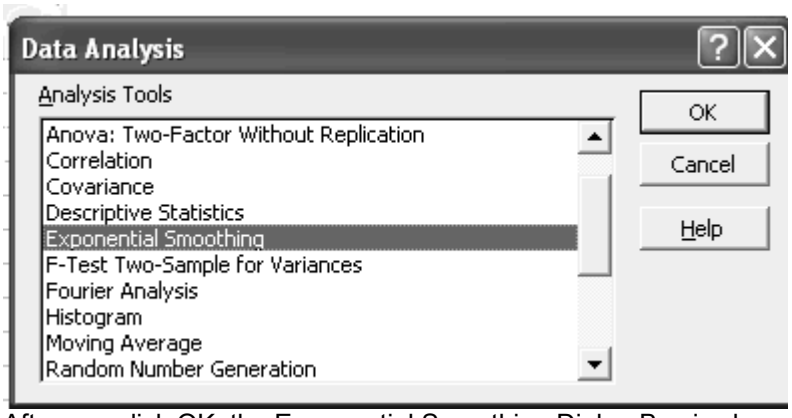
WHERE TO APPLY EXPONENTIAL SMOOTHING
What kinds of situations require the application of Exponential Smoothing?
What are good values ofα?
The accepted Criterion is Mean Square Error (MSE).
You can find MSE for by squaring all and including the present one and dividing by the number of periods included.
Sign of good forecast is when MSE stabilizes.
Generally alpha between 0.1 and 0.3 performs best.

Example
The slide below shows the calculation of MSE. Detailed formulas can be seen in the Worksheet for Lecture 34.

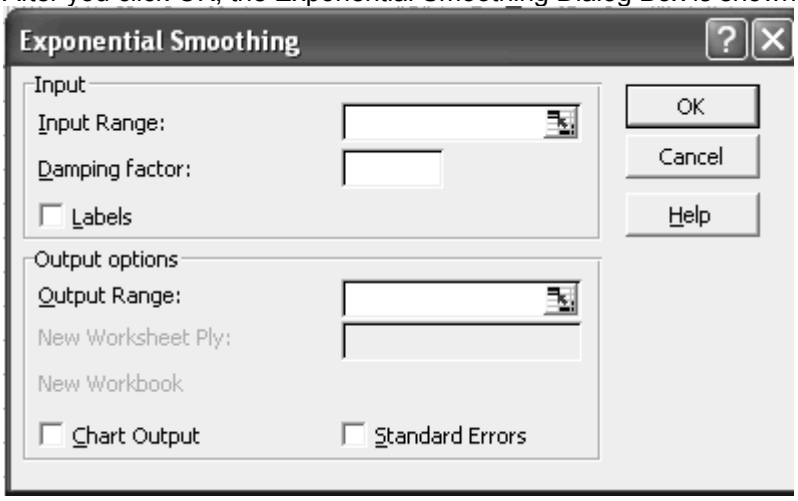


	B	C	D	E	F	G	H
172	Forecast =E164+H164				Error=D164-E164		
173	ErrorAlpha = 0.3						
174							
175	Week	Actual	Forecast	Error	0.3xError	Error^2	MSE
176	22	2200	2200	0	0	0	0
177	23	2400	2200	200	60	40000	20000
178	24	2600	2260	340	102	115600	51867
179	25	2800	2362	438	131.4	191844	86861
180	26	3000	2493.4	506.6	152.0	256644	120818
181		Forecast	=D179+F179		MSE	=SUM(H176:H180)/5	
182							

EXCEL EXPONENTIAL SMOOTHING TOOL
It is possible to use the Exponential Smoothing Tool included in the EXCEL Tools.



After you click OK, the Exponential Smoothing Dialog Box is shown as below:



Different items in the Dialog Box are described below:

Input Range

Enter the cell reference for the range of data you want to analyze. The range must contain a single column or row with four or more cells of data.

Damping factor

Enter the damping factor you want to use as the exponential smoothing constant. The damping factor is a corrective factor that minimizes the instability of data collected across a population. The default damping factor is 0.3.

Note Values of 0.2 to 0.3 are reasonable smoothing constants. These values indicate that the current forecast should be adjusted 20 to 30 percent for error in the prior forecast. Larger constants yield a faster response but can produce erratic projections. Smaller constants can result in long lags for forecast values

Labels

Select if the first row and column of your input range contain labels. Clear this check box if your input range has no labels; Microsoft Excel generates appropriate data labels for the output table.

Output Range

Enter the reference for the upper-left cell of the output table. If you select the Standard Errors check box, Excel generates a two-column output table with standard error values in the right column. If there are insufficient historical values to project a forecast or calculate a standard error, Excel returns the #N/A error value.

Note The output range must be on the same worksheet as the data used in the input range. For this reason, the New Worksheet Ply and New Workbook options are unavailable.

Chart Output

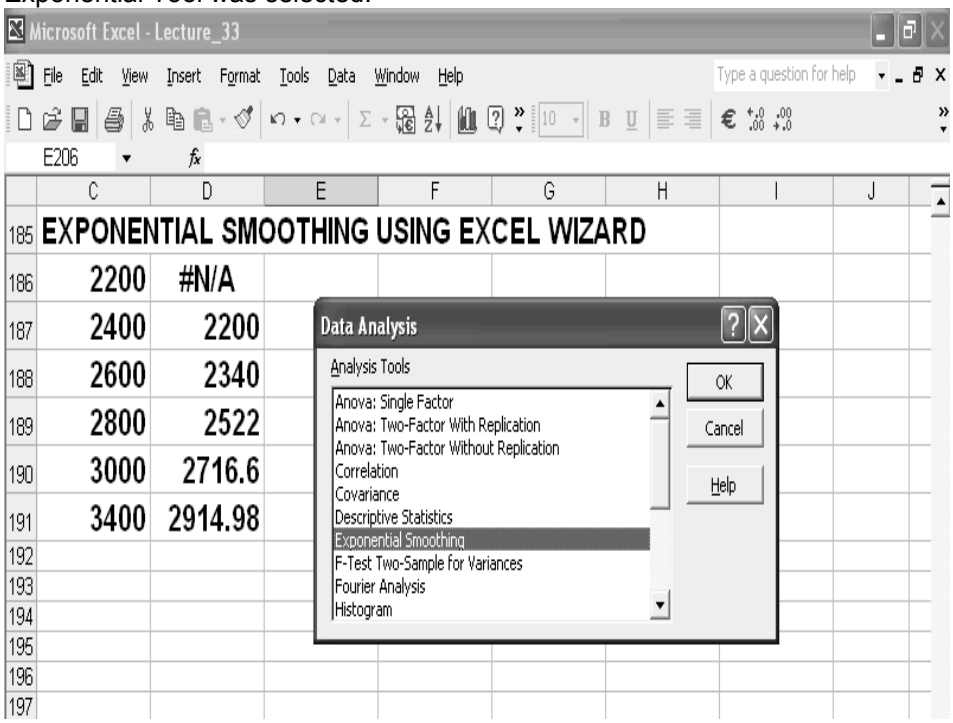
Select to generate an embedded chart for the actual and forecast values in the output table.

Standard Errors

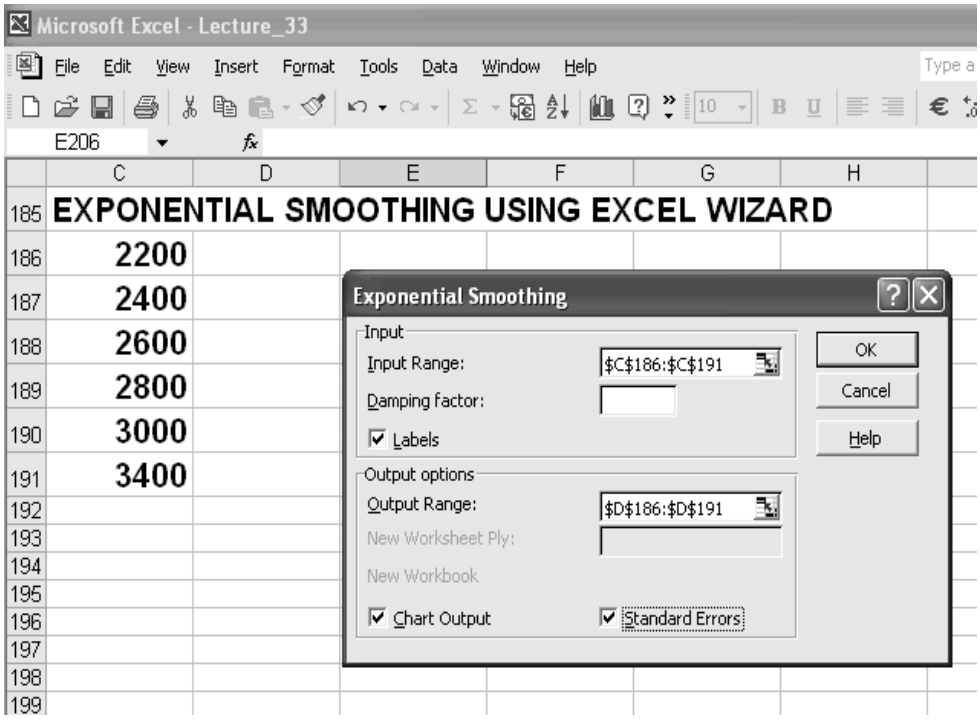
Select if you want to include a column that contains standard error values in the output table. Clear if you want a single-column output table without standard error values.

Example

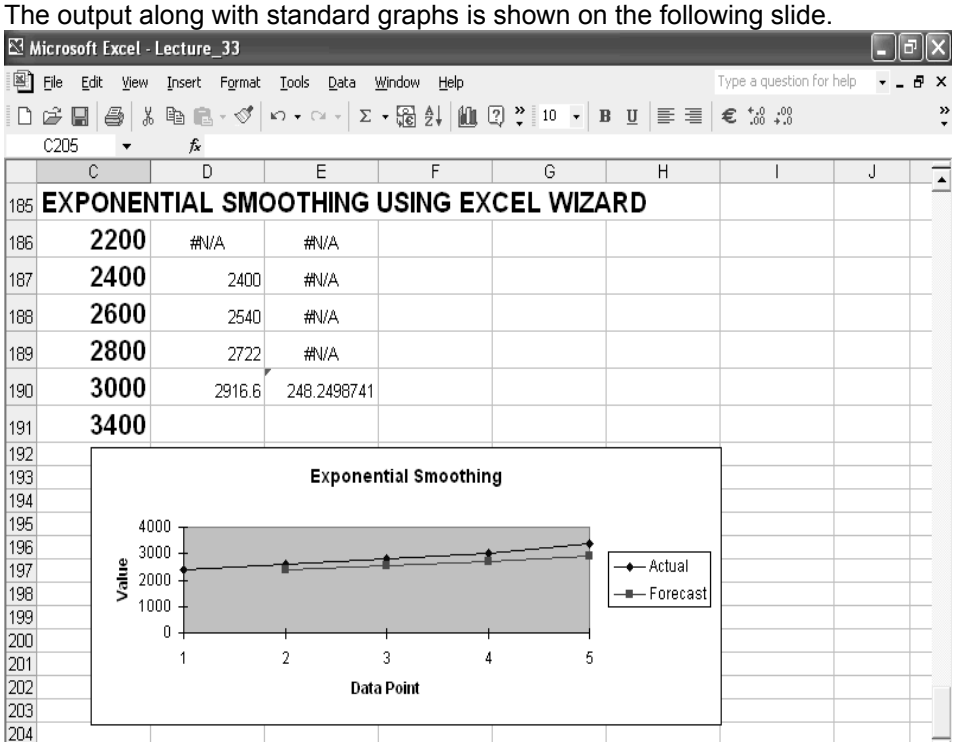
Use of the Exponential Smoothing Tool is shown in the following slides. First the Exponential Tool was selected.



Next the Input and Output Range were specified. Labels, Chart Output and Standard Errors were ticked as options in check boxes.



The output along with standard graphs is shown on the following slide.



FACTORIAL
Let us look at natural numbers.
Natural Numbers
1, 2, 3,...
Let us now define a factorial of natural numbers, say factorial of 5.
Five Factorial
5! = 5.4.3.2.1 or 1.2.3.4.5
Similarly factorial of 10 is:
Ten Factorial
= 1.2.3.4.5.6.7.8.9.10 = 10.9.8.7.6.5.4.3.2.1
In general
n! = n(n-1)(n-2)..3.2.1 or
n! = n(n-1)(n-2)!
= n(n-1)!
FACTORIAL EXAMPLES
10! = 10.9.8.7.6.5.4.3.2.1=3,628,800
8!/5! = 8.7.6.5! /5!= 8.7.6 = 336
12!/9! = 12.11.10.9!/9! = 12.11.10 = 1320
10!8!/9!5! =10.9!.8.7.6.5!/9!5! = 10.8.7.6 = 3360
WAYS
If operation A can be performed in m ways and B in n ways, then the two operations can be performed together in m.n ways.
Example
A coin can be tossed in 2 ways. A die can be thrown in 6 ways. A coin and a die together can be thrown in 2.6 = 12 ways

PERMUTATIONS
An arrangement of all or some of a set of objects in a definite order is called permutation.

Example 1
There are 4 objects A, B, C and D
Permutations of 2 object A & B:
AB, BA
Permutations in three objects A, B and C:
ABC, ACB, BCA, BAC, CAB, CBA

Example 2
Number of permutations of 3 objects taken 2 at a time = 3P2
= 3!/(3-2)! = 3.2 = 6
= AB, BA, AC, CA, BC, CB
Number of permutations of n objects taken r at a time =
nPr = n!/(n-r)!

Example 3

Let's say you and a friend love going to movies, and you get a Saturday afternoon free so you can indulge yourselves. You go to a multiplex that is showing 6 movies simultaneously, each starting at 2:00 pm, 4:00 pm, and 6:00 pm, after which you have to get back home. How many different ways can you watch the most different movies?

Answer: You have a choice of 6 movies, so this is your set. You can watch one movie at

Answer: You have a choice of 6 movies, so this is your set. You can watch one movie at 2:00, one at 4:00, and one at 6:00, therefore you can watch 3 movies and you're looking for the number of 3-permutations. We have then

P(6, 3) = 6! / (6-3) ! = 6! / 3! = 6.5.4.3! / 3! = 6.5.4 = 120

There are 120 different ways for you to watch 3 of the 6 movies that Saturday afternoon.

Example:

Suppose there are 100 numbers to choose from (00 to 99), you must choose 5 numbers in a specific order, and you can only choose a number once. What are your chances of winning the grand prize?

There are 100 choices, and we're only picking 5 of those, so we have $P(100,5) = 100! / (100-5)! = 9,034,502,400$ ways to pick 5 numbers in a specific order. Since only one of those is the winning sequence, your chances of winning are 1 in 9,034,502,400.

Example:

A government keeps some confidential information in a heavily guarded room that is locked with a 5-card mechanism. Eight different officials each carry a card, and to get access, the cards must be inserted in a specific order. The order changes daily, and 3 of the 8 cards will not be used on any given day. A novice spy needs to acquire some documents in this room. He manages to acquire all eight cards and slip past the guards, but doesn't realize until he gets to the door that only five cards are used and they must be inserted in the correct order. A wrong entry brings with it a mass of large mean guys with big guns. What are his chances of getting the right sequence?

Solution

The spy has a set of 8 cards to choose 5 from, therefore $n=8$ and $r=5$ and $P(8,5)= 8!/3! = 6720$. Only one of those 6720 possibilities is correct, so he has a 1 in 6720 chance, or $1/6720 = 0.00015 = 0.015\%$.

PERMUTATIONS OF n OBJECTS

Number of n permutations of n different objects taken n at a time is n!

nPn = n!/(n-n)! = n!/ 0! = n! /1 = n!= n(n-1)(n-2)..3.2.1

Number of permutations of n objects of which n_1 are alike of one kind, n_2 are alike of one kind and n_k are alike.

$N!/n_1!n_2!...n_k1$

Example 3

How many possible permutations can be formed from the word STATISTICS?

S=3, A =1, T =3, I =2, C = 1

Formula

nPr = n!/n1!n2!..nk!

= 10!/3!1!3!2!1! = 10.9.8.7.6.5.4.3!/3!3!2!

= 50400

PERMUT

EXCEL function PERMUT can be used to calculate number of permutations.

Returns the number of permutations for a given number of objects that can be selected from number objects. A permutation is any set or subset of objects or events where internal order is significant. Permutations are different from combinations, for which the internal order is not significant. Use this function for lottery-style probability calculations.

Syntax

PERMUT(**number**,**number_chosen**)

Number is an integer that describes the number of objects.

Number_chosen is an integer that describes the number of objects in each permutation.

Remarks

Both arguments are truncated to integers.

If number or number_chosen is nonnumeric, PERMUT returns the #VALUE! error value.

If number ≤ 0 or if number_chosen < 0, PERMUT returns the #NUM! error value.

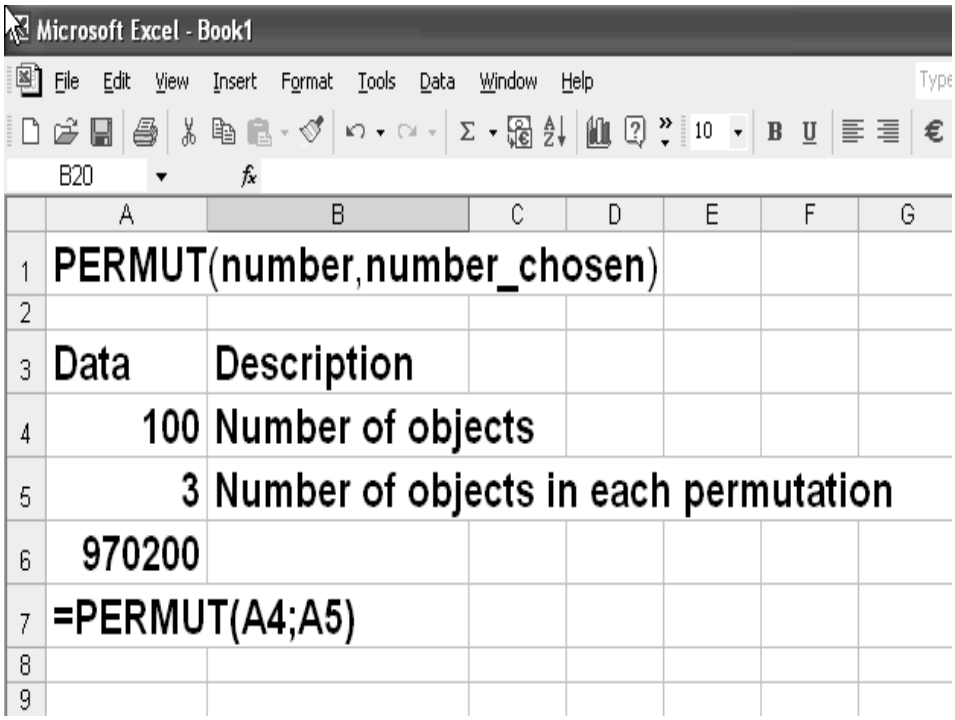
If number < number_chosen, PERMUT returns the #NUM! error value.

The equation for the number of permutations is:

$$P_{k,n} = \frac{n!}{(n-k)!}$$

Example

Suppose you want to calculate the odds of selecting a winning lottery number. Each lottery number contains three numbers, each of which can be between 0 (zero) and 99, inclusive. The following function calculates the number of possible permutations:


Microsoft Excel - Book1
File Edit View Insert Format Tools Data Window Help
B20
PERMUT(number,number_chosen)
Data
Description
100
Number of objects
3
Number of objects in each permutation
970200
=PERMUT(A4;A5)

LECTURE 35
COMBINATIONS
ELEMENTARY PROBABILITY
PART 1

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 34
- Combinations
- Elementary Probability

COMBINATIONS

Arrangements of objects without caring for the order in which they are arranged are called Combinations.
Number of n objects taken r at a time, denoted by nCr or (n) given by (r)

nCr = n!/r!(n-r)!

Difference between Combination and Permutation

Suppose we have to form a number of consisting of three digits using the digits **1,2,3,4**, To form this number the digits have to be *arranged*. Different numbers will get formed depending upon the order in which we arrange the digits. This is an example of *Permutation*.

Now suppose that we have to make a team of 11 players out of 20 players, This is an example of *combination*, because the order of players in the team will not result in a change in the team. No matter in which order we list out the players the team will remain the same! For a different team to be formed at least one player will have to be changed

Example

Number of combinations of 3 different objects A, B, C taken two at a time
= 3!/2!(3-2)! = 6/2 = 3.
These combinations are: AB, AC, and BC.

COMBINATIONS EXAMPLES

Here are a few examples of combinations which are based on the above formula.

Example 3

In how many ways a team of 11 players be chosen from a total of 15 players?
n= 15, r = 11

$$\overset{15}{C}_{11} = \frac{15!}{11!(15-11)!} = \frac{15.14.13.12.11!}{11!(4!)} = \frac{15.14.13.12}{4.3.2.1} = 1365\ Ways$$

Example 4

There are 5 white balls and 4 black balls. In how many ways can we select 3 white and 2 black balls?

$$\overset{5}{C}_{3} \times \overset{4}{C}_{2} = \frac{5!}{3!(5-3)!} \times \frac{4!}{2!(4-2)!} = 60$$

Example 5

If a committee of 3 people is to be selected from among 5 married couples so that the committee does not include two people who are married to each other, how many such committees are possible?

Solution:

Total number of ways of picking 3 out 5 married couples or 10 people = 10C3 = 120

- A set can have any of the five married couples ------> 5
- The third person can be any one of the remaining eight ----> 8 (one married couple is already part of the chosen set).

so total Number of ways in which the set of three can have a married couple = 5*8 = 40

number of combinations which don't have any of the married couples. =
10C3 - (5*8) = 80;

RESULTS OF SOME COMBINATIONS

Here are some important combinations that can simplify the process of calculations for Binomial Expansion.

nC0 = nCn = 1
e.g., 4C0 = 4C4 = 1
nC1 = nCn-1 = n
e.g., 4C1 = 4C3 = 4
nCr = nCn-r
e.g., 5C2 = 5C3

BINOMIAL EXPANSION

An expression consisting of two terms joined by + or – sign is called a Binomial Expression. Expressions such as (a+b), (a-b), (x+y)^2 are examples of Binomial Expressions

We can verify that:

(x+y)^1 = x + y

(x+y)^2 = x^2 + 2xy + y^2

(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3

(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 +4xy^3 + y^4

Expressions on the right hand side are called Binomial Expansions.

COEFFICIENTS OF BINOMIAL EXPANSION

The coefficients of the binomial expansion for any binomial expression can be written in combinatorial notation:

(x+y)^5 = 5C0.x^5 + 5C1x^4y + 5C2x^3y^2 + 5C3x^2y^3 + 5C4xy^4 + 5C5y^5

Solving:

= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5

CALCULATION OF BINOMIAL EXPANSION COEFFICIENTS

Coefficient of first and last term is always 1

Coefficient of any other term = (coefficient of previous term).(power of x from previous term)/number of that term

Example

First term = x^5

Last term = y^5

Second coefficient = 5/1 = 8

Third coefficient = 5*4/2 = 10

Fourth coefficient = 10*3/3= 10

Fifth coefficient = 10*2/4= 5

PROJECT DEVELOPMENT MANAGER'S PROBLEM

A toys manufacturer intends to start development of new product lines. A new toy is to be developed. Development of this toy is tied with a new TV series with the same name. There is 40% chance of TV series. The production in such a case is estimated at 12,000 units. The Profit per toy would be Rs. 2.

Without TV series-sale there may be demand for 2,000 units.

Already 500,000 Rs. has been invested.

A rival may bring to the market a similar toy. If so the sale may be 8000 units. The chance of rival bringing this toy to the market is 50%.

Choices:

The company has two choices:

- Abandon new product
- Risk new development

How should the company tie it all to financial results?

Sample space, Event

The set of collection of all possible outcomes of an experiment is called the sample space. Each possible outcome of an experiment is called event. Thus an event is a subset of the sample space.

For example all six faces of a die make a sample space. By rolling the dice, occurrence of number 1 is an event.

Probability

Probability is the numerical measure of the chance that an uncertain event will occur. The probability that the event A will occur is usually denoted by p(A). The probability of any event must be between zero and one inclusive.

For any event A
$0 \leq p(A) \leq 1$
$p(A) = 1$ means certain
$p(A) = 0$ means impossible.

PROBABILITY EXAMPLE 1
How can we make assessment of chances? Look at a simple example.
A worker out of 600 gets a prize by lottery.
What is the chance of any one individual say Rashid being selected?
Solution:
Chance of any one individual say Rashid being selected = 1/600
The probability of the event "Rashid is selected" is the probability of an event occurring=p(Rashid = 1/600)
This is a' priori method of finding probability as we can assess the probability before the event occurred.

PROBABILITY EXAMPLE 2
When all outcomes are equally likely a' priori probability is defined as:
p(event) = Number of ways that event can occur/Total number of possible outcomes
If out of 600 persons 250 are women, then the chance of a women being selected =
p(woman) = 250/600

PROBABILITY - EMPIRICAL APPROACH
In many situations, there is no prior knowledge to calculate probabilities.
What is the probability of a machine being defective?

Method:
1. Monitor the machine over a period of time.
2. Find out how many times it becomes defective.

This experimental or empirical approach

EXPERIMENTAL AND THEORETICAL PROBABILITY
p(event) = Number of times event occurs/Total number of experiments.
Larger the number of experiments, more accurate the estimate.
Experimental probability approaches theoretical probability as the number of experiments becomes very large**.**

OR RULE
Consider two events A and B.
What is the probability of either A or B happening?
What is the probability of A and B happening?
What is the number of possibilities?
Probability of A or B happening = Number of ways A or B can happen/ Total number of possibilities
= Number of ways A can happen + number of ways B can happen/ Total number of possibilities
Or
= Number of ways A can happen/ Total number of possibilities + Number of ways B can happen/ Total number of possibilities
= Probability of A happening + Probability of B happening

Condition for Or Rule
A and B must be mutually exclusive.
When A and B are mutually exclusive:
p(A or B) = p(A) + p(B)

OR RULE EXAMPLE

If a dice is thrown what is the chance of getting an **even number** or a number **divisible by three**?
p(even) = 3/6
p(div by 3) = 2/6
p(even or div by 3) = 3/6 + 2/6 = 5/6
The number 6 is not mutually exclusive.
Hence:
Correct answer = 4/6

AND RULE
Probability of A and B happening = Probability of A x Probability of B
Example
In a factory 40% workforce are women. Twenty five percent females are in management grade. Thirty percent males are in management grade. What is the probability that a worker selected is a women from management grade?
Solution
p(woman chosen) = 2/5
25% females = management grade
30% of males = management grade

p(woman & Management grade) = p(woman) x p(management)
Assume that the total workforce = 100
p(woman) = 0.4
p(management) = 0.25
p(woman) x p(management) = 0.4 x 0.25 = 0.1 or 10%
SET OF MUTUALLY EXCLUSIVE EVENTS
To cover all possibilities between mutually exclusive events add up all the probabilities.
Probabilities of all these events together add up to 1.
p(A) + p(B) + p(C) +....p(N) = 1

EXHAUSTIVE EVENTS
A happens or A does not happen then A and B are Exhaustive Events.
p(A happens) + A (does not happen) = 1

The sum of the probabilities of all mutually exclusive and collectively exhaustive events is always equal to 1. That is,

$$p(A) + p(B) + p(C) = 1$$

if A, B, C are mutually exclusive and collectively exhaustive events

Example 1
p(you pass) = 0.9
p(you fail) = 1 – 0.9 = 0.1

EXAMPLE1 - EXHAUSTIVE EVENTS
A production line uses 3 machines. The Chance that 1st machine breaks down in any week is 1/10. The Chance for 2nd machine is 1/20. Chance of 3rd machine is 1/40.What is the chance that at least one machine breaks down in any week?
Solution
p(at least one not working) + p(all three working) = 1
p(at least one not working) = 1- p(all three working)
p(all three working) = p(1st working) x p(2nd working) x p(3rd working)
p(1st working) = 1 - p(1st not working) = 1- 1/10 = 9/10
p(2nd working) = 19/20
p(3rd working) = 39/40
p(all working) 9/10 x 19/20 x 39/40 = 6669/8000
p(at least 1 working) = 1- 6669/8000 = 1331/8000

APPLICATION OF RULES
A firm has the following rules:
When a worker comes late there is ¼ chance that he is caught.
First time he is given a warning.
Second time he is dismissed.
What is the probability that a worker is late three times is not dismissed?
Solution
Let us use the denominations:
1C: Probability of being Caught first time
1NC: Probability of being Not Caught first time
2C: Probability of being Caught 2nd time
2NC: Probability of being Not Caught 2nd time
3C: Probability of being Caught 3rd time
3NC: Probability of being Not Caught 3rd time
Probabilities of different events can be calculated by applying the AND Rule.
1C(1/4) & 2C(1/4) (Dismissed 1) =(1/16 = 4/64)
1C(1/4) & 2NC(3/4) & 3C(1/4)(Dismissed 2)(3/64)
1C(1/4) & 2NC(3/4) & 3NC(3/4)(Not dismissed 1)(9/64)
1NC(3/4) & 2C(1/4) & 3C(1/4)(Dismissed 3)(3/64)
1NC(3/4) & 2C(1/4) & 3NC(3/4)(Not dismissed 2)(9/64)
1NC(3/4) & 2NC(3/4) & 3C(1/4)(Not dismissed 3)(9/64)
1NC(3/4) & 2NC(3/4) & 3NC(3/4)(Not dismissed 4)(27/64)
p(caught first time but not the second or third time) = ¼ x ¾ x ¾ = 9/64
p(caught only on second occasion) = ¾ x ¼ x ¾ = 9/64
p(late three times but not dismissed) = p(not dismissed 1) + p(not dismissed 2) + p(not dismissed 3) + p(not dismissed 4) = 9/64 + 9/64 + 9/64 + 27/64 = 54/64
p(caught) using OR Rule
p(caught) =
p(dismissed 1) + p(dismissed 2) + p(dismissed 3) = 4/64 + 3/64 + 3/64
= 10/64
p(caught) and p(not caught) using rule about Exhaustive events
p(not caught) = 1-p(not caught)
= 1 – 10/64
= 54/64

LECTURE 36
ELEMENTARY PROBABILITY
PART 2

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 35
- Elementary Probability

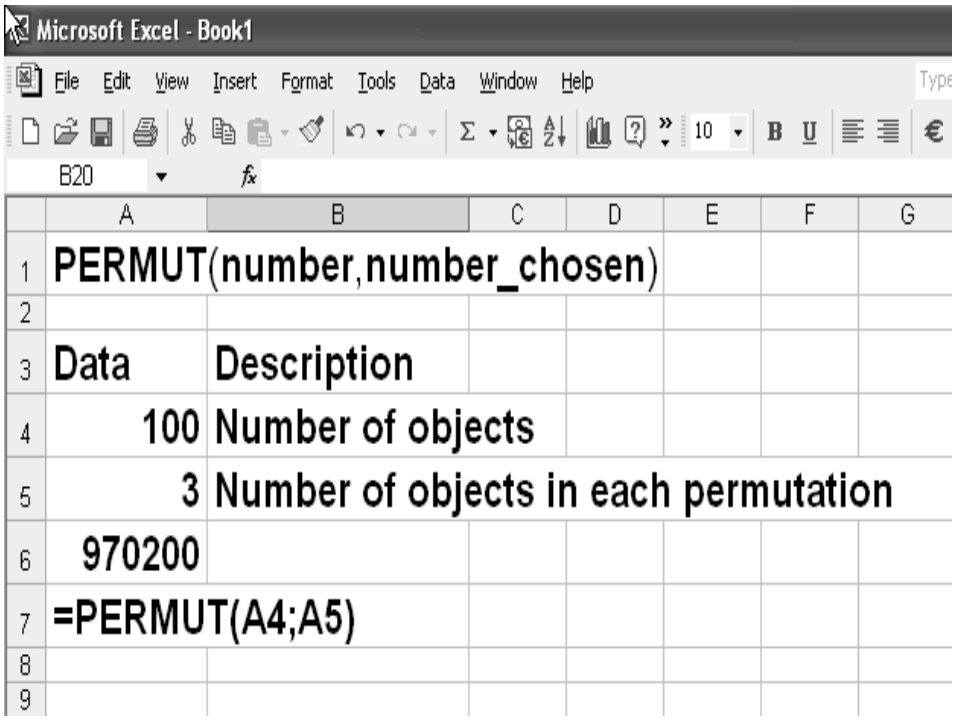
PROBABILITY CONCEPTS REVIEW

Most of the material on probability theory along with examples was included in the handout for lecture 35. You are advised to refer to handout 35. Some of the concepts and examples have been further elaborated in this handout.

Probability means making assessment of chances. The simplest example was the probability of Rashid getting the lottery when he was one of 600. The probability of the event was 1/600.

PERMUT EXAMPLE

In handout for lecture 35, we looked at the function PERMUT, that can be used for calculations of permutations. An example is shown in the slide below.



OR RULE REVIEW

When two events are mutually exclusive, the probability of either one of those occurring is the sum of individual probabilities. This is the OR Rule. This is a very extensively used rule.

A and B must be mutually exclusive. The formula for the OR rule is as under.

p(A or B) = p(A) + p(B)

Example

If a dice is thrown what is the chance of getting an odd number or a number divisble by three?

P(odd) = 3/6

p(div by 3) = 2/6

p(odd or div by 3) = 3/6 + 2/6 = 5/6

The number 6 is not mutually exclusive

Hence correct answer = 5/6

AND RULE REVIEW

The AND Rule requires that the events occur simultaneously**.**

Example

60% workforce are men.

p(man chosen) = 3/5

25% females = management grade

30% of males = management grade

What is the probability that a worker selected is a man from management grade?

Example

p(man & management grade) = p(man) x p(management)

Total workforce = 100

p(man) = 0.6

p(management) = 0.3

p(man) x p(management) = 0.6 x 0.3 = 0.18 or 18%

SET OF MUTUALLY EXCLUSIVE EVENTS REVIEW

Between them they cover all possibilities.Probabilities of all these events together add up to 1. Exhaustive Events are events that happen or do not happen.

p(it rains) = 0.9

p(it does not rain) = 1 – 0.9 = 0.1

Example

In Handout for lecture 35 we studied the problem of the three machines.

A production line uses 3 machines.

Chance that 1st machine breaks down in any week was 1/10. Chance for 2nd machine was 1/20. Chance of 3rd machine was 1/40. What is the chance that at least one machine breaks down in any week?

What are the probabilities?

Probability that one or two or three machines are not working (in other words at least one not working) and that all three areworking add up to 1 as exhaustive events.

P(at least one not working) + p(all three working) = 1

From the above, the probability that at least one is not working is worked out.

P(at least one not working) = 1- p(all three working)

Now to work out the probability that all three are working, we need to think in terms of machine 1 and machine 2 and machine 3 working. This means application of the AND Rule.

p(all three working) = p(1st working) x p(2nd working) x p(3rd working) Now the probability of machine 1 working is not known. The probability that machine 1 is not working is given. These two events (working and not working) are exhaustive events and add up to 1. Thus, the event that machine 1 is working, p(1st working), can be calculated as:

= 1 - p(1st not working) = 1- 1/10 = 9/10

The calculations for the other machines are:

p(2nd working) = 1-1/20 = 19/20

p(3rd working) = 1 - 1/40 = 39/40

Now the combined probability of p(all working) is a product of their individual probabilities using the AND Rule:

= 9/10 x 19/20 x 39/40 = 6669/8000

Finally P(at least 1 working or) = 1- 6669/8000 = 1331/8000

LECTURE 37
PATTERNS OF PROBABILITY: BINOMIAL, POISSON AND NORMAL DISTRIBUTIONS
PART 1

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 36
- Patterns of Probability: Binomial, Poisson and Normal Distributions

MODULE 7

Module 7 covers the following:
Factorials
Permutations and Combinations
(Lecture 34)
Elementary Probability
(Lectures 35- 36)
Patterns of probability: Binomial, Poisson and Normal Distributions
Part 1- 4
(Lectures 37- 40)

MODULE 8

Module 8 covers the following.
Estimating from Samples: Inference
(Lectures 41- 42)
Hypothesis Testing: Chi-Square Distribution (Lectures 43 - 44)
Planning Production Levels: Linear Programming (Lecture 45)
Assignment Module 7- 8
End-Term Examination

EXAMPLE 1

We covered in the past two lectures Elementary Probability. Most of the material was included in Handout 35. Some questions were discussed in detail in handout 36. In lecture 37, the example where the employee was warned on coming late and dismissed if late twice will be discussed. The material for this example is given in handout 35. Here we shall cover the main points and the method.
A firm has the following rules:
When a worker comes late there is ¼ chance that he is caught First time he is given a warning. Second time he is dismissed.
What is the probability that a worker is late three times is not dismissed?

Solution

How do we solve a problem of this nature? The answer is to develop the different options first. Let us see how it can be done.

First time

There are two options:
Caught: 1C
Not Caught: 1NC

2nd time

Caught: 2C
Not Caught: 2NC

3rd time

Caught: 3C
Not Caught: 3NC

Look at combinations up to 2nd stage

1C&2C
1C& 2NC
1NC& 2C
1NC& 2NC

Look at combinations up to 3rd stage

1C&2C&3C

1C & 2NC & 3C
1C & 2NC & 3NC
1NC & 2C & 3C
1NC & 2C & 3NC
1NC & 2NC & 3C
1NC & 2NC & 3NC

You saw that the first case is 1C & 2C. Here the employee was caught twice and was dismissed. He can not continue. Hence this case was closed here.
In other cases, the combinations were as given above.
Now the probability of being caught was ¼. As an exhaustive event the probability of not being caught was 1- ¼ = ¾.
Now the probabilities can be calculated as follows:
1C & 2C (1/4X1/4 = 1/16)
1C & 2NC & 3C (1/4X 3/4X1/4 = 3/64)
1C & 2NC & 3NC (1/4X3/4X3/4 = 9/64)
1NC & 2C & 3C (3/4x1/4x1/4 = 3/64)
1NC & 2C & 3NC (3/4x1/4X3/4 = 9/64)
1NC & 2NC & 3C (3/4x3/4x1/4 = 9/64)
1NC & 2NC & 3NC (3/4x3/4x3/4 = 27/64)
The probabilities for each combination of events are now summarized below:
First Caught, Second Caught, Dismissed**:**
1C (1/4) & 2C (1/4) (Dismissed 1) (1/16 = 4/64)
First caught, Second Not Caught, 3rd Caught, Dismissed**:**
1C (1/4) & 2NC (3/4) & 3C (1/4) (Dismissed 2) (3/64)
First caught, Second Not Caught, 3rd Not Caught, Not Dismissed
1C (1/4) & 2NC (3/4) & 3NC (3/4) (Not dismissed 1) (9/64)
First Not Caught, Second Caught, 3rd Caught, Dismissed
1NC (3/4) & 2C (1/4) & 3C (1/4) (Dismissed 3) (3/64)
First Not caught, Second Caught, 3rd Not Caught, Not Dismissed
1NC (3/4) & 2C (1/4) &3NC (3/4) (Not dismissed 2) (9/64)
First caught, Second Not Caught, 3rd Caught, Not Dismissed
1NC (3/4) & 2NC (3/4) & 3C (1/4) (Not dismissed 3) (9/64)
First caught, Second Not Caught, 3rd Not Caught, Not Dismissed
1NC (3/4) & 2NC (3/4) & 3NC (3/4) (Not dismissed 4) (27/64)

Probabilities

p(caught) =

The probability of being caught can be calculated by thinking that these are mutually events. All situations where there was a dismissal can be considered.
Probability(caught) =
p(dismissed 1) + p(dismissed 2) + p(dismissed 3) = 4/64 + 3/64 + 3/64
= 10/64

p(not caught) =

Once we have the probability of being caught we can find out the probability of not being caught as an exhaustive event. Thus:
p(not caught)
= 1- p(caught)
= 1 – 10/64
= 54/64

EXAMPLE 2
Two firms compete for contracts.
A has probability of ¾ of obtaining one contract.
B has probability of ¼.
What is the probability that when they bid for two contracts, firm A will obtain either the first or second contract?
Solution:
P(A gets first or A gets second) = ¾ + ¾ = 6/4
Wrong! Probability greater than 1!
We ignored the restriction: events must be mutually exclusive.
We are looking for probability that A gains the first or second or both.
We are not interested in B getting both the contracts
p(B gets first) x p(B gets both) = ¼ x ¼ = 1/16.
p(A gets one or both) = 1 - 1/16 = 15/16.

Alternative Method
Split "A gets first or the second or both" into 3 parts
A gets first but not second = ¾ x ¼ = 3/16
A does not get first but gets second = ¼ x ¾ = 3/16
A gets both = ¾ x ¾ = 9/16
P(A gets first or second or both) = 3/16 + 3/16 + 9/16 = 15/16

EXAMPLE 3
In a factory 40% workforce is female.
25% females belong to the management cadre.
30% males are from management cadre.
If management grade worker is selected, what is the probability that it is a female?
Draw up a table first.

	Male	Female	Total
Management	?	?	?
Non-Management	?	?	?
Total	?	40	100

Calculate
Total male = 100 – 40 = 60
Management female = 0.25 x 40 = 10
Non-Management female = 40 – 10 = 30
Management male = 0.3 x 60 = 18
Non-Management male = 60 – 18 = 42
Management total = 18 + 10 = 28
Non-Management total = 42 + 30 = 72
Summary

	Male	**Female**	**Total**
Management	**18**	**10**	**28**
Non-Management	**42**	**30**	**72**
Total	**60**	**40**	**100**

p(management grade worker is female) =10/28

EXAMPLE 4

A pie vendor has collected data over sale of pies. This data is organized as follows:

No. Pies sold		Income(X)	% Days(f)	fX Rs.
40	x 35 =	1400	20	28000
50		1750	20	35000
60		2100	30	63000
70		2450	20	49000
80		2800	10	28000
	Total		100	203000

Mean/day = 203000/100 = 2030

The selling price per pie was Rs. 35. What was the mean sale per day?

Such a question can be solved by calculating the sale in each slab and then dividing the total sale by number of pies.

% days is the probability. If multiplied with the income from each pie, the expected sale from all pies can be calculated. The overall expected value was 203,000. When divided by the number of days (100) an average of 2,030 Rs. Per day was obtained as average sale per day.

EXPECTED VALUE

EMV = ∑ (probability of outcome x financial result of outcome)

Example

In an insurance company 80% of the policies have no claim.
In 15% cases the Claim is 5000 Rs.
For the remaining 5% the Claim is 50000 Rs.
What is the Expected value of claim per policy?
Applying the formula above:
EMV = 0.8 x 0 + 0.15 x 5000 + 0.05 x 50000
= 0 + 750 + 2500
= 3250 Rs.

TYPICAL PRODUCTION PROBEM

In a factory producing biscuits, the packing machine breaks 1 biscuit out of twenty (p = 1/20 = 0.05).
What proportion of boxes will contain more than 3 broken biscuits?
This is a typical Binomial probability situation!
The individual biscuit is broken or not
= two possible outcomes

Conditions for Binomial Situation

1. Either or situation
2. Number of trials (n) known and fixed
3. Probability for success on each trial (p) is known and fixed

CUMULATIVE BINOMIAL PROBABILITIES

The Cumulative Probability table gives the probability of r or more successes in n trials, with the probability p of success in one trial
In the table:
The total number of trials n = 1 to 10
The number of successes r = 1 to 10
The probability p = 0.05, 0.1, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5

LECTURE 38
PATTERNS OF PROBABILITY: BINOMIAL, POISSON AND NORMAL DISTRIBUTIONS
PART 2

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 37
- Patterns of Probability: Binomial, Poisson and Normal Distributions

CUMULATIVE BINOMIAL PROBABILITIES

Probability of r or more successes in n trials with the probability of success in each trial

- Look in column for n
- Look in column for r
- Look at column for value of p(0.05 to 0.5)

In other words a cumulative binomial probability refers to the probability that the binomial random variable falls within a specified range (e.g., is greater than or equal to a stated lower limit and less than or equal to a stated upper limit).
For example, we might be interested in the cumulative binomial probability of obtaining 45 or fewer heads in 100 tosses of a coin (see Example 1 below). This would be the sum of all these individual binomial probabilities.

$b(x \leq 45; 100, 0.5) = b(x = 0; 100, 0.5) + b(x = 1; 100, 0.5) + \ldots + b(x = 44; 100, 0.5) + b(x = 45; 100, 0.5)$

You can calculate these values by using the formula $P(x \leq c) = \sum_{c=0}^{x} \binom{n}{x} P^x (1-P)^{n-x}$

Or directly from the table.

Example:

The probability that a student is accepted to a prestigious college is 0.3. If 5 students from the same school apply, what is the probability that at most 2 are accepted?

Solution:

To solve this problem, we compute 3 individual probabilities,

Using the binomial formula. The sum of all these probabilities is the answer we seek. Thus,

$b(x \leq 2; 5, 0.3) = 0.837$

Table: Cumulative Binomial probabilities

$$P[X \le c] = \sum_{x=0}^{c} \binom{n}{x} p^x (1-p)^{n-x}$$

		p										
	c	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95
n = 1	0	0.950	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100	0.050
	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 2	0	0.903	0.810	0.640	0.490	0.360	0.250	0.160	0.090	0.040	0.010	0.003
	1	0.998	0.990	0.960	0.910	0.840	0.750	0.640	0.510	0.360	0.190	0.098
	2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 3	0	0.857	0.729	0.512	0.343	0.216	0.125	0.064	0.027	0.008	0.001	0.000
	1	0.993	0.972	0.896	0.784	0.648	0.500	0.352	0.216	0.104	0.028	0.007
	2	1.000	0.999	0.992	0.973	0.936	0.875	0.784	0.657	0.488	0.271	0.143
	3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 4	0	0.815	0.656	0.410	0.240	0.130	0.063	0.026	0.008	0.002	0.000	0.000
	1	0.986	0.948	0.819	0.652	0.475	0.313	0.179	0.084	0.027	0.004	0.000
	2	1.000	0.996	0.973	0.916	0.821	0.688	0.525	0.348	0.181	0.052	0.014
	3	1.000	1.000	0.998	0.992	0.974	0.938	0.870	0.760	0.590	0.344	0.185
	4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 5	0	0.774	0.590	0.328	0.168	0.078	0.031	0.010	0.002	0.000	0.000	0.000
	1	0.977	0.919	0.737	0.528	0.337	0.188	0.087	0.031	0.007	0.000	0.000
	2	0.999	0.991	0.942	0.837	0.683	0.500	0.317	0.163	0.058	0.009	0.001
	3	1.000	1.000	0.993	0.969	0.913	0.813	0.663	0.472	0.263	0.081	0.023

BINOMDIST

Returns the individual term binomial distribution probability. Use BINOMDIST in problems with a fixed number of tests or trials, when the outcomes of any trial are only success or failure, when trials are independent, and when the probability of success is constant throughout the experiment. For example, BINOMDIST can calculate the probability that two of the next three babies born are male.

Syntax

BINOMDIST(**number_s**,**trials**,**probability_s**,**cumulative**)

Number_s is the number of successes in trials.

Trials is the number of independent trials.

Probability_s is the probability of success on each trial.

Cumulative is a logical value that determines the form of the function. If cumulative is TRUE, then BINOMDIST returns the cumulative distribution function, which is the probability that there are at most number_s successes; if FALSE, it returns the probability mass function, which is the probability that there are number_s successes.

Remarks

Number_s and trials are truncated to integers.

If number_s, trials, or probability_s is nonnumeric, BINOMDIST returns the #VALUE! error value.

If number_s < 0 or number_s > trials, BINOMDIST returns the #NUM! error value.

If probability_s < 0 or probability_s > 1, BINOMDIST returns the #NUM! error value.

The binomial probability mass function is:

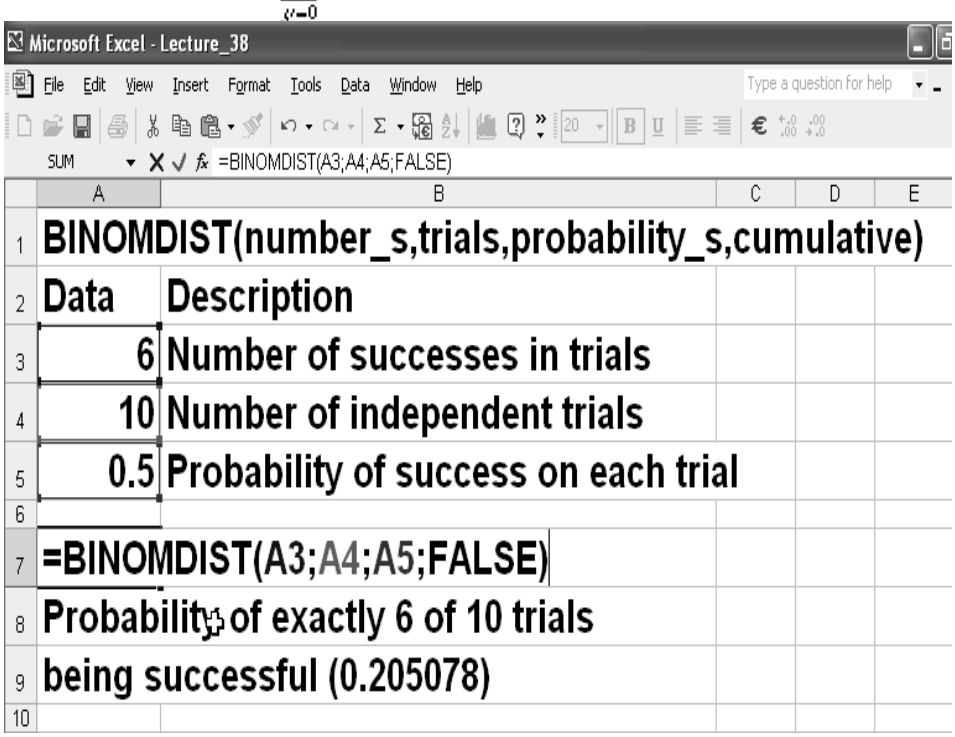
$$b(x;n,p) = \binom{n}{x} p^x (1-p)^{n-x}$$

where:

$$\binom{n}{x},$$

The cumulative binomial distribution is:

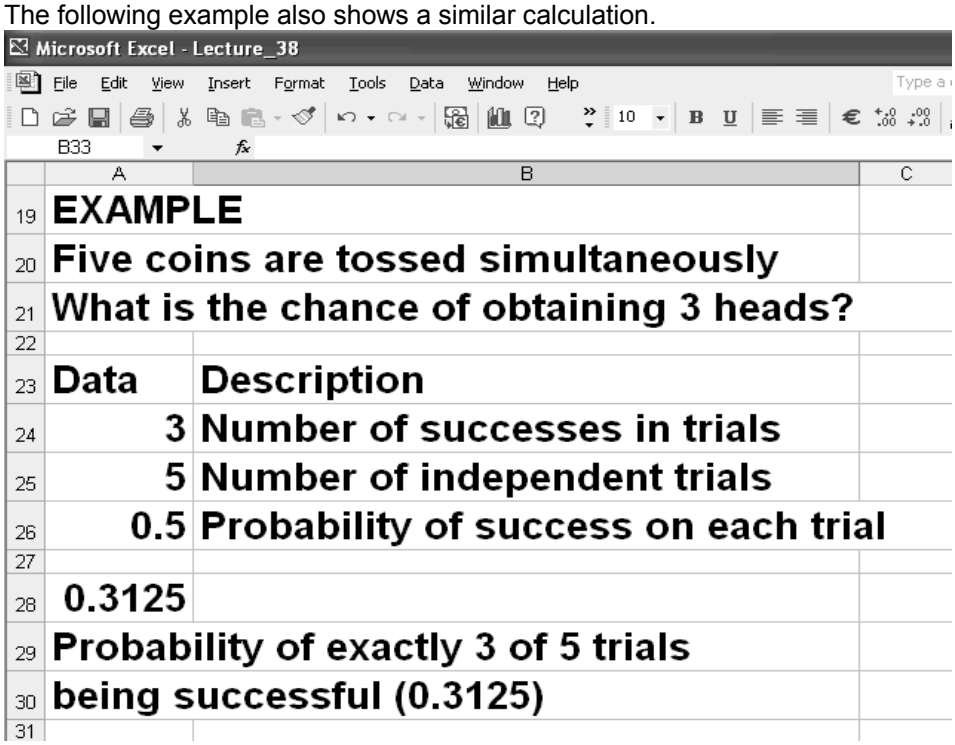
$$B(x;n,p)=\sum_{y=0}^{x} b(y;n,p)$$



Example

In the above example, the BINOMDIST function was used to calculate the probability of exact 6 out of 10 trials being successful. Here the value of Cumulative was set as False.

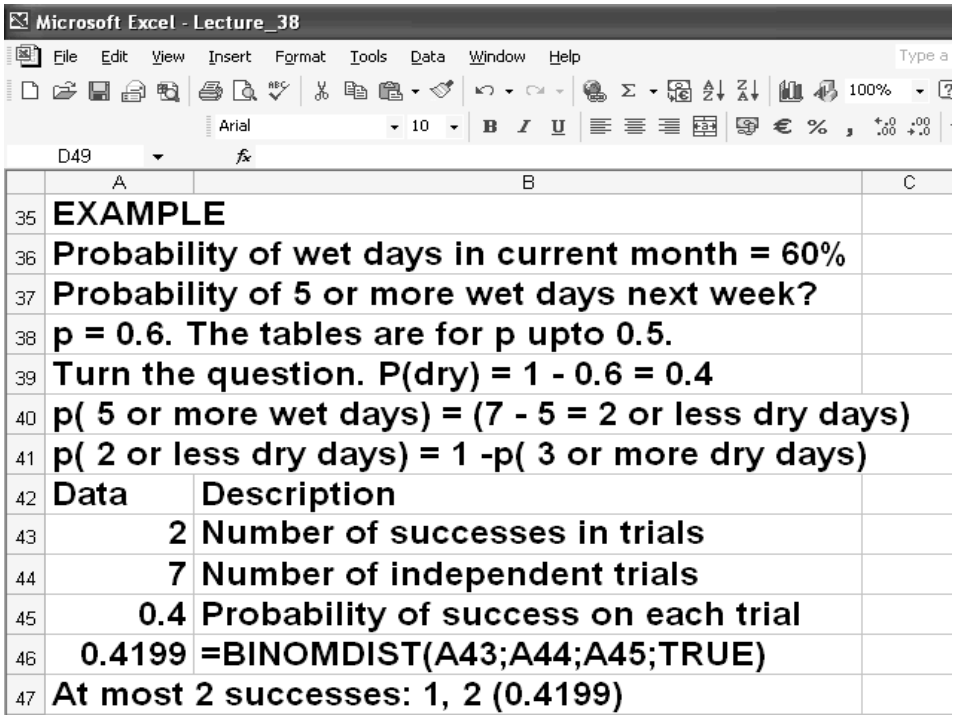
The following example also shows a similar calculation.



EXAMPLE 1

The probability of wet days is 60%. Note that the figure 0.6 is beyond the maximum value 0.5 as given in the tables. Let us first convert our problem to p(dry) = 1 - 0.6 = 0.4. Now p(5 or more wet days) can be restated as p(2 or less dry days). The BINOMDIST function is for p(r or more). Let us convert p(2 or less dry days) to 1 – p(3 or more days). Now the value of n = 7, r = 3 and p = 0.4.

Using BINOMDIST, the answer is 0.4199. Note that the value of cumulative was TRUE.



	A	B	C
35	EXAMPLE		
36	Probability of wet days in current month = 60%		
37	Probability of 5 or more wet days next week?		
38	p = 0.6. The tables are for p upto 0.5.		
39	Turn the question. P(dry) = 1 - 0.6 = 0.4		
40	p(5 or more wet days) = (7 - 5 = 2 or less dry days)		
41	p(2 or less dry days) = 1 -p(3 or more dry days)		
42	Data	Description	
43	2	Number of successes in trials	
44	7	Number of independent trials	
45	0.4	Probability of success on each trial	
46	0.4199	=BINOMDIST(A43;A44;A45;TRUE)	
47	At most 2 successes: 1, 2 (0.4199)		

EXAMPLE 2

In a transmission where 8 bit message is transmitted electronically there is 10% probability of one bit being transmitted erroneously? What is the chance that entire message is transmitted correctly)?

We can state that the probability required is for 0 successes (errors) in 8 trials (bits).

p(one bit transmitted erroneously) = 0.1

$$P(x=0;\ 8;\ p=0.1) = 0.430$$

For exact binomial distribution

$$P(x=n) = \binom{n}{x} P^x (1-P)^{n-x} = \binom{8}{0} 0.1^0 (1-0.1)^{8-0} = 0.430$$

Table: Cumulative Binomial probabilities *(continued)*

		p										
	c	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95
n = 8	0	0.663	0.430	0.168	0.058	0.017	0.004	0.001	0.000	0.000	0.000	0.000
	1	0.943	0.813	0.503	0.255	0.106	0.035	0.009	0.001	0.000	0.000	0.000
	2	0.994	0.962	0.797	0.552	0.315	0.145	0.050	0.011	0.001	0.000	0.000
	3	1.000	0.995	0.944	0.806	0.594	0.363	0.174	0.058	0.010	0.000	0.000
	4	1.000	1.000	0.990	0.942	0.826	0.637	0.406	0.194	0.056	0.005	0.000
	5	1.000	1.000	0.999	0.989	0.950	0.855	0.685	0.448	0.203	0.038	0.006
	6	1.000	1.000	1.000	0.999	0.991	0.965	0.894	0.745	0.497	0.187	0.057
	7	1.000	1.000	1.000	1.000	0.999	0.996	0.983	0.942	0.832	0.570	0.337
	8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 9	0	0.630	0.387	0.134	0.040	0.010	0.002	0.000	0.000	0.000	0.000	0.000
	1	0.929	0.775	0.436	0.196	0.071	0.020	0.004	0.000	0.000	0.000	0.000
	2	0.992	0.947	0.738	0.463	0.232	0.090	0.025	0.004	0.000	0.000	0.000
	3	0.999	0.992	0.914	0.730	0.483	0.254	0.099	0.025	0.003	0.000	0.000
	4	1.000	0.999	0.980	0.901	0.733	0.500	0.267	0.099	0.020	0.001	0.000
	5	1.000	1.000	0.997	0.975	0.901	0.746	0.517	0.270	0.086	0.008	0.001
	6	1.000	1.000	1.000	0.996	0.975	0.910	0.768	0.537	0.262	0.053	0.008
	7	1.000	1.000	1.000	1.000	0.996	0.980	0.929	0.804	0.564	0.225	0.071

8.50 x 11.00 in 2 of 7

Using BINOMDIST

The data was for 0 or more successes. BINOMDIST function gives the value for at most r successes. Hence the answer was obtained directly.

Microsoft Excel - Lecture_38

File Edit View Insert Format Tools Data Window Help

B64

	A	B	C	D
51	EXAMPLE			
52	Probability of 1 erroneous bit = 0.1			
53	Probability of 8 correct bits? = 0 erroneous bits			
54	Data	Description		
55	0	Number of successes in trials		
56	8	Number of independent trials		
57	0.1	Probability of success on each trial		
58	0.4305	=BINOMDIST(B55;B56;B57;TRUE)		
59	At most 0 successes: (0.4305)			
60				

EXAMPLE 3

A surgery is successful for 75% patients. What is the probability of its success in at least 7 cases out of randomly selected 9 patients?

p(success in at least 7 cases in randomly selected 9 patients)?

Here

n = 9; p(success) = 0.75; p(at lease 7 cases)?

p = 0.75 is outside the table

Let us invert the problem.

p(failure) = 1 –0.75 = 0.25

Success at least 7 = Failure 2 or less

P(x>=7; n=9;p=0.75) = 1- p(x<7; n=9;p=0.75) = 1 –p(x<=6;n=9;p=0.75)
= 1- 0.3995
= 0.6005
=60%

		p										
	c	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95
n = 8	0	0.663	0.430	0.168	0.058	0.017	0.004	0.001	0.000	0.000	0.000	0.000
	1	0.943	0.813	0.503	0.255	0.106	0.035	0.009	0.001	0.000	0.000	0.000
	2	0.994	0.962	0.797	0.552	0.315	0.145	0.050	0.011	0.001	0.000	0.000
	3	1.000	0.995	0.944	0.806	0.594	0.363	0.174	0.058	0.010	0.000	0.000
	4	1.000	1.000	0.990	0.942	0.826	0.637	0.406	0.194	0.056	0.005	0.000
	5	1.000	1.000	0.999	0.989	0.950	0.855	0.685	0.448	0.203	0.038	0.006
	6	1.000	1.000	1.000	0.999	0.991	0.965	0.894	0.745	0.497	0.187	0.057
	7	1.000	1.000	1.000	1.000	0.999	0.996	0.983	0.942	0.832	0.570	0.337
	8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
n = 9	0	0.630	0.387	0.134	0.040	0.010	0.002	0.000	0.000	0.000	0.000	0.000
	1	0.929	0.775	0.436	0.196	0.071	0.020	0.004	0.000	0.000	0.000	0.000
	2	0.992	0.947	0.738	0.463	0.232	0.090	0.025	0.004	0.000	0.000	0.000
	3	0.999	0.992	0.914	0.730	0.483	0.254	0.099	0.025	0.003	0.000	0.000
	4	1.000	0.999	0.980	0.901	0.733	0.500	0.267	0.099	0.020	0.001	0.000
	5	1.000	1.000	0.997	0.975	0.901	0.746	0.517	0.270	0.086	0.008	0.001
	6	1.000	1.000	1.000	0.996	0.975	0.910	0.768	0.537	0.262	=0.3995	0.008
	7	1.000	1.000	1.000	1.000	0.996	0.980	0.929	0.804 2	0.564		0.071
	8	1.000	1.000	1.000	1.000	1.000	0.998	0.990	0.960	0.866	0.613	0.370

Calculation using BINOMDIST

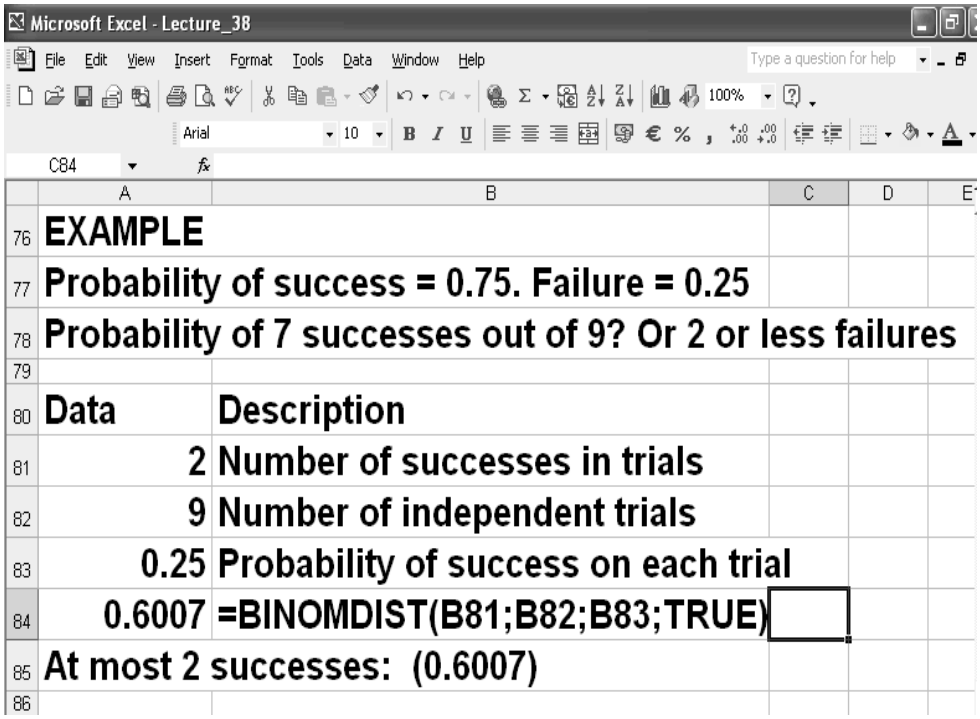
Here the question was inverted.

We had to find 7 successes out of 9. The probability was 75% for success. It becomes 25% for failure. Now let us restate the problem in terms of failure.

We are interested in 7 or more successes. It means 2 or less failures.

Now the BINOMDIST function gives us at most r successes. In other words 2 or less.

Hence if we specify r = 2, we get the answer 0.6007 directly.



NEGATIVE BINOMIAL DISTRIBUTION

A **negative binomial experiment** is a statistical experiment that has the following properties:

- The experiment consists of *x* repeated trials.
- Each trial can result in just two possible outcomes. We call one of these outcomes a success and the other, a failure.
- The probability of success, denoted by *P*, is the same on every trial.
- The trials are independent; that is, the outcome on one trial does not affect the outcome on other trials.
- The experiment continues until *r* successes are observed, where *r* is specified in advance.

Consider the following statistical experiment. You flip a coin repeatedly and count the number of times the coin lands on heads. You continue flipping the coin until it has landed 5 times on heads. This is a negative binomial experiment because:

- The experiment consists of repeated trials. We flip a coin repeatedly until it has landed 5 times on heads.
- Each trial can result in just two possible outcomes - heads or tails.
- The probability of success is constant - 0.5 on every trial.
- The trials are independent; that is, getting heads on one trial does not affect whether we get heads on other trials.
- The experiment continues until a fixed number of successes have occurred; in this case, 5 heads.

Negative Binomial Formula. Suppose a negative binomial experiment consists of *x* trials and results in *r* successes. If the probability of success on an individual trial is *P*, then the negative binomial probability is:

$$b^*(x; r, P) = {}_{x-1}C_{r-1} * P^r * (1 - P)^{x - r}$$

Example

Bob is a high school basketball player. He is a 70% free throw shooter. That means his probability of making a free throw is 0.70. During the season, What is the probability that Bob makes his first free throw on his fifth shot?

Solution:

This is an example of a geometric distribution, which is a special case of a negative binomial distribution. Therefore, this problem can be solved using the negative binomial formula or the geometric formula. We demonstrate each approach below, beginning with the negative binomial formula.

The probability of success (*P*) is 0.70, the number of trials (*x*) is 5, and the number of successes (*r*) is 1. We enter these values into the negative binomial formula.

$$b^*(x; r, P) = {}_{x-1}C_{r-1} * P^r * Q^{x-r}$$

$$b^*(5; 1, 0.7) = {}_4C_0 * 0.7^1 * 0.3^4$$

$$b^*(5; 3, 0.7) = 0.00567$$

NEGBINOMDIST

Returns the negative binomial distribution. NEGBINOMDIST returns the probability that there will be number_f failures before the number_s-th success, when the constant probability of a success is probability_s. This function is similar to the binomial distribution, except that the number of successes is fixed, and the number of trials is variable. Like the binomial, trials are assumed to be independent.

For example, you need to find 10 people with excellent reflexes, and you know the probability that a candidate has these qualifications is 0.3. NEGBINOMDIST calculates the probability that you will interview a certain number of unqualified candidates before finding all 10 qualified candidates.

Syntax

NEGBINOMDIST(**number_f**,**number_s**,**probability_s**)

Number_f is the number of failures.

Number_s is the threshold number of successes.

Probability_s is the probability of a success.

Remarks

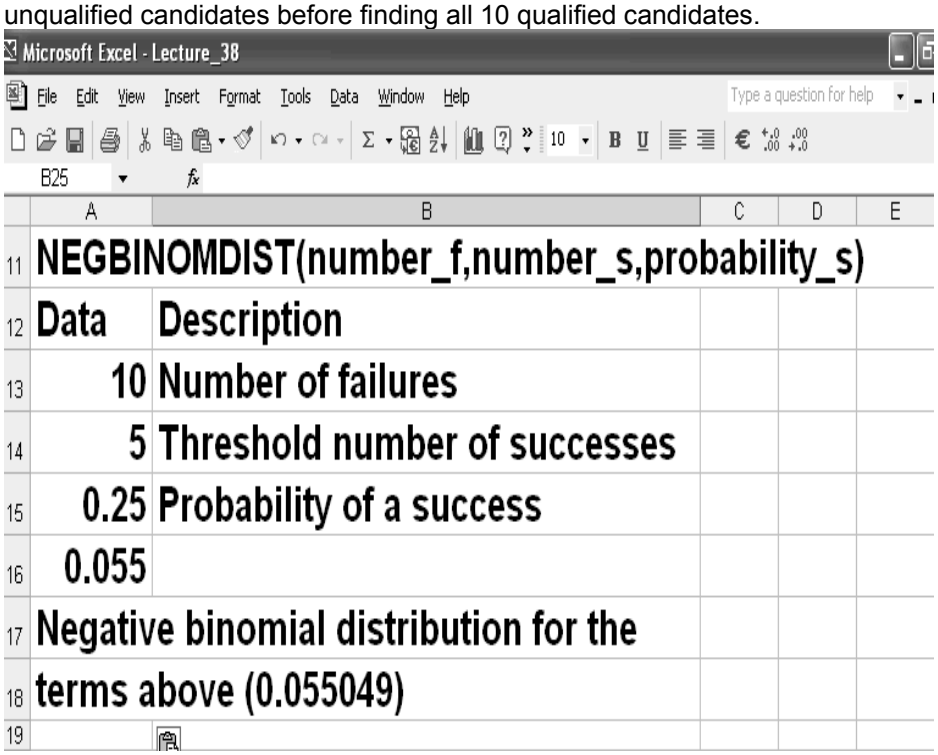
- **Number_f** and **number_s** are truncated to integers.
- If any argument is nonnumeric, NEGBINOMDIST returns the #VALUE! error value.
- If probability_s < 0 or if probability > 1, NEGBINOMDIST returns the #NUM! error value.
- If (number_f + number_s - 1) ≤ 0, NEGBINOMDIST returns the #NUM! error value.
- The equation for the negative binomial distribution is:

$$nb(x; r, p) = \binom{x + r - 1}{r - 1} p^r (1 - p)^x$$

where:
x is number_f, r is number_s, and p is probability_s.

NEGBINOMDIST- EXAMPLE

You need to find 10 people with excellent reflexes, and you know the probability that a candidate has these qualifications is 0.3
NEGBINOMDIST calculates the probability that you will interview a certain number of unqualified candidates before finding all 10 qualified candidates.



CRITBINOM

Returns the smallest value for which the cumulative binomial distribution is greater than or equal to a criterion value. Use this function for quality assurance applications. For example, use CRITBINOM to determine the greatest number of defective parts that are allowed to come off an assembly line run without rejecting the entire lot.

Syntax

CRITBINOM(**trials**,**probability_s**,**alpha**)

Trials is the number of Bernoulli trials.

Probability_s is the probability of a success on each trial.

Alpha is the criterion value.

Remarks

If any argument is nonnumeric, CRITBINOM returns the #VALUE! error value.
If trials is not an integer, it is truncated.
If trials < 0, CRITBINOM returns the #NUM! error value.
If probability_s is < 0 or probability_s > 1, CRITBINOM returns the #NUM! error value.
If alpha < 0 or alpha > 1, CRITBINOM returns the #NUM! error value.

Example

	A	B
1	**Data**	**Description**
2	6	Number of Bernoulli trials
3	0.5	Probability of a success on each trial
4	0.75	Criterion value
	Formula	**Description (Result)**
	=CRITBINOM(A2,A3,A4)	Smallest value for which the cumulative binomial distribution is greater than or equal to a criterion value (4)

LECTURE 39
PATTERNS OF PROBABILITY: BINOMIAL, POISSON AND NORMAL DISTRIBUTIONS
PART 3

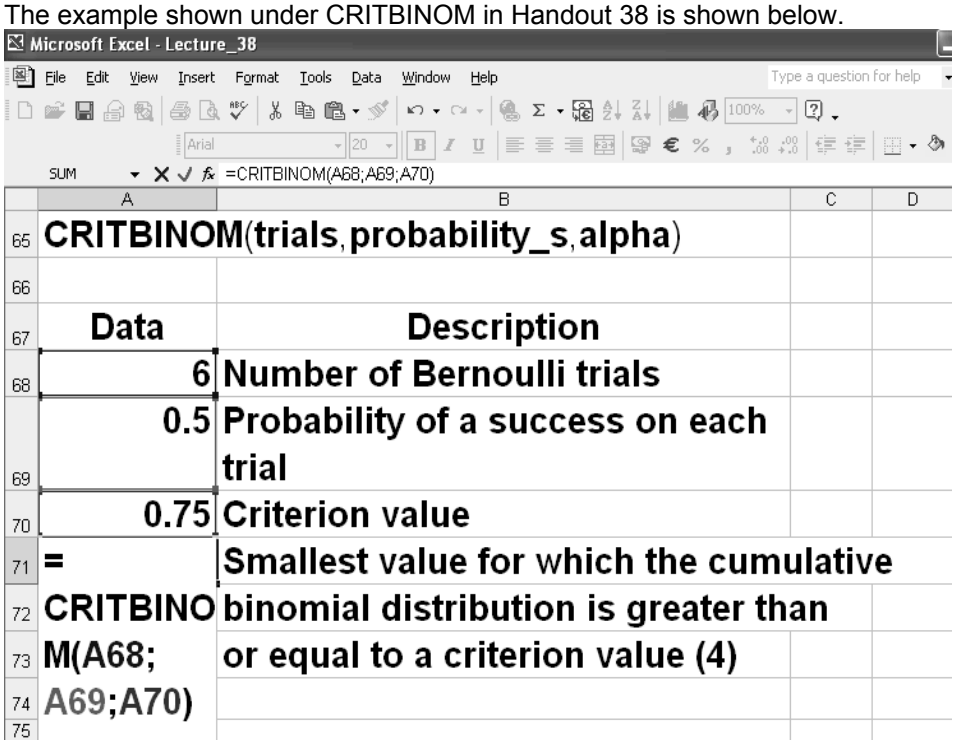
OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 38
- Patterns of Probability: Binomial, Poisson and Normal Distributions

CRITBINOM EXAMPLE

The example shown under CRITBINOM in Handout 38 is shown below.



	A	B	C	D
65	CRITBINOM(trials,probability_s,alpha)			
66				
67	Data	Description		
68	6	Number of Bernoulli trials		
69	0.5	Probability of a success on each trial		
70	0.75	Criterion value		
71	=	Smallest value for which the cumulative		
72	CRITBINO	binomial distribution is greater than		
73	M(A68;	or equal to a criterion value (4)		
74	A69;A70)			
75				

EXPECTED VALUE EXAMPLE

A lottery has 100 Rs. Payout on average 20 turns.
Is it worthwhile to buy the lottery if the ticket price is 10 Rs.?
Expected win per turn = p(winning) x gain per win + p(losing) x loss if you loose
= 1/20 x (100 – 10) + 19/20 x (- 10) Rs.
= 90/20 –190/20 Rs.
= 4.5 - 9.5 = - 5 Rs.
So on an average you stand to loose 5 Rs.

DECISION TABLES

Look at the data in the table below:

No. of Pies demanded	% Occasions
25	10
30	20
35	25
40	20
45	15
50	10

Price per pie = Rs. 15
Refund on return = Rs. 5
Sale price = Rs. 25
Profit per pie = Rs. 25 – 15 = Rs. 10
Loss on each return = Rs. 15 – 5 = Rs. 10
How many pies should be bought for best profit?
To solve such a problem, a decision table is set up as shown below. The values in the first column are number of pies to be purchased. Figures in columns are the sale with % share of sale within brackets. If the number of pies bought is less than the number that can be sold, the number of pies sold remains constant. If the number of pies bought exceeds the number of pies sold then the remaining are returned. This means a loss. For every value the sum of profit for sale and loss for pies returned is calculated.
The average sale for each row is calculated by multiplying the profit for each sale with % sale in the column. An example calculation is given as a guide for 30 pies.

DECISION TABLES

	25(0.1)	30(0.2)	35(0.25)	40(0.2)	45(0.15)	50(0.1)	EMV
25	250	250	250	250	250	250	250
30	200	300	300	300	300	300	290
35	150	250	350	350	350	350	310
Buy							
40	100	200	300	400	400	400	305
45	50	150	250	350	450	450	280
		0	100	200	300	400	500	240

Expected profit 30 pies
= 0.1 x 200 + 0.2 x 300 + 0.25 x 300 + 0.2 x 300 + 0.15 x 300 + 0.1 x 300
= 20 + 60 + 75 + 60 + 45 + 30
= 290 Rs.
Best Profit
It may be noted that the best profit is for 35 Pies = Rs. 310

DECISION TREE TOY MANUFACTURING CASE
The problem of the manufacturer intending to start manufacturing a new toy under the conditions that the TV series may or may not appear, that the rival may or may not sell a similar toy is now solved below.
Here a Decision tree has been developed with the possible branches as shown below. Each sequence represents an application of the AND rule.
1A Abandon
1B Go ahead >2A: Series appears (60%)
>2B: No series (40%)
>2A>3A: Rival markets (50%)
>2A>3B: No Rival (50%)

Production
Series, no rival = 12000 units
Series, rival = 8000 units
No series = 2000 units
Investment = Rs. 500000
Profit per unit = Rs. 200
Loss if abandon = Rs. 500000
What is the best course of action?

Decision Tree

Profit if rival markets, series appears = 8000 x 200 – 500000 = 1600000 – 500000 = 1100000 Rs.
Profit if no rivals = 12000 x 200 – 500000 = 2400000 – 500000 = 1900000 Rs.
Profit/Loss if no series = 2000 x 200 – 500000 = 400000 – 500000 = -100000 Rs. (No series)
EMV = Rival markets and no rivals = 0.5 x 1100000 + 0.5 x 1900000 = 1500000 (Series)
EMV = 0.6 x 1500000 + 0.4 x – 100000 = 900000 – 40000 = 860000 Rs.

Conclusion

It is clear that in spite of the uncertainty, there is a likelihood of a reasonable profit.
Hence the conclusion is:
Go ahead

THE POISSON DISTRIBUTION

The Poisson distribution is most commonly used to model the number of random occurrences of some phenomenon in a specified unit of space or time. For example,

- The number of phone calls received by a telephone operator in a 10-minute period.
- The number of flaws in a bolt of fabric.
- The number of typos per page made by a secretary.

It has the following characteristics:
Either or situation
No data on trials
No data on successes
Average or mean value of successes or failures
This is a typical Poisson Situation.

Characteristics

- Either/or situation
- Mean number of successes per unit, m, known and fixed
- p, chance, unknown but small, (event is unusual)

For a Poisson random variable, the probability that X is some value x is given by the formula

$$P(X = x) = \frac{\mu^x e^{-\mu}}{x!}, \quad x = 0,1,...$$

where μ is the average number of occurrences in the specified interval. For the Poisson distribution,

$$E(X) = \mu, \quad Var(X) = \mu$$

Example:

The number of false fire alarms in a suburb of Houston averages 2.1 per day. Assuming that a Poisson distribution is appropriate, the probability that 4 false alarms will occur on a given day is given by

$$P(X=4)=\frac{2.1^4 e^{-2.1}}{4!}=0.0992$$

THE POISSON TABLES OF PROBABILITIES

Gives cumulative probability of r or more successes
Knowledge of m is required.
Table gives the probability of that r or more random events are contained in an interval when the average number of events per interval is m

Example 2

Attendance in a factory shows 7 absences.
What is the probability that on a given day there will be more than 8 people absent?

Solution

Method 1

$$P(X>8)=1-P(X\le 8)=1-[P(x=1)+P(x=2)+P(x=3)+P(x=4)+P(x=5)+P(x=6)+P(x=7)+P(x=8)]$$

$$=1-[\frac{7^1e^{-7}}{1!}+\frac{7^2e^{-7}}{2!}+\frac{7^3e^{-7}}{3!}+\frac{7^4e^{-7}}{4!}+\frac{7^5e^{-7}}{5!}+\frac{7^6e^{-7}}{6!}+\frac{7^7e^{-7}}{7!}+\frac{7^8e^{-7}}{8!}]$$

$$=1-[0.0064+0.0223+0.0521+0.0912+0.1277+0.149+0.149+0.1304]$$

$$=0.2709$$

Method 2

$$P(X>8)=1-P(X\le 8)=1-0.7291=0.2709$$

p(9 or more successes) = 0.2709

Cumulative Poisson Distribution Table

Table shows cumulative probability functions of Poisson Distribution with various α. Example: to find the probability $P(X \leq 3)$ where X has a Poisson Distribution with $\alpha = 2$, look in row 4 and column 4 to find $P(X \leq 3)$=0.8571 where X is Poisson(2).

x	α 0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
0	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067
1	0.9098	0.7358	0.5578	0.4060	0.2873	0.1991	0.1359	0.0916	0.0611	0.0404
2	0.9856	0.9197	0.8088	0.6767	0.5438	0.4232	0.3208	0.2381	0.1736	0.1247
3	0.9982	0.9810	0.9344	0.8571	0.7576	0.6472	0.5366	0.4335	0.3423	0.2650

x	α 5.5	6	6.5	7	7.5	8	8.5	9	9.5	10
0	0.0041	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001	0.0000
1	0.0266	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008	0.0005
3	0.0884	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042	0.0028
3	0.2017	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149	0.0103
4	0.3575	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744	0.0550	0.0403	0.0293
5	0.5289	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885	0.0671
6	0.6860	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649	0.1301
7	0.8095	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687	0.2202
8	0.8944	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918	0.3328
9	0.9462	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218	0.4579
10	0.9747	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453	0.5830
11	0.9890	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.8030	0.7520	0.6968
12	0.9955	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364	0.7916
13	0.9983	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981	0.8645
14	0.9994	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400	0.9165
15	0.9998	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665	0.9513
16	0.9999	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823	0.9730

Example 3

An automatic production line breaks down every 2 hours.
Special production requires uninterrupted operation for 8 hours.
What is the probability that this can be achieved?

Solution

α = 8/2 = 4

x = 0 (no breakdown)

p(x=0) = $\frac{4^0 e^0}{0!}$ = 0.0183 =1.83%

From Table

Cumulative Poisson Distribution Table

Table shows cumulative probability functions of Poisson Distribution with various α. Example: to find the probability $P(X \leq 3)$ where X has a Poisson Distribution with $\alpha = 2$, look in row 4 and column 4 to find $P(X \leq 3)$=0.8571 where X is Poisson(2).

	α									
x	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
0	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067
1	0.9098	0.7358	0.5578	0.4060	0.2873	0.1991	0.1359	0.0916	0.0611	0.0404
2	0.9856	0.9197	0.8088	0.6767	0.5438	0.4232	0.3208	0.2381	0.1736	0.1247
3	0.9982	0.9810	0.9344	0.8571	0.7576	0.6472	0.5366	0.4335	0.3423	0.2650
4	0.9998	0.9963	0.9814	0.9473	0.8912	0.8153	0.7254	0.6288	0.5321	0.4405
5	1.0000	0.9994	0.9955	0.9834	0.9580	0.9161	0.8576	0.7851	0.7029	0.6160
6	1.0000	0.9999	0.9991	0.9955	0.9858	0.9665	0.9347	0.8893	0.8311	0.7622
7	1.0000	1.0000	0.9998	0.9989	0.9958	0.9881	0.9733	0.9489	0.9134	0.8666

Example 4

An automatic packing machine produces on an average one in 100 underweight bags.

What is the probability that 500 bags contain less than three underweight bags?

Solution

m = 1 x 500/100 = 5

p(x<3, α =5) = p(x<=2, α =5) = 0.1247=12.47%

Cumulative Poisson Distribution Table

Table shows cumulative probability functions of Poisson Distribution with various α. Example: to find the probability $P(X \leq 3)$ where X has a Poisson Distribution with $\alpha = 2$, look in row 4 and column 4 to find $P(X \leq 3)$=0.8571 where X is Poisson(2).

	α									
x	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
0	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067
1	0.9098	0.7358	0.5578	0.4060	0.2873	0.1991	0.1359	0.0916	0.0611	0.0404
2	0.9856	0.9197	0.8088	0.6767	0.5438	0.4232	0.3208	0.2381	0.1736	0.1247
3	0.9982	0.9810	0.9344	0.8571	0.7576	0.6472	0.5366	0.4335	0.3423	0.2650
4	0.9998	0.9963	0.9814	0.9473	0.8912	0.8153	0.7254	0.6288	0.5321	0.4405
5	1.0000	0.9994	0.9955	0.9834	0.9580	0.9161	0.8576	0.7851	0.7029	0.6160
6	1.0000	0.9999	0.9991	0.9955	0.9858	0.9665	0.9347	0.8893	0.8311	0.7622
7	1.0000	1.0000	0.9998	0.9989	0.9958	0.9881	0.9733	0.9489	0.9134	0.8666
8	1.0000	1.0000	1.0000	0.9998	0.9989	0.9962	0.9901	0.9786	0.9597	0.9319
9	1.0000	1.0000	1.0000	1.0000	0.9997	0.9989	0.9967	0.9919	0.9829	0.9682

LECTURE 40
PATTERNS OF PROBABILITY: BINOMIAL, POISSON AND NORMAL DISTRIBUTIONS
PART 4

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 39
- Patterns of Probability: Binomial, Poisson and Normal Distributions

Part 4

POISSON WORKSHEET FUNCTION

Returns the Poisson distribution. A common application of the Poisson distribution is predicting the number of events over a specific time, such as the number of cars arriving at a toll plaza in 1 minute.

Syntax

POISSON(**x**,**mean**,**cumulative**)

X is the number of events.

Mean is the expected numeric value.

Cumulative is a logical value that determines the form of the probability distribution returned. If cumulative is TRUE, POISSON returns the cumulative Poisson probability that the number of random events occurring will be between zero and x inclusive; if FALSE, it returns the Poisson probability mass function that the number of events occurring will be exactly x.

Remarks

- If x is not an integer, it is truncated.
- If x or mean is nonnumeric, POISSON returns the #VALUE! error value.
- If x ≤ 0, POISSON returns the #NUM! error value.
- If mean ≤ 0, POISSON returns the #NUM! error value.
- POISSON is calculated as follows.

For cumulative = FALSE:

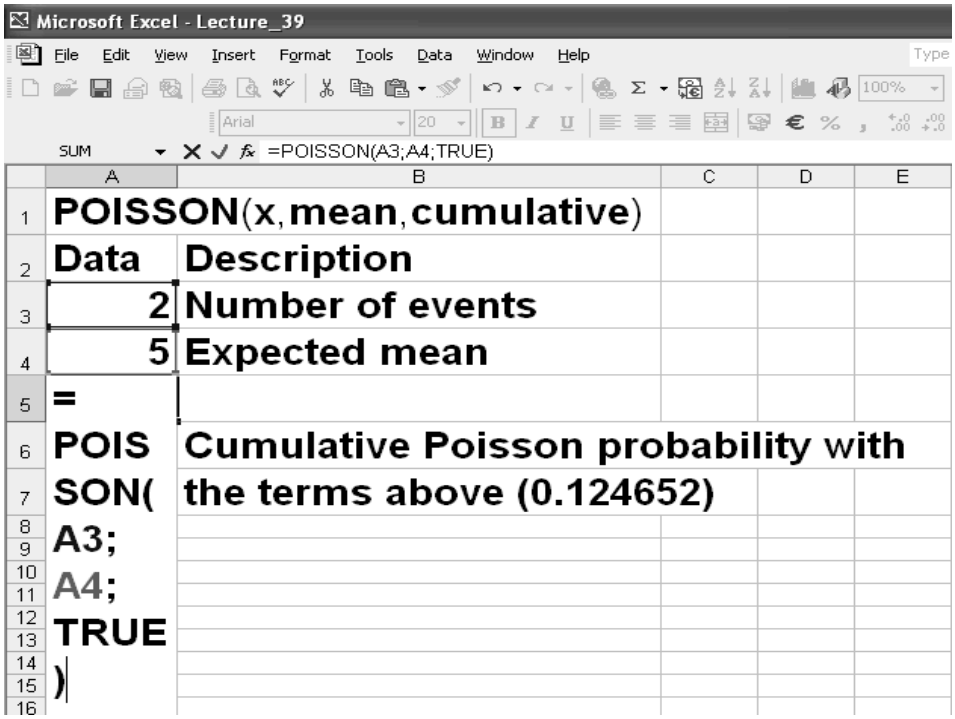
$$POISSON = \frac{e^{-\lambda}\ \lambda^{x}}{x!}$$

For cumulative = FALSE:

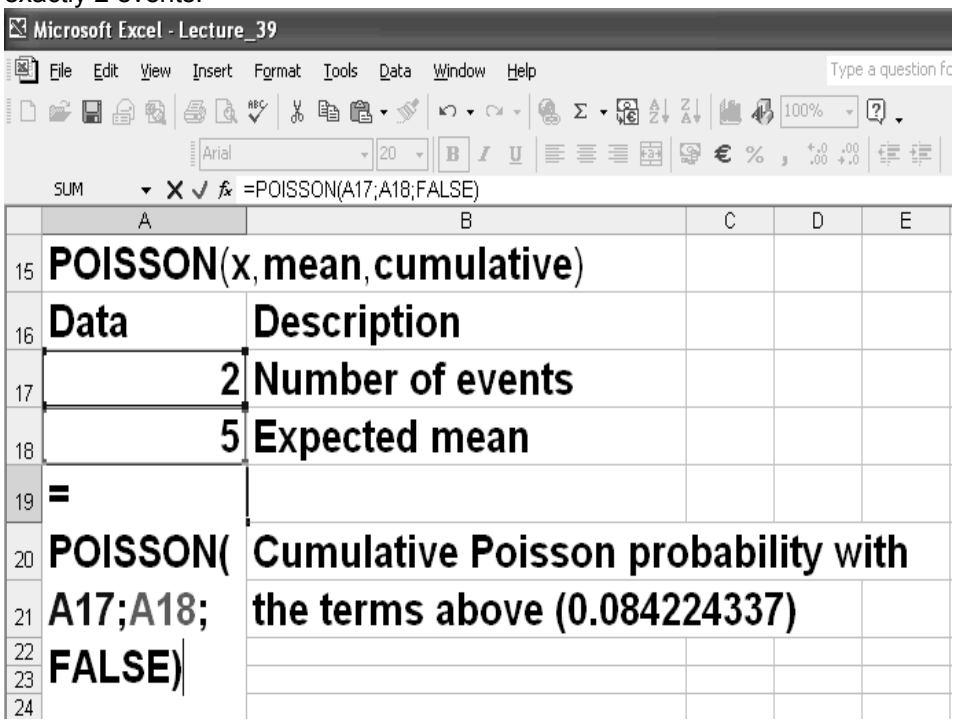
$$CUMPOISSON = \sum_{k=0}^{x} \frac{e^{-\lambda}\ \lambda^{x}}{k!}$$

Example

An application of the POISSON function is shown below. In this slide the value of Cumulative was TRUE. It means that the probability is for at the most case.



In the slide below the Cumulative is FALSE, which means that the probability is for exactly 2 events.



THE PATTERN

In Binomial and Poisson the situations are: either/or
Number of times could be counted.
In the Candy problem with underweight boxes, there is measurement of weight.
Binomial and Poisson are discrete probability distributions.
Candy problem is a Continuous probability distribution. Such problems need a different treatment.

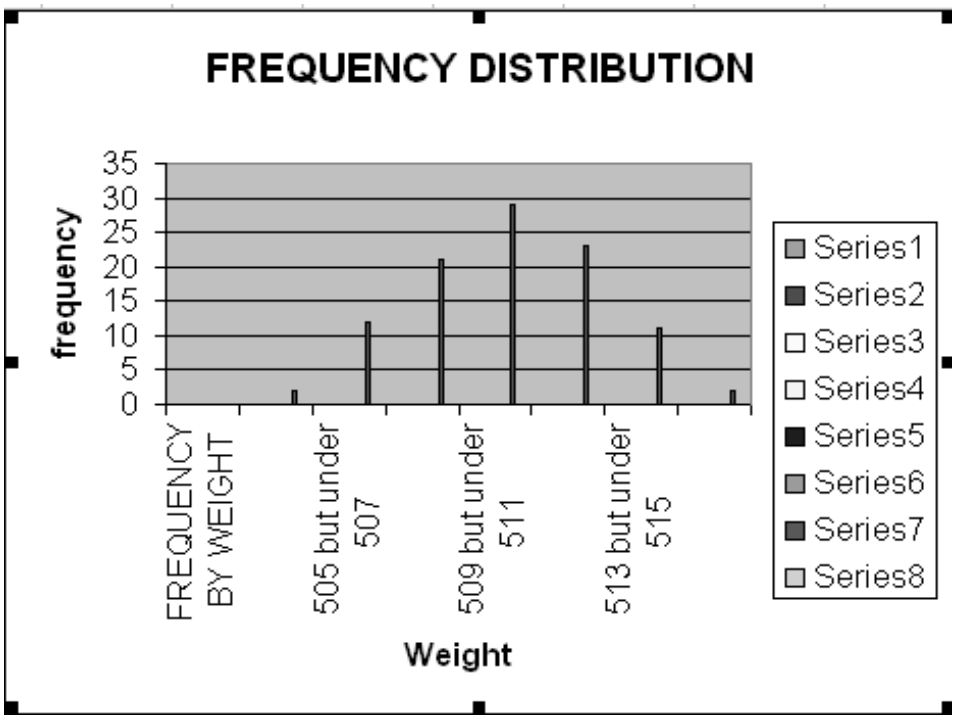
FREQUENCY BY WEIGHT

Look at the frequency distribution of weight of sample bags.

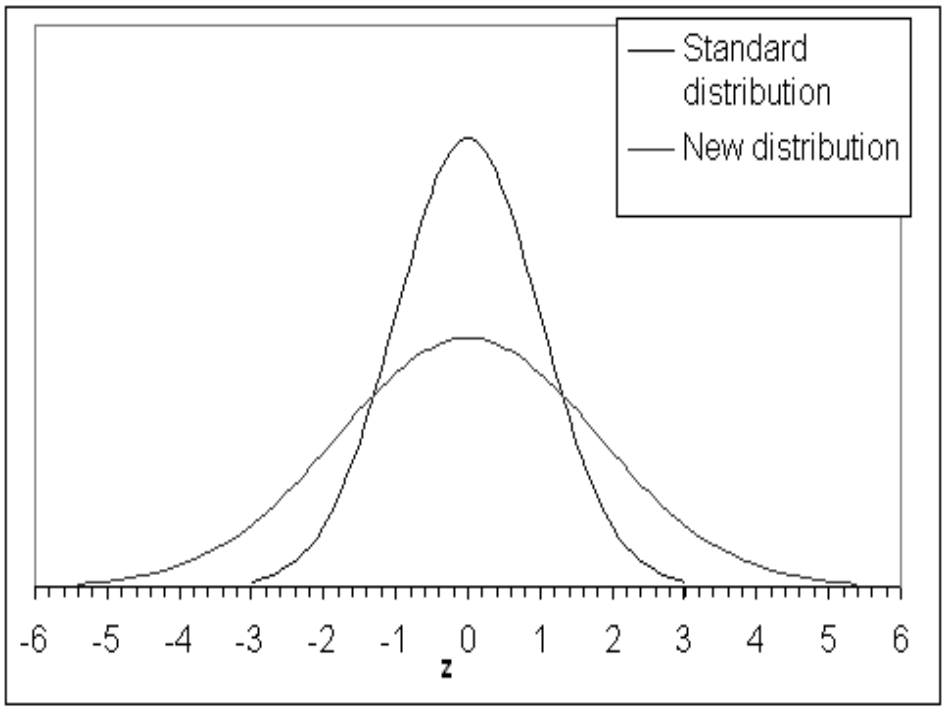
Microsoft Excel - Lecture_39

FREQUENCY BY WEIGHT	No. of bags
503 but under 505	2
505 but under 507	12
507 but under 509	21
509 but under 511	29
511 but under 513	23
513 but under 515	11
515 but under 517	2

Frequency distribution graph of the sample is shown below. You may see a distinct shape in the graph. It appears to be symmetrical.



The shape of the distribution is that of a Normal Distribution as shown as New distribution in the slide below. On this slide you also see a Standard Normal Distribution with 0 mean and standard deviations 1, 2, 3, 4 etc.


Standard distribution
New distribution
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6
z

NORMAL DISTRIBUTION

The blue Curve is a typical Normal Distribution.
A standard normal distribution is a distribution with mean = 0 and standard deviation = 1.
The Y-axis gives the probability values.
The X-axis gives the z (measurement) values.
Each point on the curve corresponds to the probability p that a measurement will yield a particular z value (value on the x-axis.).

Probability is a number from 0 to 1.
Percentage probabilities –multiply p by 100.
Area under the curve must be one.
Note how the probability is essentially zero for any value z that is greater than 3 standard deviations away from the mean on either side.

Mean gives the peak of the curve.
Standard deviation gives the spread.

Weight distribution case

Mean = 510 g
StDev = 2.5 g
What proportion of bags weighs more than 515 g?
Proportion of area under the curve to the right of 515 g gives this probability

AREA UNDER THE STANDARD NORMAL CURVE

The normal distribution table gives the area under one tail only.
z-value
Ranges between 0 and 4 in first column.
Ranges between 0 and 0.09 in other columns.

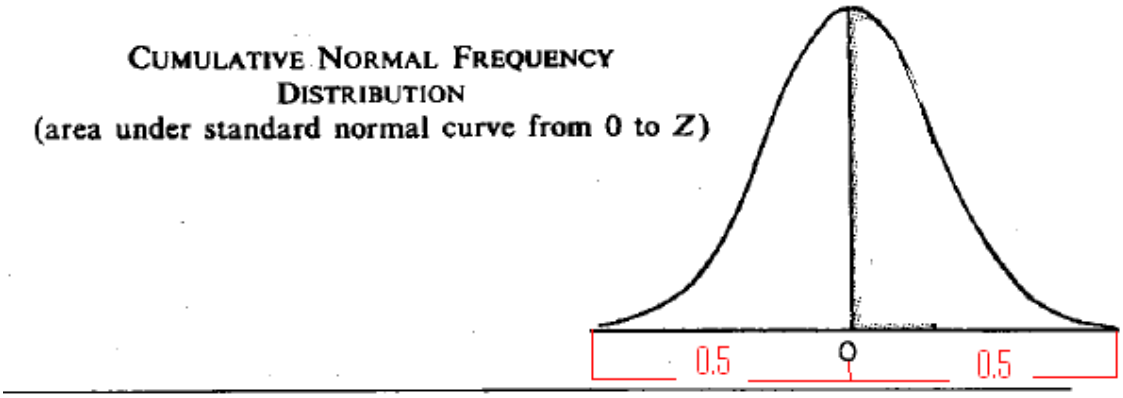
Example

Find area under one tail for z-value of 2.05.

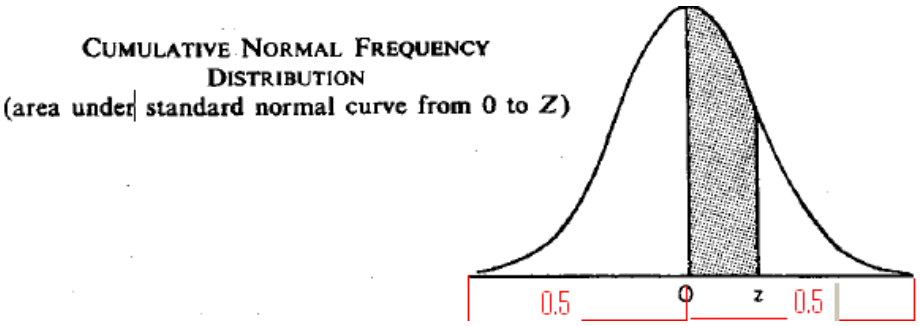
- Look in column 1. Find 2.0.
- Look in column 0.05 and go to intersection of 2.0 and 0.05.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890

You can find the probability 0.4798 at the intersection of 2.0 and 0.05. (i.e. 2.5 which is corresponding z-value).
As the probability at right from the center of the curve is 0.5 and also at left is 0.5.



Now if we want to find the probability corresponding to z value greater than equal to 2.5, then we have to subtract the value of probability correspoing to given z value from 0.5.



0.5-0.4798 = 0.0202 = 2.02%

CALCULATING Z- VALUES

z = (Value x – Mean)/StDev

Process of calculating z from x is called Standardization.

z indicates how many standard deviations the point is from the mean

Example 1

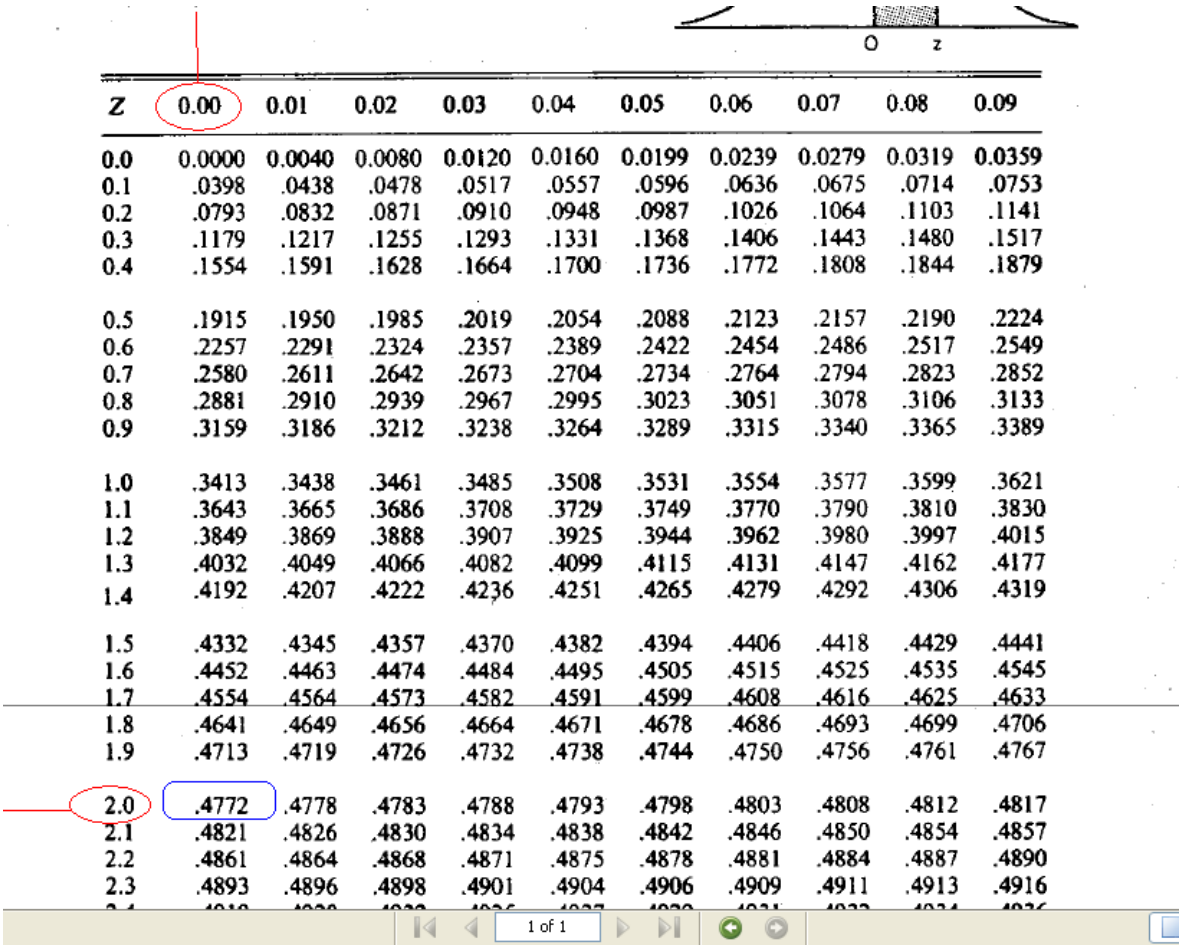
Find proportion of bags which have weight in excess of 515 g.

Mean = 510. StDev = 2.5 g

Solution

z = (515 – 510)/2.5 = 2

From tables: probability corresponding to z value is 0.4772



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916

= 0.5 – 0.4772 = 0.0228

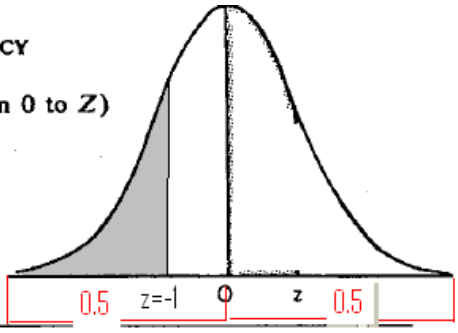
Example 2

What percentage of bags filled by the machine will weigh less than 507.5 g?
Mean = 510 g; StDev = 2.5 g

Solution

z = (507.5 – 510)/2.5 = -1
Look at value of z= +1

CUMULATIVE NORMAL FREQUENCY DISTRIBUTION
(area under standard normal curve from 0 to Z)



By table

= 0.5 -0.3413 = 0.1587

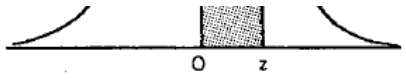
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936

Example 3

What is the probability that a bag filled by the machine weighs less than 512 g?

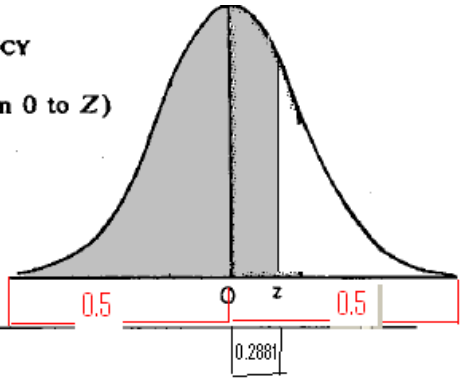
z = (512 – 510)/2.5 = 0.8

Solution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706

CUMULATIVE NORMAL FREQUENCY DISTRIBUTION
(area under standard normal curve from 0 to Z)



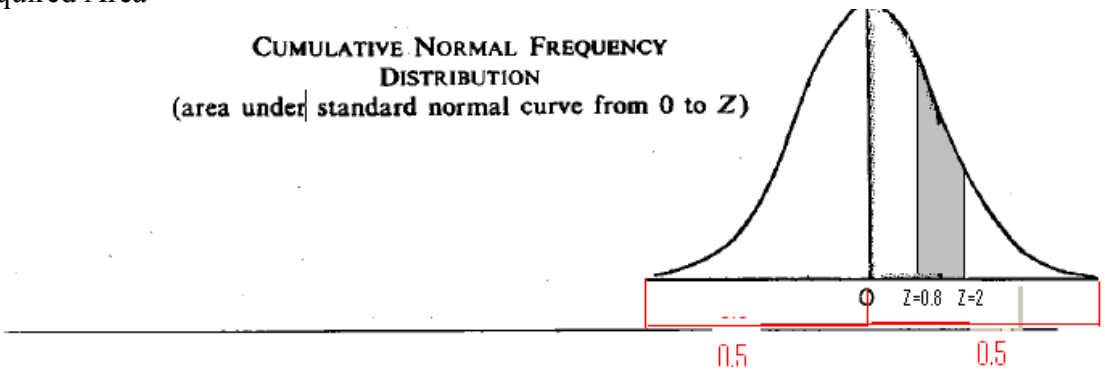
=0.5+0.2881 = 0.7881

]Example 4

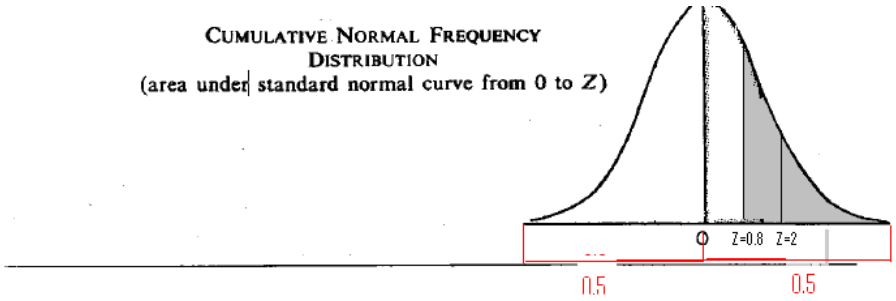
What percentage of bags weigh between 512 and 515?
z1 = (512 – 510)/2.5 = 0.8

Solution
Area 1 = 0.2119
z2 = (515 – 510)/2.5 = 2
Area 2 = 0.02275
p(bags weighs between 512 and 515) =
Area 1 – Area 2
= 0.2119 – 0.02275
= 0.18915 = 18.9%

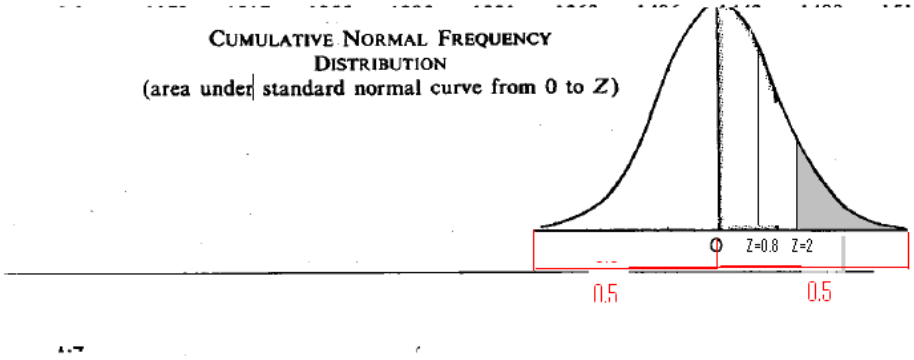
Required Area =



Area1 = 0.5-0.2881=0.2119

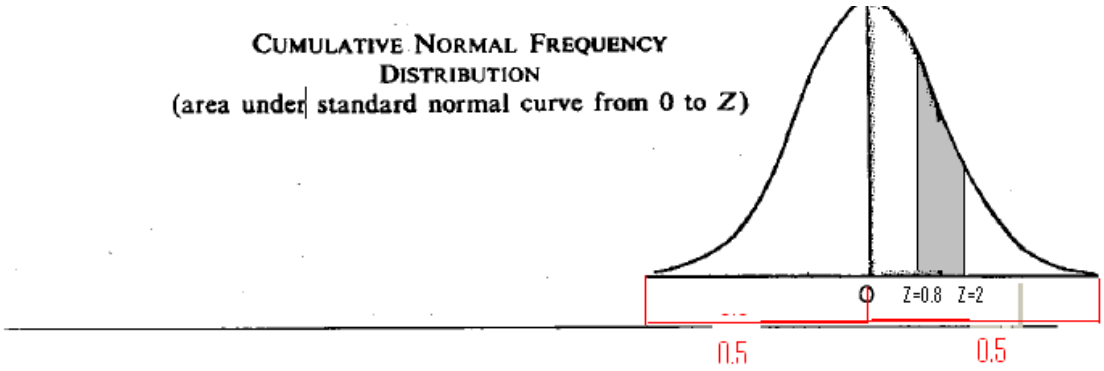


Area 2 =0.5-0.4772=0.0228



= 0.2119-0.0228 = 0.1891 = 18.91%

So we obtain


CUMULATIVE NORMAL FREQUENCY
DISTRIBUTION
(area under standard normal curve from 0 to Z)
0
Z=0.8
Z=2
0.5
0.5

LECTURE 41
ESTIMATING FROM SAMPLES: INFERENCE
PART 1

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 40
- Estimating from Samples: Inference

NORMDIST

Returns the normal distribution for the specified mean and standard deviation. This function has a very wide range of applications in statistics, including hypothesis testing.

Syntax

NORMDIST(**x**,**mean**,**standard_dev**,**cumulative**)

X is the value for which you want the distribution.

Mean is the arithmetic mean of the distribution.

Standard_dev is the standard deviation of the distribution.

Cumulative is a logical value that determines the form of the function. If cumulative is TRUE, NORMDIST returns the cumulative distribution function; if FALSE, it returns the probability mass function.

Remarks

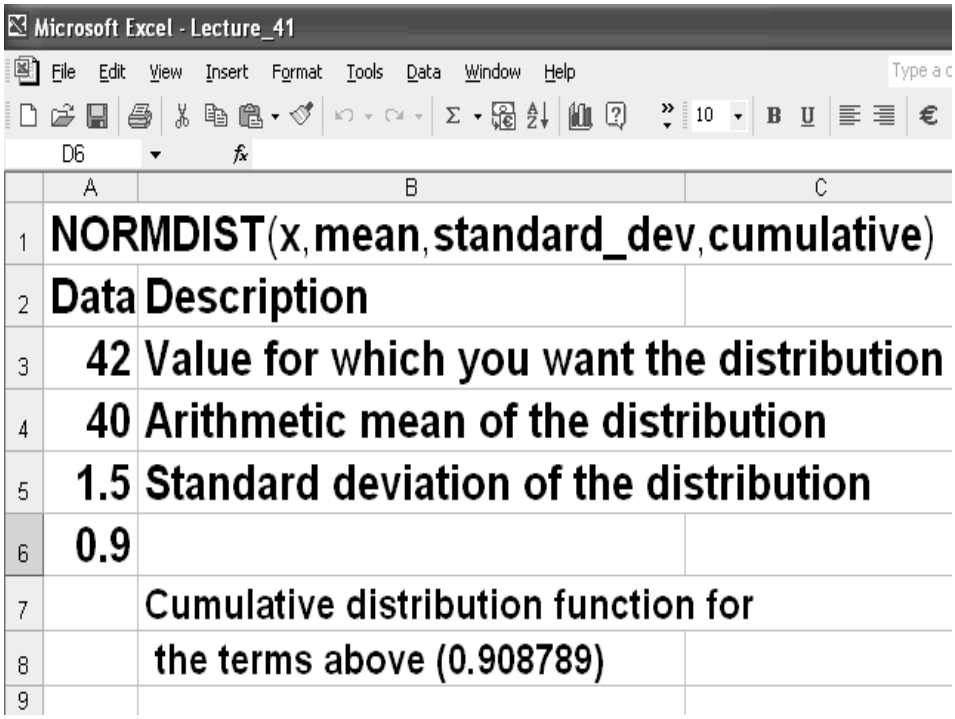
- If mean or standard_dev is nonnumeric, NORMDIST returns the #VALUE! error value.
- If standard_dev ≤ 0, NORMDIST returns the #NUM! error value.
- If mean = 0, standard_dev = 1, and cumulative = TRUE, NORMDIST returns the standard normal distribution, NORMSDIST.
- The equation for the normal density function (cumulative = FALSE) is:

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

- When cumulative = TRUE, the formula is the integral from negative infinity to x of the given formula.

Example

In the slide the x value is 42. Arithmetic mean is 40. Standard deviation is 1.5. The cumulative distribution is 0.9.



NORMSDIST

Returns the standard normal cumulative distribution function. The distribution has a mean of 0 (zero) and a standard deviation of one. Use this function in place of a table of standard normal curve areas.

Syntax

NORMSDIST(**z**)

z is the value for which you want the distribution.

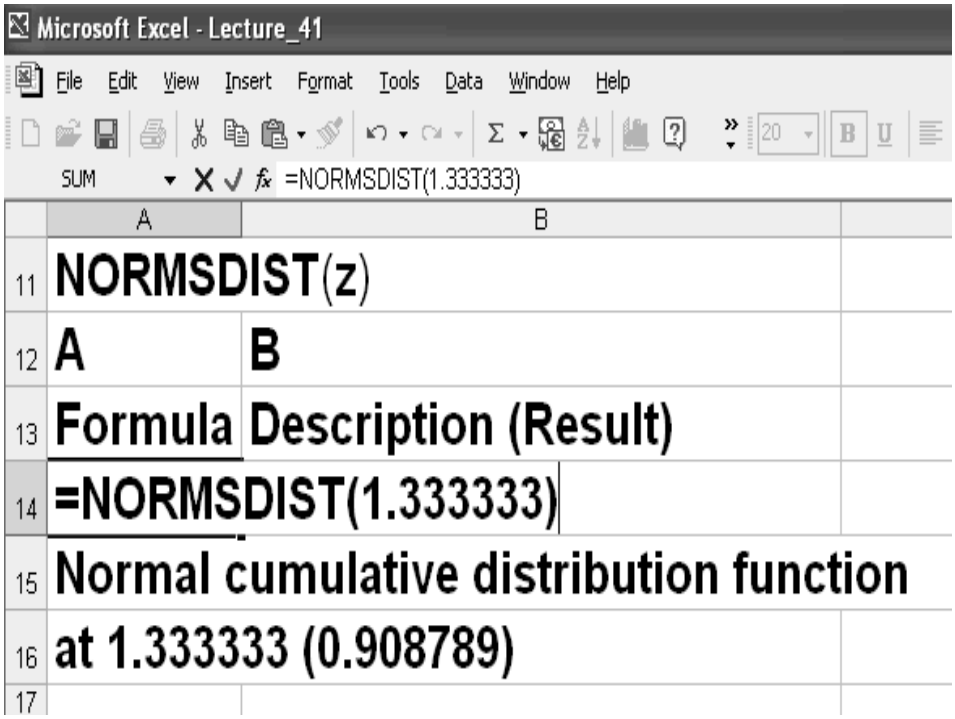
Remarks

If z is nonnumeric, NORMSDIST returns the #VALUE! error value.

The equation for the standard normal density function is:

Example

The input to the NORMSDIST function is the z-value. The output is the cumulative probability distribution. In the example z = 1.333333. The normal cumulative probability function is 0.908789.



NORMINV

Returns the inverse of the normal cumulative distribution for the specified mean and standard deviation.

Syntax

NORMINV(**probability**,**mean**,**standard_dev**)

Probability is a probability corresponding to the normal distribution.

Mean is the arithmetic mean of the distribution.

Standard_dev is the standard deviation of the distribution.

Remarks

If any argument is nonnumeric, NORMINV returns the #VALUE! error value.

If probability < 0 or if probability > 1, NORMINV returns the #NUM! error value.

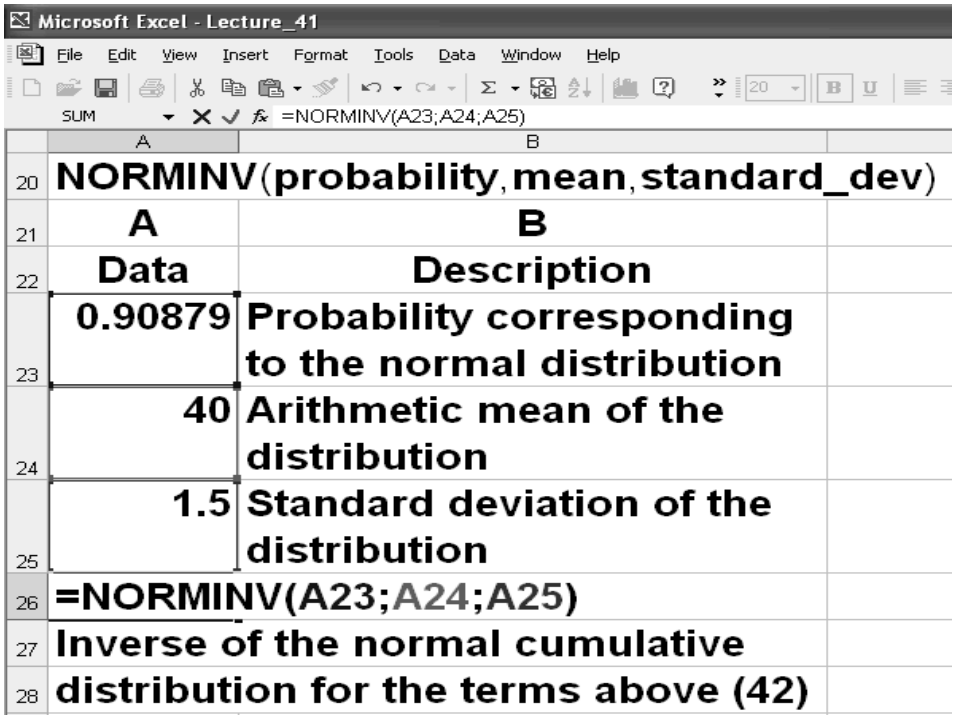
If standard_dev ≤ 0, NORMINV returns the #NUM! error value.

If mean = 0 and standard_dev = 1, NORMINV uses the standard normal distribution (see NORMSINV).

NORMINV uses an iterative technique for calculating the function. Given a probability value, NORMINV iterates until the result is accurate to within ± 3x10^-7. If NORMINV does not converge after 100 iterations, the function returns the #N/A error value.

Example

Here the probability value, arithmetic mean and standard deviation are given. The answer is the x-value.



NORMSINV

Returns the inverse of the standard normal cumulative distribution. The distribution has a mean of zero and a standard deviation of one.

Syntax

NORMSINV(**probability**)

Probability is a probability corresponding to the normal distribution.

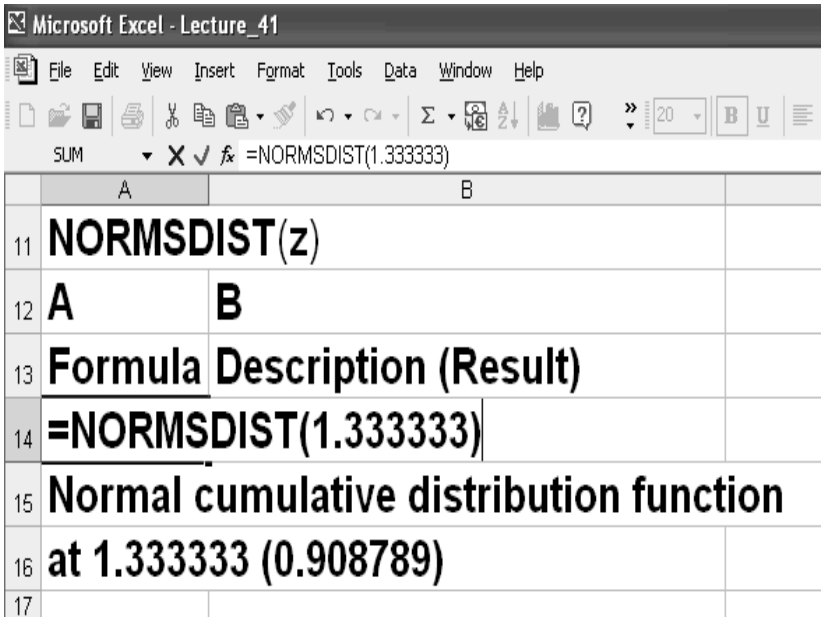
Remarks

- If probability is nonnumeric, NORMSINV returns the #VALUE! error value.
- If probability < 0 or if probability > 1, NORMSINV returns the #NUM! error value.

NORMSINV uses an iterative technique for calculating the function. Given a probability value, NORMSINV iterates until the result is accurate to within ± 3x10^-7. If NORMSINV does not converge after 100 iterations, the function returns the #N/A error value.

Example

In this case, the input is the z-value. The corresponding cumulative distribution is calculated.



SAMPLING VARIATIONS

Electronic components are despatched by a manufacturer in boxes of 500.
A small number of faulty components are unavoidable.
Customers have agreed to a defect rate of 2%.
One customer recently found 25 faulty components (5%) in a box.
Was this box representative of production as a whole?
The box represents a sample from the whole output. In such a case sampling variations are expected
If overall proportion of defective items has not increased, just how likely is it that a box of 500 with 25 defective components will occur?

SAMPLING VARIATIONS EXAMPLE 1

In a section of a residential colony there are 6 households say Household A, B, C, D, E and F. A survey is to be carried out to determine % of households who use corn flakes (cf) in breakfast.
Survey data exists and the following information is available:
Households A, B, C and D: Use corn flakes
Households E and F: Do not
It was decided to take random samples of 3 households
The first task is to list all possible samples and find % of each sample using corn flakes. Possible Samples

Sample	% cf users	Sample	% cf users
ABC	100	BCD	100
ABD	100	BCE	67
ABE	67	BCF	67
ABF	67	BDE	67
ACD	100	BDF	67
ACE	67	BEF	33
ACF	67	CDE	67
ADE	67	CDF	67
ADF	67	CEF	33
AEF	33	DEF	33

Percentage In Sample

Out of 20 samples:
4 contain 100% cf users,
12 contain 67% cf users,
4 contain 33% cf users,
with required characteristic
If the samples are selected randomly, then each sample is likely to arise.
The probability of getting a sample
with 100% cf users is: 4/20 or o.2
with 67% : 12/20 or 0.6
with 33% : 4/20 or 0.2
This is a **Sampling Distribution**.

SAMPLING DISTRIBUTION

The sampling distribution of percentages is the distribution obtained by taking all possible samples of fixed size n from a population, noting the percentage in each sample with a certain characteristic and classifying these into percentages

Mean of the Sampling Distribution

Using the above data:
Mean = 100% x 0.2 + 67% x 0.6 + 33% x 0.2 = 67%
Mean of the sampling distribution is the true percentage for the population as a whole.
You must make allowance for variability in samples.

Conditions For Sample Selection

- Number of items in the sample, n, is fixed and known in advance
- Each item either has or has not the desired characteristic
- The probability of selecting an item with the characteristic remains constant and is known to be P percent

If n is large (>30) then the distribution can be approximated to a normal distribution

STANDARD ERROR OF PERCENTAGES

Standard deviation of the sampling distribution tells us how the sample values differ from the mean P.
It gives us an idea of error we might make if we were to use a sample value instead of the population value.
For this reason it is called STandard Error of Percentages or STEP.

STEP

The sampling distribution of percentages in samples of n items (n>30) taken at random from an infinite population in which P percent of items have characteristic X will be:
A **Normal Distribution**
with **mean P%**
and **standard deviation (STEP)=** [P(100-P)/n]^1/2 %
The mean and StDev of the sampling distribution of percentages will also be percentages.

LECTURE 42
Estimating from Samples: Inference
Part 2

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 41
- Estimating from Samples: Inference

EXAMPLE 1

In a factory 25% workforce is women.
How likely is it that a random sample of 80 workers contains 25 or more women?

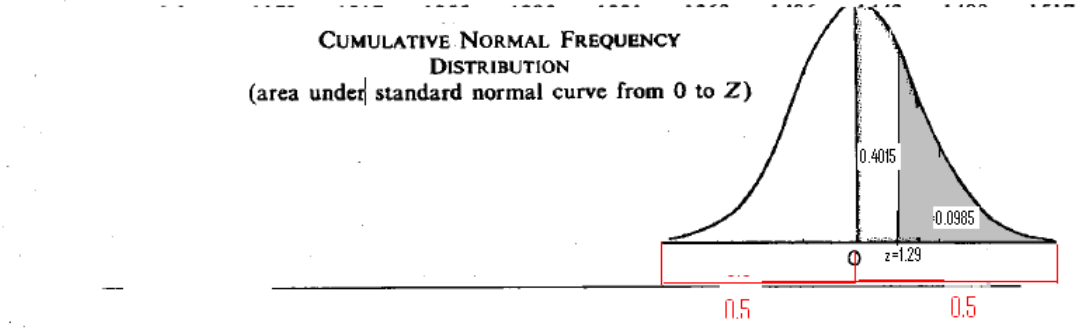
Solution:

standard deviation (STEP)= [P(100-P)/n]^(1/2) %
Mean = P = 25
N = 80
STEP = [25(100 – 25)/80]^(1/2)%
= [(25 x 75)/80]^(1/2)%
= 4.84%
% women in sample = (25/80) x 100 %= 31.25%
z = (31.25 – 25)/4.84
= 1.29
Look for p against z = 1.29 in the table, you will get 0.4015.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545

Since we want to find p(sample contains 25 or more women), so



p(sample contains 25 or more women) = 0.5 – 0.4015
=0.0985 or about 10%.

APPLICATIONS OF STEP

Some important issues are:

- What is the probability that such a sample will arise?
- How to estimate the percentage P from information obtained from a single sample?
- How large a sample will be required in order to estimate a population percentage with a given degree of accuracy?
- To obtain answers to these questions, let us solve some typical problems.

CONFIDENCE LIMITS

A market researcher wishes to conduct a survey to determine % consumers buying the company's products.
He selects a sample of 400 consumers at random.
He finds that 280 of these (70%) are purchasers of the product.
What can he conclude about % of all consumers buying the product?
First let us decide some limits.
It is common to use 95% confidence limits.
These will be symmetrically placed around the 70% buyers.
In a normal sampling distribution 2.5% corresponds to a z-value of 1.96 on either side of 70%.

With 95% confidence limit we have 5% chance of errors (level of significance)
That mean 2.5% on each side of curve.

2.5% = 0.025

0.5-0.025 = 0.475

We have subtracted 0.025 from 0.5 because we find out the probability of acceptance region (as the total probability under the curve is always 1, that mean 0.5 on right side of mean and 0.5 on left side of mean. And 0.25 is rejection region so 0.475 is acceptance region).

So 0.475 is the probability of acceptance region.
Corresponding z value is 1.96

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767

Now the sample percentage of 70% can be used as an approximation for. population percentage P.

Hence:

STEP = [(70(100 – 70)/400]^1/2 = 2.29%

Confidence Limits

Estimate for population percentage = 70 +/- 1.96 x STEP

Or

70 +/- 1.96 x 2.29

= 65.515% and 74.49% as the two limits for 95% confidence interval.

We can round off 1.96 to 2

Then with 95% confidence we estimate the population percentage with that characteristic as lying in the interval

P +/- 2 x STEP

EXAMPLE 2

A sample of 60 students contains 12 (20%) who are left handed.

Find the range with 95% confidence in which the entire left handed students fall.

Range = 20 +/- 2 x STEP

= 20 +/- 2 x [(20 x (100-20))/60]^1/2

= 9.67% and 30.33%

ESTIMATING PROCESS SUMMARY

1. Identify n and P (the sample size and percentage) in the sample.
2. Calculate STEP using these values.

- The 95% confidence interval is approximately P +/- 2 STEP.

99% confidence
For 99% confidence limits:
z-value = 2.58.

With 99% confidence limit we have 1% chance of errors (level of significance)
That mean 0.5% on each side of curve.

0.5% = 0.005

0.5-0.005 = 0.495

We have subtracted 0.005 from 0.5 because we find out the probability of acceptance region (as the total probability under the curve is always 1, that mean 0.5 on right side of mean and 0.5 on left side of mean. And 0.05 is rejection region so 0.495 is acceptance region).

So 0.495 is the probability of acceptance region

Corresponding z value is 2.58 (from table, as we did in example stated above).

FINDING A SAMPLE SIZE
To satisfy 95% confidence:
2 x STEP = 5
STEP = 2.5
Pilot survey value of P = 30%.
STEP = [(30 x 70)/n]^1/2 = 2.***5***
Solving
n = 336
We must interview 336 persons to be 95% confident that our estimate is within 5% of the true answer.

DISTRIBUTION OF SAMPLE MEANS
The standard deviation of the Sampling Distribution of means is called STandard Error of the Mean STEM.

$$STEM = \frac{s.d}{\sqrt{n}}$$

s.d denotes standard deviation of the population.
n is the size of the sample.

EXAMPLE 3
What is the probability that if we take a random sample of 64 children from a population whose mean IQ is 100 with a StDev of 15, the mean IQ of the sample will be below 95?
Solution:
s = 15; n= 64; population mean = 100
STEM = 15/(64)^1/2 = 15/(64)^1/2 = 15/8 = 1.875
z = 100 – 95 /STEM
= 5/1.875 = 2.67
This gives a probability of 0.0038.
So the chance that the average IQ of the sample is below 95 is very small.

LECTURE 43
HYPOTHESIS TESTING: CHI-SQUARE DISTRIBUTION PART 1

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 42
- Hypothesis testing: Chi-Square Distribution

EXAMPLE 1

An inspector took a sample of 100 tins of beans. The sample weight is 225 g.
Standard deviation is 5 g.
Calculate with 95% confidence the range of the population mean.

Solution:

$$STEM = \frac{s.d}{\sqrt{n}}$$

s.d is not known

Use s.d of sample as an approximation.

$$STEM = \frac{5}{\sqrt{100}} = 0.5$$

95% confidence interval = 225 +/- 2 x 0.5 or From 224 to 226 g

PROBLEM OF FAULTY COMPONENTS REVISITED

Box of 500 components may have 25 or 5% faulty components.
Overall faulty items = 2%
P = 2%; n= 500;
STEP = [(2 x 98)/500]^1/2
= 0.626

To find the probability that the sample percentage is 5% or over:

z = (5 – 2)/STEP = 3/0.626 = 4.79
Area against z = 4.79 is negligible.
Chance of such a sample is very small

FINITE POPULATION CORRECTION FACTOR

If population is very large compared to the sample then multiply STEM and STEP by the:

Finite Population Correction Factor = [1 – (n/N)]^1/2

Where
N = Size of the population
n = Size of the sample
n = less than 0.1N

TRAINING MANAGER'S PROBLEM

New refresher course for training of workers was completed.
The Training Manager would like to assess the effect of retraining if any.

Particular questions:

- Is quality of product better than produced before retraining?
- Has the speed of machines increased?
- Do some classes of workers respond better to retraining than others?

Training Manager hopes to:

- Compare the new position with established
- Test a theory or hypothesis about the course

Case Study

Before the course:

Worker X produced 4% rejects.

After the course:

Out of 400 items 14 were defective = 3.5%

An improvement?

The 3.5% figure may not demonstrate overall improvement.

It does not follow that every single sample of 400 items contains exactly 4% rejects.

To draw a sound conclusion:

Sampling variations must be taken into account.

We do not begin by assuming what we are trying to prove NULL HYPOTHESIS.

We must begin with the assumption that there is no change at all.

This initial assumption is called

NULL HYPOTHESIS

Implication of Null Hypothesis:

That the sample of 400 items taken after the course was drawn from a population in which the percentage of reject items is still 4%.

NULL HYPOTHESIS EXAMPLE

Data:

P = 4%; n = 400

STEP

= [P(100 – P)/n]^1/2

= [4(100 – 4)/400]^1/2 = 0.98%

At 95% confidence limit:

Range = 4 +/- 2 x 0.98 = 2.04 to 5.96 %

Conclusion:

Sample with 3.5% rejects is not inconsistent.

No ground to assume that % rejects has changed at all.

On the strength of sample there were no grounds for rejecting Null Hypothesis.

ANOTHER EXAMPLE

Before the course:

5% rejects

After the course:

2.5% rejects (10 out of 400)

P = 5

STEP = [5(100 – 5)/400]^1/2

= [5 x 95/400]^1/2

= 1.09

Range at 95 % Confidence Limits

= 5 +/- 2 x 1.09

= 2.82 % to 7.18 %

Conclusion:

Doubt about Null Hypothesis most of the time

Null hypothesis to be rejected

PROCEDURE FOR CARRYING OUT HYPOTHESIS TEST

1. Formulate null hypothesis
2. Calculate STEP & P +/- 2 x STEP
3. Compare the sample % with this interval to see whether it is inside or outside
   If the sample falls outside the interval, reject the null hypothesis (sample differs significantly from the population %)
   If the sample falls inside the interval,

do not reject the null hypothesis (sample does not differ significantly from the population % at 5% level)

HOW THE RULE WORKS?

Bigger the difference between the sample and population percentages, less likely it is that the population percentages will be applicable.

- When the difference is so big that the sample falls outside the 95% interval, then the population percentages cannot be applied.
  Null Hypothesis must be rejected
- If sample belongs to majority and it falls within 95% interval,

then there are no grounds for doubting the Null Hypothesis

FURTHER POINTS ABOUT HYPOTHESIS TESTING

- 99% interval requires 2.58 x STEP. Interval becomes wider. It is less likely to conclude that something is significant.
- (A) We might conclude there is a significant difference when there is none. Chance of error = 5% (type 1 Error)
  (B) We might decide that there is no significant difference when there is one (Type 2 Error)

LECTURE 44
HYPOTHESIS TESTING: CHI-SQUARE DISTRIBUTION PART 2

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 43
- Hypothesis Testing : Chi-Square Distribution

FURTHER POINTS ABOUT HYPOTHESIS TESTING

This is a continuation of the points covered under Handout 43.

3. We cannot draw any conclusion regarding the direction the difference is in

(A) Possible to do 1-tailed test

Null Hypotheis: P >= 4% against the alternative P> 4%

z = 1.64 for 5% significance level

Range = P - 1.64 x STEP (0.98%)

Example

Range = 4 – 1.64 x 0.98 = 2.39%

New figure = 3.5%.

Hence:

There is no reason to conclude that things have improved.

4. We cannot draw any conclusion regarding the direction the difference is in.

(B) Possible to do 2-tailed test

Null Hypothesis: P >= 4% against the alternative P> 4%

z = 1.96 for 5% significance level

Range = P +/- 1.96 x STEP (0.98%)

Example

Range = 4 +/– 2 x 1.96 x 0.98 = 2.08% to 5.92%

New figure = 3.5%

There is no reason to conclude that things have improved

HYPOTHESES ABOUT MEANS

Let us go back to the problem of retraining course discussed earlier.

Before the course:

Worker X took 2.5 minutes to produce 1 item.

StDev = 0.5 min

After the course:

Foe a sample of 64 items, mean time = 2.58 min

Null hypothesis

No change after the course.

STEM

= s.d/(n)^1/2 = 0.5/(64)^1/2 = 0.0625

Range

= 2.5 +/- 2 x 0.0625 = 2.375 to 2.625 min

Conclusion:

No grounds for rejecting the Null Hypothesis.

There is no change significant at 5% level.

ALTERNATIVE HYPOTHESIS TESTING USING Z-VALUE

z = (sample percentage – population mean)/STEP
= (3.5 – 4)/0.98 = 0.51
Compare it with z-value which would be needed to ensure that our sample falls in the 5% tails of distribution (1.96 or about 2).
z is much less than 2.

We conclude that the probability of getting by random chance a sample which differs from the mean of 4% or more is quite high.
Certainly it is greater than the 5% significance level.
Sample is quite consistent with null hypothesis.
Null hypothesis should not be rejected.

PROCESS SUMMARY

1. State Null Hypothesis (1-tailed or 2-tailed)
2. Decide on a significance level and find corresponding critical value of z
3. Calculate sample z(sample value – population value divided by STEP or STEM as appropriate)
4. Compare sample z with critical value of z
5. If sample z is smaller, do not reject the Null Hypothesis
6. If sample z is greater than critical value of z, sample provides ground for rejecting the Null Hypothesis.

TESTING HYPOTHESES ABOUT SMALL SAMPLES

Whatever the form of the underlying distribution the means of large samples will be normally distributed.
This does not apply to small samples.
We can carry out hypothesis testing using the methods discussed only if the underlying distribution is normal.
If we only know the Standard Deviation of sample and have to approximate population Standard Deviation then we use **Student's t-distribution.**

STUDENT'S t-DISTRIBUTION

Student's T-Distribution is very much like normal distribution.
In fact it is a whole family of t-distributions.
As n gets bigger, t-distribution approximates to normal distribution.
t-distribution is wider than normal distribution.
95% confidence interval reflects greater degree of uncertainty in having to approximate the population Standard Deviation by that of the sample.

EXAMPLE

Mean training time for population = 10 days.
Sample mean for 8 women = 9 days.
Sample Standard Deviation = 2 days.
To approximate population Standard Deviation by a sample divide the sum of squares by n – 1:
STEM = 2/(8)^1/2 = 0.71

Null Hypothesis:

There is no difference in overall training time between men and women.

t-value = (sample mean – population mean)/STEM
= (9 – 10)/0.71 = - 1.41
For n = 8, v = 8 – 1 = 7;
For 5%(.05) significance level looking at 0.025 (2-tailed):
t = 2.365 (Calculated table value)

Decision:

Do not reject the Null Hypothesis

SUMMARY - I
If underlying population is normal and we know the Standard Deviation
Then
Distribution of sample means is normal
with
Standard Deviation = STEM = population s.d/(n)^1/2
and
we can use a z-test.

SUMMARY - II
If underlying population is unknown but the sample is large
Then
Distribution of sample means is approximately normal
With
StDev = STEM = population s.d/(n)^1/2
and again
we can use a z-test.

SUMMARY - III
If underlying population is normal but we do not know its StDev and the sample is small Then We can use the sample s.d to approximate that of the population with n – 1 divisor in the calculation of s.d.
Distribution of sample means is a t-distribution with n – 1 degrees of freedom
With
Standard Deviation = STEM = sample s.d/(n)^1/2
And we can use a t-test**.**

SUMMARY - IV
If underlying **population is not normal** and we have a **small sample**
Then **none of the hypothesis testing procedures can be safely used.**

TESTING DIFFERENCE BETWEEN TWO SAMPLE MEANS
A group of 30 from production has a mean wage of 120 Rs. per day with
Standard Deviation = Rs. 10.
50 Workers from Maintenance had a mean of Rs. 130 with
Standard Deviation = 12
Is there a difference in wages between workers?
Difference of two sample means = s[(1/n1) + (1/n2)]^1/2
s = [(n1.s1^2 + n2.s2^2)/(n1 + n2)]^1/2
N1 = 30; n2 = 50; s1 = 10; s2 = 12
s = [(30 x 100 + 50 x 144)/(30 + 50)]^1/2 = 11.29
Standard Error of Difference in Sample Means (STEDM)
= 11.29(1/30 + 1/50)^1/2 = 2.60
z = (difference in sample means – 0)/STEDM
= 120 – 130/2.60 = - 3.85
This is well outside the critical z for 5% significance.
There are grounds for rejecting Null Hypothesis (There is difference in the two samples).

PROCEDURE SUMMARY
1. State Null Hypothesis and decide significance level
2. Identify information (no. of samples, large or small, mean or proportion) and decide what standard error and what distribution are required
3. Calculate standard error
4. Calculate z or t as difference between sample and population values divided by standard error
5. Compare your z or t with critical value from tables for the selected significance level; if z or t is greater than critical value, reject the Null Hypothesis

MORE THAN ONE PROPORTION

Look at a problem, where after the course some in different age groups shows improvement while others did not.

Let us assume that the expected improvement was uniform. An improvement of 40%, if applied to 21, 24 and 15 would give 14, 16 and 10 respectively, who improved. Let us write these values within brackets. Subtracting 14, 16 and 10 from the totals 21, 24 and 15 gives us 7, 8 and 5 respectively, who did not improve. This is the estimate if every person was affected in a uniform manner.

Let us write the observations as O, in one line (17 17 6 4 7 9).

Let us write down the expected as E, in the next line as (14 16 10 7 8 8).

Calculate O-E.

Next calculate (O-E)^2.

Now standardize (O-E)^2 by dividing by E.

Calculate the total and call it **χ2.**

Age	Improved	Did not improve	Total
Under 35	17(14)	4(7)	21
35 – 50	17(16)	7(8)	24
Over 50	6(10)	9(5)	15
Total	40	20	60

O		17	17	6	4	7	9
E		14	16	10	7	8	8
O-E	3	1	-4	-3	-1	4	
(O-E)^2:	9	1	16	9	1	16	
(O-E)^2:/E:	0.643	0.0625	1.6	1.286	0.125	3.2	= 6.92

Measurement of disagreement = Sum [(O-E)^2/E]

is known as **Chi-squared (χ2)**

Degrees of freedom v = (r-1) x (c-1) = (3-1)(2-1)= 2

There are tables that give Critical value of chi-squared at different confidence limits and degrees of freedom v (columns-1) x (rows-1). In the above case

v = 2-1 x 3-1 = 2

In the present case, the Critical value of chi-squared at 5% (and v = 2) = 5.991.

The value 6.92 is greater than 5.991.

This means that the Sample falls outside of 95% interval.

Null hypothesis should be rejected.

CHI-SQUARED SUMMARY

1. Formulate null hypothesis (no association form)
2. Calculate expected frequencies
3. Calculate χ2
4. Calculate degrees of freedom (rows minus 1) x (columns minus 1); look up the critical χ2 under the selected significance level
5. Compare the calculated value of χ2 from the sample with value from the table; if the sample χ2 is smaller (within the interval) don't reject the null hypothesis; if it is bigger (outside) reject the null hypothesis

Example

Look at the data in the slide below.

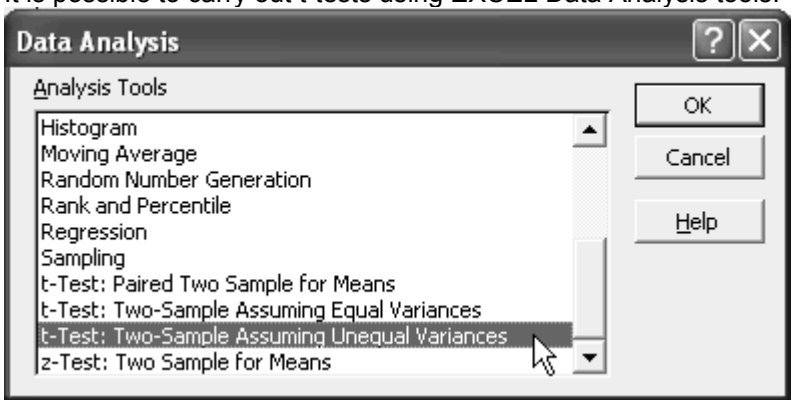
Microsoft Excel - Emp

File Edit View Insert Format Tools Data Window Help

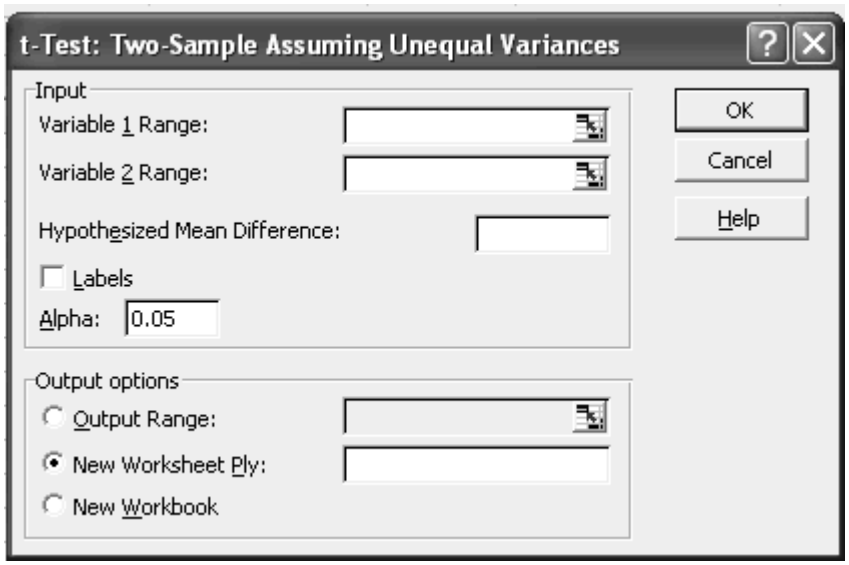
F17

	A	B	C
1	STATUS	SEX	AGE
2	0	0	38
3	0	1	37
4	0	1	30
5	0	1	40
6	1	0	36
7	0	1	38
8	0	1	47
9	1	0	32
10	1	0	38
11	0	1	49
12	1	0	44
13	0	1	31

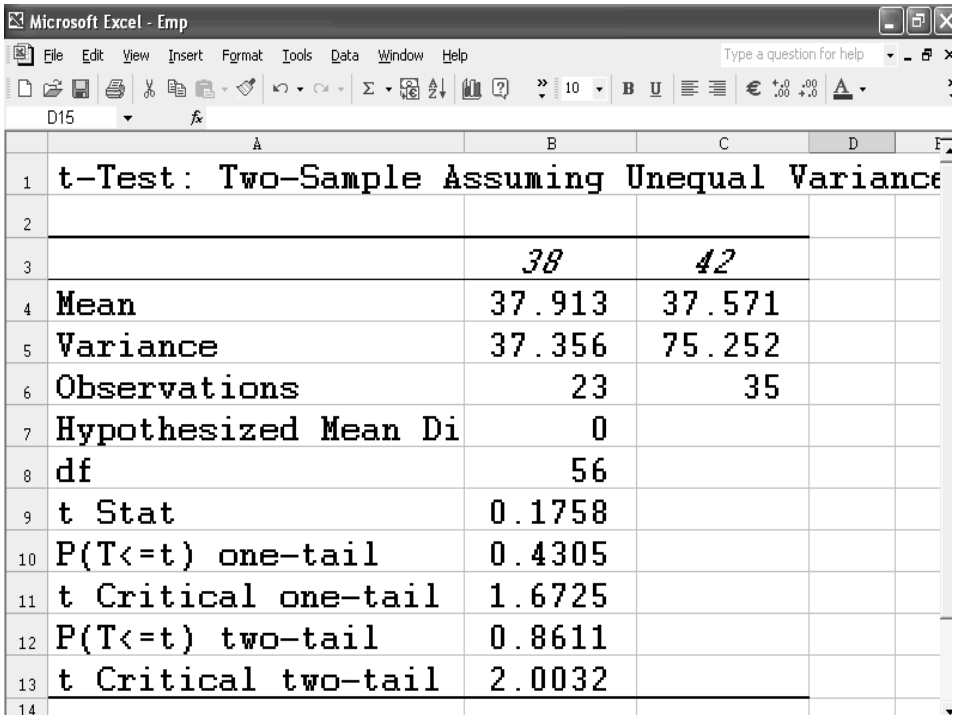
It is possible to carry out t-tests using EXCEL Data Analysis tools.



When you select the tool and press OK, the t-test dialog box is opened as below.



The ranges for the two variables, labels and output options are specified. For the above data the output was as follows:


Microsoft Excel - Emp

File Edit View Insert Format Tools Data Window Help

D15

	A	B	C	D	E
1	t-Test: Two-Sample Assuming Unequal Variance				
2					
3		38	42		
4	Mean	37.913	37.571		
5	Variance	37.356	75.252		
6	Observations	23	35		
7	Hypothesized Mean Di	0			
8	df	56			
9	t Stat	0.1758			
10	P(T<=t) one-tail	0.4305			
11	t Critical one-tail	1.6725			
12	P(T<=t) two-tail	0.8611			
13	t Critical two-tail	2.0032			
14					

CHITEST

Returns the test for independence. CHITEST returns the value from the chi-squared (γ2) distribution for the statistic and the appropriate degrees of freedom. You can use γ2 tests to determine whether hypothesized results are verified by an experiment.

Syntax

CHITEST(**actual_range**,**expected_range**)

Actual_range is the range of data that contains observations to test against expected values.

Expected_range is the range of data that contains the ratio of the product of row totals and column totals to the grand total.

Remarks

- If actual_range and expected_range have a different number of data points, CHITEST returns the #N/A error value.
- The γ2 test first calculates a γ2 statistic and then sums the differences of actual values from the expected values. The equation for this function is CHITEST=p(X>γ2), where:

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(A_{ij} - E_{ij}\right)^2}{E_{ij}}$$

and where:

Aij = actual frequency in the i-th row, j-th column

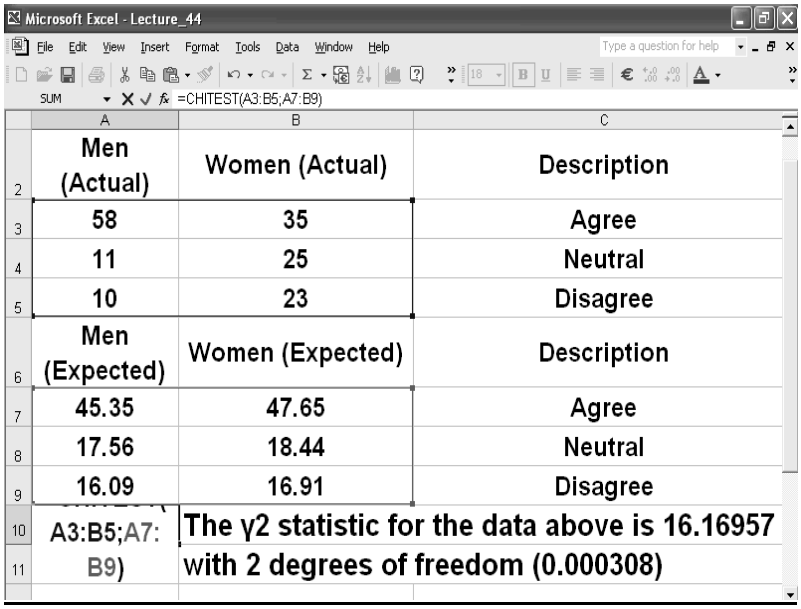
Eij = expected frequency in the i-th row, j-th column

r = number or rows

c = number of columns

CHITEST returns the probability for a γ2 statistic and degrees of freedom, df, where df = (r - 1)(c - 1).

Example



	A	B	C
2	Men (Actual)	Women (Actual)	Description
3	58	35	Agree
4	11	25	Neutral
5	10	23	Disagree
6	Men (Expected)	Women (Expected)	Description
7	45.35	47.65	Agree
8	17.56	18.44	Neutral
9	16.09	16.91	Disagree
10	A3:B5;A7:	The γ2 statistic for the data above is 16.16957	
11	B9)	with 2 degrees of freedom (0.000308)	

The above example shows two different groups. The calculation shows that the probability for chi-squared 16.16957 with 2 degrees of freedom was 0.000308, which is negligible.

LECTURE 45
Planning Production Levels: Linear Programming

OBJECTIVES

The objectives of the lecture are to learn about:

- Review Lecture 44
- Planning Production Levels: Linear Programming

INTRODUCTION TO LINEAR PROGRAMMING

A Linear Programming model seeks to maximize or minimize a linear function, subject to a set of linear constraints.

The linear model consists of the following components:

A set of decision variables, x_j.

An objective function, $\sum c_j x_j$.

A set of constraints, $\Sigma\ a_{ij} x_j \leq b_i$.

THE FORMAT FOR AN LP MODEL

Maximize or minimize $\sum c_j x_j = c_1 x_1 + c_2 x_2 + ... + c_n x_n$

Subject to

$a_{ij} x_j \leq b_i$, i = 1,,,,,m

Non-negativity conditions: all $x_j \geq 0$, j = 1, ,n

Here n is the number of decision variables.

Here m is the number of constraints.

(There is no relation between n and m)

THE METHODOLOGY OF LINEAR PROGRAMMING

1. Define decision variables
2. Hand-write objective
3. Formulate math model of objective function
4. Hand-write each constraint
5. Formulate math model for each constraint
6. Add non-negativity conditions

THE IMPORTANCE OF LINEAR PROGRAMMING

Many real world problems lend themselves to linear programming modeling.

Many real world problems can be approximated by linear models.

There are well-known successful applications in:

- Operations
- Marketing
- Finance (investment)
- Advertising
- Agriculture

There are efficient solution techniques that solve linear programming models.

The output generated from linear programming packages provides useful “what if” analysis.

ASSUMPTIONS OF THE LINEAR PROGRAMMING MODEL

1. The parameter values are known with certainty
2. The objective function and constraints exhibit constant returns to scale
3. There are no interactions between the decision variables (the additivity assumption)

   The Continuity assumption: Variables can take on any value within a given feasible range.

A PRODUCTION PROBLEM – A PROTOTYPE EXAMPLE

A company manufactures two toy doll models:

Doll A

Doll B

Resources are limited to:
1000 kg of special plastic.
40 hours of production time per week.

Marketing requirement:
Total production cannot exceed 700 dozens.
Number of dozens of Model A cannot exceed number of dozens of Model B by more than 350.
The current production plan calls for:

- Producing as much as possible of the more profitable product, Model A (Rs. 800 profit per dozen).
- Use resources left over to produce Model B (Rs. 500 profit per dozen), while remaining within the marketing guidelines.

Management is seeking:
a production schedule that will increase the company's profit
A linear programming model
can provide:
an insight and
an intelligent solution to this problem

Decisions variables:
X_1 = Weekly production level of Model A (in dozens)
X_2 = Weekly production level of Model B (in dozens).

Objective Function:
Weekly profit, to be maximized
Maximize $800X_1 + 500X_2$ (Weekly profit)
subject to
$2X_1 + 1X_2 \leq 1000$ (Plastic)
$3X_1 + 4X_2 \leq 2400$ (Production Time)
$X_1 + X_2 \leq 700$ (Total production)
$X_1 - X_2 \leq 350$ (Mix)
$X_j >= 0, \ j = 1,2$ (Nonnegativity)

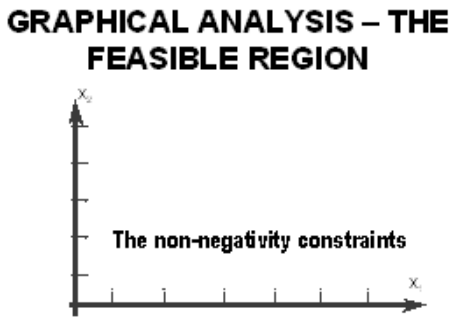
ANOTHER EXAMPLE
A dentist is faced with deciding:
how best to split his practice
between the two services he offers—general dentistry
and pedodontics?
(children's dental care)
Given his resources,
how much of each service should he provide
to maximize his profits?
The dentist employs three assistants and uses two operatories.
Each pedodontic service requires .75 hours of operatory time, 1.5 hours of an assistant's time and .25 hours of the dentist's time
A general dentistry service requires .75 hours of an operatory, 1 hour of an assistant's time and .5 hours of the dentist's time.
Net profit for each service is Rs. 1000 for each pedodontic service and Rs. 750 for each general dental service.
Time each day is: eight hours of dentist's, 16 hours of operatory time, and 24 hours of assistants' time.

THE GRAPHICAL ANALYSIS OF LINEAR PROGRAMMING

Using a graphical presentation,
we can represent:
all the constraints, the objective function, and the three types of feasible points.

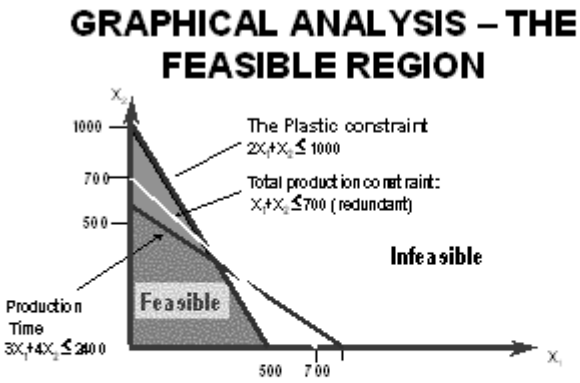
GRAPHICAL ANALYSIS – THE FEASIBLE REGION

The slide shows how a feasible region is defined with non-negativity constraints.

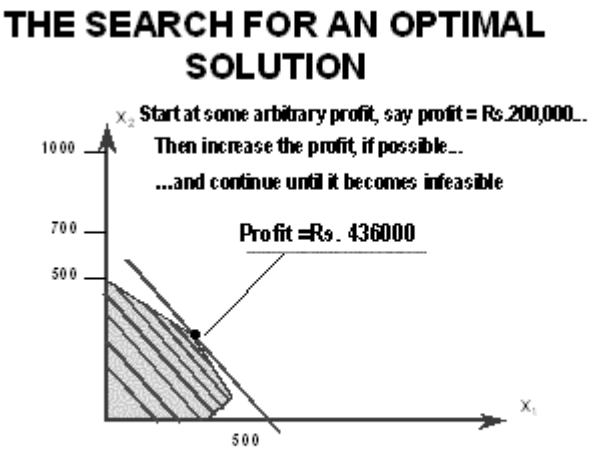


THE SEARCH FOR AN OPTIMAL SOLUTION

The figure shows how different constraints can be represented by straight lines to define a feasible region. There is an area outside the feasible region that is infeasible.



It may be seen that each of the constraints is a straight line. The constraints intersect to form a point that represents the optimal solution. This is the point that results in maximum profit of 436,000 Rs. As shown in the slide below. The procedure is to start with a point that is the starting point say 200,000 Rs. Then move the line upwards till the last point on the feasible region is reached. This region is bounded by the lines representing the constraints.

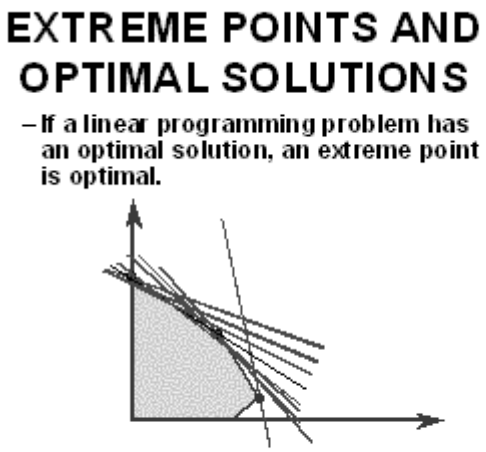


SUMMARY OF THE OPTIMAL SOLUTION

Model A = 320 dozen
Model B = 360 dozen
Profit = Rs. 436000
This solution utilizes all the plastic and all the production hours.
Total production is only 680 (not 700).
Model a production does not exceed Model B production at all.

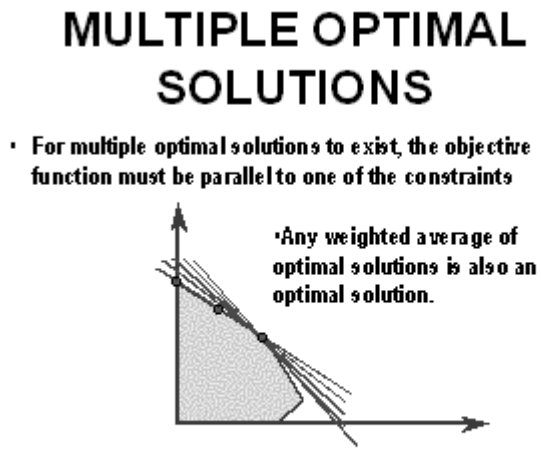
EXTREME POINTS AND OPTIMAL SOLUTIONS

If a linear programming problem has an optimal solution, an extreme point is optimal.



MULTIPLE OPTIMAL SOLUTIONS

There may be more than one optimal solutions. However, the condition is that the objective function must be parallel to one of the constraints. If a weightage average of different optimal solutions is obtained, it is also an optimal solution.



Some useful functions of Excel

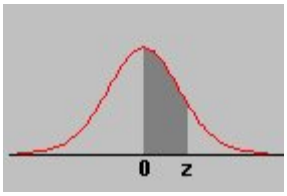
AMORDEGRC	Returns the depreciation of an asset, for each accounting period by using depreciation coefficient (French accounting system)
AMORLINC	Returns the depreciation of an asset, for each accounting period (French accounting system)
AVERAGE	Returns the average of its arguments
AVERAGEA	Returns the average of its arguments, including numbers, text, and logical values
BINOMDIST	Returns the Binomial Distribution Probability
CHITEST	Returns the test for independence. CHITEST returns the value from the chi-squared (χ^2) distribution for the statistic and the appropriate degrees of freedom.
COMBIN	Returns Number of Combinations for a Given Number of Items
CORREL	Returns the correlation coefficient between two data sets
COUNT	Counts how many numbers are in the list of arguments
COUNTA	Counts the number of cells that are not empty and the values within the list of arguments.
COUNTBLANK	Counts the number of blank cells within a range
COUNTIF	Counts the number of nonblank cells within a range that meet the given criteria
COVAR	Returns covariance
CRITBINOM	Returns smallest value for which the Cumulative Binomial Distribution is less than or equal to a criterion value.
CUMIPMT	Returns cumulative interest paid between two periods
CUMPRINC	Returns cumulative principal paid on a loan between two periods
DAVERAGE	Averages the values that match specified conditions
DB	Returns depreciation of an Asset for a specified period using fixed-declining balance method.
DDB	Returns the depreciation of an asset for a specified period by using the double-declining balance method or some other method that you specify.
DPRODUCT	Multiplies the values in a list that match the specified condition.
DSUM	Adds the numbers in a list that match specified conditions.
EFFECT	Returns effective annual interest rate
EVEN	Rounds Up to the Nearest Even Integer
EXP	e Raised to the Power of a Given Number

FACT	Returns factorial of a Number
FORECAST	Prediction by Trend.
FREQUENCY	Returns a frequency distribution as a vertical array
FV	Returns future Value of an Investment
FVSCHEDULE	Returns Future value of an initial principal with variable interest rate.
GEOMEAN	Returns Geometric Mean of Positive Numeric Data
HARMEAN	Returns Harmonic Mean of Positive Numbers
INT	Rounds to the Nearest Integer
INTERCEPT	Calculates Point Where Line Will Intersect Y Axis
IPMT	Returns the interest payment for an investment for a given period
IRR	Returns the internal rate of return for a series of cash flows
ISPMT	Calculates the interest paid during a specific period of an investment
LN	Returns Natural Logarithm of a number
LOG	Returns Logarithm of a Number to a Specified Base
LOG10	Returns Base 10 Logarithm of a number
MEDIAN	Gives Median or Number in Middle
MDETERM	Matrix Determinant of an Array
MINVERSE	Gives Inverse Matrix for the Matrix Stored in an Array
MIRR	Returns the internal rate of return where positive and negative cash flows are financed at different rates
MMULT	Matrix Product of Two Arrays
MODE	Returns Most Frequent Value of an Array
NEGBINOMDIST	Returns Negative Binomial Distribution
NOMINAL	Returns annual nominal interest rate
NORMDIST	Returns Normal Cumulative Distribution
NORMSINV	Returns Inverse of Normal Cumulative Distribution
NPER	Returns Number of Periods for an Investment
NPV	Returns the net present value of an investment based on a series of periodic cash flows and a discount rate
ODD	Rounds Number Up to the Nearest Odd Integer
PERCENTILE	Returns K-th Percentile of Values in a Range
PERMUT	Returns Number of Permutations for a Given Number of Objects

PMT	Returns the periodic payment for an annuity
POISSON	Returns Poisson Distribution
POWER	Returns the result of a Number Raised to a Power
PPMT	Returns the payment on the principal for an investment for a given period
PRODUCT	Multiplies All Numbers
PV	Returns Present Value of an Investment
QUARTILE	Returns Specified Quartile of Data Set
RATE	Returns Interest Rate of a Loan or Annuity
ROUND	Rounds Number to a Specific Number of Digits
ROUNDDOWN	Rounds Number Down Towards Zero
ROUNDUP	Rounds Number Up Away From Zero
RSQ	Returns Square of Pearson Product Moment Correlation Coefficient
SLN	Returns the straight-line depreciation of an asset for one period
SLOPE	Returns Slope of a Linear Regression Line
SQRT	Returns Square Root of a Number
STDEV	Estimates standard deviation based on a sample
STDEVA	Estimates standard deviation based on a sample, including numbers, text, and logical values
STDEVP	Calculates standard deviation based on the entire population
SUBTOTAL	Returns a Subtotal in a List
SUM	Add all numbers in a range of cells.
SUMIF	Adds Specified Cells
SUMPRODUCT	Multiplies Array and Gives Sum
SUMSQ	Square Numbers and Then Add
SYD	Returns the sum-of-years' digits depreciation of an asset for a specified period
TREND	Return Values Along Linear Trend
TRUNC	Truncates to an Integer by Removing Fractional Part
VAR	Estimate Variance from Sample
VARP	Calculates Variance from entire Population
VDB	Returns the depreciation of an asset for a specified or partial period by

	using a declining balance method. VDB stands for variable declining balance.
XIRR	Returns internal rate of return for a schedule of cash flows that is not necessarily periodic.
XNPV	Returns net present value for a schedule of cash flows that is not necessarily periodic.

Area under Standard Normal Curve from 0 to Z



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0159	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2083	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2380	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3880
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3990	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4430	0.4441
1.6	0.4452	0.4463	0.4474	0.4485	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4690	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4758	0.4762	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4865	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4980	0.4980	0.4981
2.9	0.4981	0.4982	0.4983	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.49865	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.49903	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993